# External Economies of Scale and International Trade: Further Analysis

Kar-yiu Wong[1]
University of Washington

August 9, 2000

[1]Department of Economics, Box 353330, University of Washington, Seattle, WA 98195-3330, U. S. A.; Tel. 1-206-685-1859; Fax: 1-206-685-7477; e-mail: karyiu@u.washington.edu; http://faculty.washington.edu/karyiu/

**Abstract**

This paper examines the validity of the five fundamental theorems of international trade and some other issues in a general model of externality. The model allows the possibility of own-sector externality and cross-sector externality. This paper derives conditions under which some of these theorems are valid, and explains what the government may choose to correct the distortionary effects of externality. Conditions under which a economy with no optimality policies may gain from trade are also derived.

# 1 Introduction

Ever since the work of Marshall (1879, 1890), external economies of scale has been an important topic in the economics literature. Marshall considered economies of scale external to firms while the firms remained competitive. This assumption provides a flexible model that is compatible with perfect competition while it can be used to handle variable returns to scale. Models of external economies of scale are also common in the theory of international trade.[1] However, most of the results obtained are mixed and are sensitive to the structures of the models assumed. In most cases, people limit their analysis to some special cases, and derive results that may or may not be generalized.[2]

Wong (2000b, 2000c) develop a basic, two-factor, two-sector model of external economies of scale. It has externality in one sector, but no cross-sector externality is assumed. The advantage of the model is that it is different from the neoclassical framework in only one aspect. Thus the results derived from the model can be compared with the well known results in the neoclassical framework, and any difference can be attributed to the existence of external economies of scale. Another advantage of the present model is that it reduces to some more special cases considered in the literature: the case of one factor and the case of two countries with the same capital-labor ratio but different sizes.

It is recognized that the basic model has its own limitations, and some of the results may not be valid in a more general model. It is thus the purpose of this paper to examine the implications of relaxing some of the assumptions in the basic model.

The model considered in the present paper has externality in both sectors and cross-sector externality between the sectors. This means that an increase in the output in one of the sectors will have a feedback effect on its output and also will affect the output of the other sector (in addition to the effects due to reallocation of resources). As a result, effects of externality on resource allocation, income distribution, and foreign trade will no longer be so pure, as in general several forces may occur at the same time. Therefore we cannot expect to have results as clean as those obtained in the basic model.

The purpose of developing the present more general model is to see how much we can say about foreign trade when both own-sector externality and cross-sector externality are present. Perhaps one may say that in such a general model anything can happen. However, as shown in this paper, there are cases in which definite results

[1] See Wong (2000a) for an introductory note on externality and international trade.

[2] A general model has been examined in Tawada (1989).

can be obtained.

As is done in Wong (2000b), we will examine the validity of the five fundamental theorems of international trade, but we are also using the present model to derive more implications of externality. Noting that externality is a form of distortion, meaning that a competitive equilibrium in the absence of any corrective policies is in general suboptimal, we do two more things in the present paper: deriving the optimal tax-cum-subsidy policies to correct the distortion, and analyzing the gains from trade when no corrective policies have been imposed. The results we will derive are more general than those in the literature.

Section 2 of the present paper introduces the model and its basic features. The virtual system technique in analyzing comparative static results is also introduced. Section 3 derives the output effects of an increase in factor endowments. The validity of the Rybczynski Theorem will be discussed. Section 4 turns to the relationship between commodity prices and factor prices, with a careful examination of the validity of the Stolper-Samuelson Theorem. Autarkic equilibrium is examined in Section 5, where the optimal policy will be derived. Section 6 allows international trade between two countries with identical technology and preferences, but the countries may have different factor endowments. The validity of the Law of Comparative Advantage, the Heckscher-Ohlin Theorem and the Factor Price Equalization Theorem will be examined. Section 7 analyzes the gains from trade for one or both economies. Sufficient conditions for a gainful trade will be derived. The last section concludes.

# 2 The Model

The model examined in the present paper is a general version of the basic model of externality introduced in Wong (2000b, 2000c). The present model is used to examine how some of the results derived in Wong (2000b) may or may not survive in a more general model. In particular, we will examine whether the five fundamental trade theorems are still valid. The model will also be used to derive more properties of externality that have not been examined in Wong (2000b). In this section, we briefly describe the model and explain how it is different from the basic model.

## 2.1 Production Technology

There are two countries, but for the time being, we focus on the home economy. There are two goods, labeled 1 and 2, and two factors, capital and labor. The technology of sector $i$, $i = 1, 2$, is described by the following production function:

$$Q_i = h_i(Q_1, Q_2)F_i(K_i, L_i), \tag{1}$$

where $Q_i$ is the output of sector $i$, and $K_i$ and $L_i$ are the capital and labor inputs employed in the sector. Function $h_i(Q_1, Q_2)$ measures the externality effect. Firms in sector $i$ regard the value of the function as constant, but in fact it depends on the aggregate output levels of the sectors. The argument $Q_i$ in function $h_i(.,.)$ measures the own-sector externality effect while $Q_k$, $i \neq k$, represents the cross-sector externality effect. Function $F_i(.,.)$ is increasing, differentiable, linearly homogeneous, and concave. We assume that sector 1 is capital intensive at all possible factor prices. Denote the supply price of sector $i$ by $p_i^s$.

Let us define the following elasticities of function $h_i(.,.)$

$$\varepsilon_{ij} \equiv \frac{\partial h_i}{\partial Q_j}\frac{Q_j}{h_i} = h_{ij}\frac{Q_j}{h_i}, \quad i, j = 1, 2,$$

where $h_{ij} \equiv \partial h_i / \partial Q_j$. The own-sector elasticity of sector $i$, $\varepsilon_{ii}$, is sometimes called the sector's rate of variable returns to scale. The sign of $h_{ij}$, or that of $\varepsilon_{ij}$, represents how a change in the output of good $j$ may affect $h_i(.,.)$. For example, if $\varepsilon_{ii} > 0$, sector $i$ is subject to increasing returns; if $\varepsilon_{ij} > 0$, $i \neq j$, then an increase in the output of good $j$ will have a positive externality on the production of good $i$. The current model reduces to some special cases. For example, if $\varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = 0$ for all output levels, it reduces to the basic model in which only externality in sector 1 exists with no cross-sector externality.

The rest of the model has the usual properties of a neoclassical framework; for example, all sectors are competitive and there are perfect price flexibility and perfect factor mobility across sectors. In the present framework, full employment of factors is achieved

$$K = K_1 + K_2 \tag{2}$$

$$L = L_1 + L_2, \tag{3}$$

where $K$ and $L$ are the available amounts of capital and labor in the economy, and are assumed to be given exogenously.

## 2.2 Marginal Products of Factors

Since the firms in the two sectors take the value of $h_i = h_i(Q_1, Q_2)$ as given, the private marginal product (PMP) of factor $j$ in sector $i$, $i = 1, 2$, $j = K, L$, is equal to

$$PMP_{ij} = h_i F_{ij}(K_i, L_i), \tag{4}$$

where $F_{ij}(K_i, L_i)$ is the partial derivative of $F_i$ with respect to factor $j$. The private marginal products are what firms consider in choosing the optimal employment of the factors in maximizing their profits. However, the PMP is evaluated under the assumption that the aggregate output levels of the two sectors remain unchanged. That is not true for each sector or the economy. Thus we can derive the total effect of an increase in factor $j$ on the output of sector $i$ by totally differentiating the production functions:

$$Q_{ij} = F_i h_{ii} Q_{ij} - F_i h_{ik} Q_{kj} + h_i F_{ij}, \tag{5}$$

for $i$, $k$ = 1, 2, $i \neq k$, $j = K$, $L$. In deriving (5), the full employment conditions have been used to give $\mathrm{d}K_1 + \mathrm{d}K_2 = 0$, and $\mathrm{d}L_1 + \mathrm{d}L_2 = 0$. Alternatively, (5) can be written as

$$\hat{Q}_i = \varepsilon_{ii}\hat{Q}_i + \varepsilon_{ik}\hat{Q}_k + \nu_{ij}\widehat{j}_i, \tag{6}$$

where $\nu_{ij} = jF_{ij}/F_i$ is the elasticity of function $F_i(.,.)$ with respect to factor $j$, $j = K, L$. Note that $\nu_{ij}$ has the same sign as the PMP of factor $j$ in sector $i$, which is positive when competitive firms are maximizing their profits. If we define $\phi_j = j_1/j_2$ and use the full employment condition, (6) can be written in a matrix form:

$$\begin{bmatrix} 1-\varepsilon_{11} & -\varepsilon_{12} \\ -\varepsilon_{21} & 1-\varepsilon_{22} \end{bmatrix} \begin{bmatrix} \hat{Q}_1 \\ \hat{Q}_2 \end{bmatrix} = \begin{bmatrix} \nu_{1j} \\ -\phi_j \nu_{2j} \end{bmatrix} \widehat{j}_1. \tag{7}$$

Let $\Theta = (1-\varepsilon_{11})(1-\varepsilon_{22}) - \varepsilon_{12}\varepsilon_{21}$ be the determinant of the matrix in (7), which can be solved to give

$$\hat{Q}_1 = \frac{\nu_{1j}(1-\varepsilon_{22}) - \nu_{2j}\varepsilon_{12}\phi_j}{\Theta}\widehat{j}_1 \tag{8}$$

$$\hat{Q}_2 = \frac{\nu_{2j}(1-\varepsilon_{11}) - \nu_{1j}\varepsilon_{21}/\phi_j}{\Theta}\widehat{j}_2. \tag{9}$$

The social marginal product (SMP) of factor $j$ in sector $i$ is the total derivative of output $Q_i$ with respect to the factor. In other words, using (8) and (9), the social marginal product of factor $j$ in sector $i, k$ is equal to

$$Q_{ij} = \frac{h_i(1-\varepsilon_{kk})}{\Theta}F_{ij} - \frac{F_i h_k h_{ik}}{\Theta}F_{kj}, \tag{10}$$

for $i \neq k$. In general, the social marginal product of a factor is not the same as its private marginal product. Consider the following condition:

**Condition A.** $1-\varepsilon_{ii}>0\geq\varepsilon_{ik}$, and $(1-\varepsilon_{11})(1-\varepsilon_{22})>\varepsilon_{12}\varepsilon_{21}$, $i\neq k$.

**Lemma 1**. Given condition A, the social marginal product of labor or capital in any sector is positive.

The lemma can be proved by using (8) and (9). Condition A is a sufficient condition for positive social marginal products. It appears to be restrictive, but generally what it requires is that the externality terms are not too significant in magnitudes.

Note that the relationship between the efficient outputs in the present model can be described by the production possibility frontier, which is negatively sloped, but its curvature is in general ambiguous.

## 2.3 The Virtual and Real Systems

Following Wong (2000b), we introduce the concept of virtual and real systems. In the virtual system, we define $\tilde{Q}_i$ as the virtual output, which is equal to $\tilde{Q}_i=F_i(K_i,L_i)$ with the later function described in (1). Since function $F_i(.,.)$ has the properties of a neoclassical production function, the virtual system behaves like a neoclassical framework. The advantage of the present approach is that the properties of the neoclassical framework are well known.

Let us denote the price of sector $i$ in the virtual system by $\tilde{p}_i$. As it is done for the neoclassical framework, the virtual output of sector $i$ can be expressed as a function of virtual prices and factor endowments:[3] $\tilde{Q}_i=\tilde{Q}_i(\tilde{p}_1,\tilde{p}_2,K,L)$. The virtual output of sector $i$ is related to the real output in the following way:

$$Q_i=h_i(Q_1,Q_2)\tilde{Q}_i(\tilde{p}_1,\tilde{p}_2,K,L). \tag{11}$$

Similarly, the virtual prices are related to the real prices in the following way:

$$\tilde{p}_i^s=h_i(Q_1,Q_2)p_i^s. \tag{12}$$

Conditions (11) and (12) give the relationship between the virtual and real systems. The approach we take in this paper is to make use of the properties of the neoclassical framework and then apply conditions (11) and (12) to derive the properties of the real system.

[3] We can define a virtual GDP (gross domestic product) function. Partial differentiation of the function with respect to virtual prices gives the virtual output function. See Wong (1995) for more details.

The efficient virtual outputs can be described by a virtual production possibility frontier, which is negatively sloped and concave to the origin. For the analysis below, the frontier is assumed to be strictly concave to the origin.

The output function in (11) can be used to define the following elasticities:

$$\eta_{ik} = \frac{\partial \tilde{Q}_i}{\partial \tilde{p}_k} \frac{\tilde{p}_k}{\tilde{Q}_i}.$$

# 3 Output Effects

In this section, we examine the validity of the two fundamental trade theorems in the present framework: the Rybczynski Theorem and the Stolper-Samuelson Theorem.

## 3.1 General Conditions

Differentiate conditions (11) and (12) totally and rearrange the terms to give

$$\begin{aligned} \mathrm{d}Q_i &= h_{ii}\tilde{Q}_i \mathrm{d}Q_i + h_{ij}\tilde{Q}_i \mathrm{d}Q_j + h_i[\tilde{Q}_{ii}\mathrm{d}\tilde{p}_i^s + \tilde{Q}_{ij}\mathrm{d}\tilde{p}_j^s + \tilde{Q}_{iK}\mathrm{d}K + \tilde{Q}_{iL}\mathrm{d}L] \quad (13) \\ \mathrm{d}\tilde{p}_i^s &= h_{ii}p_i^s \mathrm{d}Q_i + h_{ij}p_i^s \mathrm{d}Q_j + h_i \mathrm{d}p_i^s, \quad (14) \end{aligned}$$

where $i \neq j$. Combining together equations (13) and (14), we have

$$\varphi_{ii}\mathrm{d}Q_i + \varphi_{ij}\mathrm{d}Q_j = h_i^2\tilde{Q}_{ii}\mathrm{d}p_i^s + h_i\tilde{Q}_{ij}h_j\mathrm{d}p_j^s + h_i\tilde{Q}_{iK}\mathrm{d}K + h_i\tilde{Q}_{iL}\mathrm{d}L, \quad (15)$$

where

$$\begin{aligned} \varphi_{ii} &= 1 - h_{ii}\tilde{Q}_i - h_i\tilde{Q}_{ii}h_{ii}p_i^s - h_i\tilde{Q}_{ij}h_{ji}p_j^s \\ \varphi_{ij} &= -(h_{ij}\tilde{Q}_i + h_i\tilde{Q}_{ii}h_{ij}p_i^s + h_i\tilde{Q}_{ij}h_{jj}p_j^s). \end{aligned}$$

Let us introduce the following elasticities:

$$\begin{aligned} \eta_{ij} &= \frac{\partial \tilde{Q}_i}{\partial \tilde{p}_j^s}\frac{\tilde{p}_j^s}{\tilde{Q}_i} = \frac{h_i h_j \tilde{Q}_{ij} p_j^s}{\tilde{Q}_i} \\ \eta_{im} &= \frac{\partial \tilde{Q}_i}{\partial m}\frac{m}{\tilde{Q}_i} = \frac{h_i m \tilde{Q}_{im}}{\tilde{Q}_i}, \end{aligned}$$

where $i, j \in \{1, 2\}$, and $m \in \{K, L\}$. From the properties of the neoclassical framework, we know that $\tilde{Q}_{ij} > 0$ for $i = j$, or $< 0$ for $i \neq j$. This implies that $\eta_{ij} > 0$

if $i = j$, or $< 0$ if $i \neq j$. Furthermore, due to the factor-intensity ranking assumed, $\eta_{1K}, \eta_{2L} > 0$, and $\eta_{2K}, \eta_{1L} > 0$. Using the homogeneity properties of the virtual GDP function, we can easily derive the following relations:

$$\eta_{ii} + \eta_{ij} = 0 \tag{16}$$

$$s_i\eta_{ii} + s_j\eta_{ji} = 0 \tag{17}$$

$$\eta_{iL} + \eta_{iK} = 1, \tag{18}$$

where $s_i \equiv \tilde{p}_i\tilde{Q}_i/g$ is the share of sector $i$, and $g$ is the value of the GDP of the economy.[4] Let us define $\lambda_{mi}$ as the proportion of factor $m$ employed in sector $i$, and $|\lambda| \equiv \lambda_{K2} - \lambda_{L2} \equiv \lambda_{L1} - \lambda_{K1}$. Since sector 1 is capital intensive, $|\lambda| < 0$. We know further that

$$\eta_{im} = \xi_{im}\frac{\lambda_{nj}}{|\lambda|}, \tag{19}$$

where $i, j = 1, 2$, $m, n = K, L$, $i \neq j$, $m \neq n$, and $\xi_{im}$ is an index which is equal to 1 for ($i = 1$ and $m = L$) or ($i = 2$ and $m = K$), or equal to $-1$ for ($i = 2$ and $m = L$) or ($i = 1$ and $m = K$).[5]

Let us use a "hat" to denote the proportional change of a variable; for example, $\hat{Q}_i \equiv \mathrm{d}Q_i/Q_i$. Using the output elasticities, (15) can be written in an alternative way:

$$\alpha_{ii}\hat{Q}_i + \alpha_{ij}\hat{Q}_j = \eta_{ii}\hat{p}_i^s + \eta_{ij}\hat{p}_j^s + \eta_{iK}\hat{K} + \eta_{iL}\hat{L}, \tag{20}$$

where

$$\alpha_{ii} = 1 - \varepsilon_{ii} - \eta_{ii}\varepsilon_{ii} - \eta_{ij}\varepsilon_{ji} \tag{21}$$

$$\alpha_{ij} = -(\varepsilon_{ij} + \eta_{ii}\varepsilon_{ij} + \eta_{ij}\varepsilon_{jj}), \quad i \neq j. \tag{22}$$

In (21) and (22), the signs of $\alpha_{ii}$ and $\alpha_{ij}$ are in general ambiguous. In the neoclassical framework, $\varepsilon_{ik} = 0$, $i, k = 1, 2$. This implies that $\alpha_{ii} = 1$ and $\alpha_{ij} = 0$, for $i \neq j$. For the two sectors, equation (20) can be expressed in an alternative form:

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \hat{Q}_1 \\ \hat{Q}_2 \end{bmatrix} = \begin{bmatrix} \eta_{11} \\ \eta_{21} \end{bmatrix} \hat{p}_1^s + \begin{bmatrix} \eta_{12} \\ \eta_{22} \end{bmatrix} \hat{p}_2^s + \begin{bmatrix} \eta_{1K} \\ \eta_{2K} \end{bmatrix} \hat{K} + \begin{bmatrix} \eta_{1L} \\ \eta_{2L} \end{bmatrix} \hat{L}. \tag{23}$$

[4] It can be shown that, based on the assumed factor intensity ranking,

$$\frac{\lambda_{K1}}{\lambda_{K2}} > \frac{s_1}{s_2} > \frac{\lambda_{L1}}{\lambda_{L2}}.$$

Since this result is not needed in the present paper, its proof is left to the reader.

[5] For the proof of these results, see Wong (1995, Chapter 2).

Denote the determinant of the matrix in (23) by $\Phi \equiv \alpha_{11}\alpha_{22} - \alpha_{12}\alpha_{21}$. Assuming that $\Phi \neq 0$, condition (23) can be solved for the changes in output levels:

$$\begin{aligned} \hat{Q}_1 &= [(\eta_{11}\alpha_{22} - \eta_{21}\eta_{12})\hat{p}_1^s + (\eta_{12}\alpha_{22} - \eta_{22}\eta_{12})\hat{p}_2^s \\ &\quad + (\eta_{1K}\alpha_{22} - \eta_{2K}\alpha_{12})\hat{K} + (\eta_{1L}\alpha_{22} - \eta_{2L}\alpha_{12})\hat{L}]/\Phi \qquad (24) \\ \hat{Q}_2 &= [(\eta_{21}\alpha_{11} - \eta_{11}\alpha_{21})\hat{p}_1^s + (\eta_{22}\alpha_{11} - \eta_{12}\alpha_{21})\hat{p}_2^s \\ &\quad + (\eta_{2K}\alpha_{11} - \eta_{1K}\alpha_{21})\hat{K} + (\eta_{2L}\alpha_{11} - \eta_{1L}\alpha_{21})\hat{L}]/\Phi. \qquad (25) \end{aligned}$$

Define $z \equiv Q_1/Q_2$ as the output ratio, and $p^s \equiv p_1^s/p_2^s$ as the supply price ratio, where good 2 is chosen as the numeraire. Conditions (24) and (25) can be combined to give

$$\hat{z} = \frac{\mu}{\Phi}\hat{p}^s + \frac{\sigma}{\Phi}\hat{K} + \frac{\zeta}{\Phi}\hat{L}, \qquad (26)$$

where

$$\begin{aligned} \mu &= \eta_{11}(\alpha_{21} + \alpha_{22}) - \eta_{21}(\alpha_{11} + \alpha_{12}) \qquad (27) \\ \sigma &= \eta_{1K}(\alpha_{21} + \alpha_{22}) - \eta_{2K}(\alpha_{11} + \alpha_{12}) \qquad (28) \\ \zeta &= \eta_{1L}(\alpha_{21} + \alpha_{22}) - \eta_{2L}(\alpha_{11} + \alpha_{12}). \qquad (29) \end{aligned}$$

It should be noted that if all $\alpha_{ij} > 0$, $i$, $j = 1, 2$, then $\mu$, $\sigma > 0$ and $\zeta < 0$. However, because the signs of $\alpha_{ii}$ and $\alpha_{ij}$ are ambiguous, $\mu$, $\sigma$, and $\zeta$ may be positive or negative. We note further that

$$\sigma + \zeta = (\alpha_{21} + \alpha_{22}) - (\alpha_{11} + \alpha_{12}). \qquad (30)$$

In (30), the sign of $\sigma + \zeta$ is ambiguous, even if all $\alpha_{ij} > 0$. Equation (26) shows the dependence of the output ratio on prices and factor endowments. Specifically, we say that the price-output response is normal if and only if $\mu/\Phi > 0$, and the capital-output (labor-output) response is normal if and only if $\sigma/\Phi > 0$ ($\zeta/\Phi < 0$).

Note that an alternative form of (26) is

$$\hat{z} = \frac{\mu}{\Phi}\hat{p}^s + \frac{\sigma + \zeta}{\Phi}\hat{K} - \frac{\zeta}{\Phi}\left(\hat{K} - \hat{L}\right). \qquad (31)$$

In (31), $(\sigma + \zeta)/\Phi$ can be termed the scale effect of an increase in factor endowment. To see this point, suppose that the economy expands uniformly so that $\hat{K} = \hat{L} > 0$, and that the supply price is fixed. Then (31) reduces to:

$$\hat{z} = \frac{\sigma + \zeta}{\Phi}\hat{K}. \qquad (32)$$

Condition (32) represents the scale effect of an increase in factor endowments on the output ratio.

Conditions (27) to (30) consist of the following two terms: $(\alpha_{11}+\alpha_{12})$ and $(\alpha_{21}+\alpha_{22})$. Let us for the time being focus on these two terms. Making use of (21) and (22), we have

$$\begin{aligned}\alpha_{21}+\alpha_{22} &= 1-(\varepsilon_{21}+\varepsilon_{22})-\eta_{22}(\varepsilon_{22}+\varepsilon_{21})-\eta_{21}(\varepsilon_{11}+\varepsilon_{12}) \\ &= 1-(\varepsilon_{21}+\varepsilon_{22})+\eta_{22}(\varepsilon_{11}+\varepsilon_{12}-\varepsilon_{21}-\varepsilon_{22}). \qquad (33)\end{aligned}$$

We can obtain a similar expression:

$$\alpha_{11}+\alpha_{12} = 1-(\varepsilon_{11}+\varepsilon_{12})-\eta_{11}(\varepsilon_{11}+\varepsilon_{12}-\varepsilon_{21}-\varepsilon_{22}). \qquad (34)$$

Making use of (16), (17), (33), and (34), equation (27) reduces to

$$\begin{aligned}\mu &= \eta_{11}\left[(\alpha_{21}+\alpha_{22})+\frac{s_1}{s_2}(\alpha_{11}+\alpha_{12})\right] \\ &= \frac{\eta_{11}}{s_2}\left[1-s_1(\varepsilon_{11}+\varepsilon_{12})-s_2(\varepsilon_{21}+\varepsilon_{22})\right]. \qquad (35)\end{aligned}$$

Similarly, equation (28) and (29) reduce to

$$\begin{aligned}\sigma &= -\frac{1}{|\lambda|}[1-\lambda_{L1}(\varepsilon_{11}+\varepsilon_{12})-\lambda_{L2}(\varepsilon_{21}+\varepsilon_{22}) \\ &\quad -\eta_{11}(s_2\lambda_{L1}-s_1\lambda_{L2})(\varepsilon_{11}+\varepsilon_{12}-\varepsilon_{21}-\varepsilon_{22})/s_2] \qquad (36) \\ \zeta &= \frac{1}{|\lambda|}[1-\lambda_{K1}(\varepsilon_{11}+\varepsilon_{12})-\lambda_{K2}(\varepsilon_{21}+\varepsilon_{22}) \\ &\quad -\eta_{11}(s_2\lambda_{K1}-s_1\lambda_{K2})(\varepsilon_{11}+\varepsilon_{12}-\varepsilon_{21}-\varepsilon_{22})/s_2]. \qquad (37)\end{aligned}$$

Summing up conditions (36) and (37), we have

$$\sigma+\zeta = (\varepsilon_{11}+\varepsilon_{12}-\varepsilon_{21}-\varepsilon_{22})(1+\eta_{11}/s_2). \qquad (38)$$

Condition (38) means that the scale effect is zero if $\varepsilon_{11}+\varepsilon_{12}=\varepsilon_{21}+\varepsilon_{22}$. Consider the following condition:

**Condition B**. $\varepsilon_{11}+\varepsilon_{12}>\varepsilon_{21}+\varepsilon_{22}$.

**Lemma 2**. Given condition A, $\mu>0$. Given conditions A and B, $\sigma$, $\sigma+\zeta>0$ and $\zeta<0$.

This lemma can be proved easily by making use of equations (35) to (38). Note that condition A is sufficient for $\mu$ to be positive.

## 3.2 Production Stability

As Wong (2000b) shows, comparative statics effects can be analyzed both in a local and a global ways. To analyze global changes, we need to examine stability of equilibrium and the adjustment process. Following Wong (2000b), we adopt the Ide-Takayama rule (Ide and Takayama, 1991):

$$\dot{z} = \beta(\bar{p} - p^s) = \phi(z), \tag{39}$$

where $\beta > 0$ is a constant, and $\bar{p}$ is the supply price ratio the firms are facing. Making use of (26), both sides of (39) are differentiated to give

$$\mathrm{d}\dot{z} = -\frac{\beta p \Phi}{z\mu}\mathrm{d}z = \phi'(z)\mathrm{d}z. \tag{40}$$

To have a stable equilibrium, the value of $\phi'(z)$ evaluated near an equilibrium should be negative. This requires that $\mu/\Phi$ be positive. In the basic model used in Wong (2000b), $\mu > 0$. Therefore the stability condition reduces to $\Phi > 0$. In the present model, $\mu$ may be negative, meaning a positive $\Phi$ is neither sufficient nor necessary for a stable production equilibrium.

Stability can still be analyzed in the same way. Figure 1 shows a possible relationship, represented by curve KLMN, between the supply price and the output ratio. Because of the possibility of multiple equilibria, the curve can be partly positively sloped and partly negatively sloped. Given certain conditions, the curve must be positively sloped when $z$ is small or very large.[6] An equilibrium on a positively- (negatively-) sloped segment means that the local price-output response is normal (perverse). For example, given the supply price ratio $p^1$, three equilibria exist, with equilibria A and C stable, and B unstable. This means that sign($\mu$) = sign($\Phi$) at points A and C while at point B sign($\mu$) $\neq$ sign($\Phi$). However, from the diagram we cannot know whether $\mu$ and $\Phi$ are positive or negative.

Wong (2000b) shows that in the basic model, worry about instability of an equilibria is misplaced (a) because an unstable equilibrium is nearly never observed, and (b) because even if initially the system is at an unstable equilibrium, after a small disturbance the system will adjust to a stable equilibrium. In the present more general model, can these two results still exist? The answer is in the affirmative. For example, suppose that the initial equilibrium is at B, with the output ratio at $z^B$. Consider a small disturbance that lowers the output ratio to, say, $z^{B\prime}$. Because the

[6] See Wong (2000b).

corresponding supply price ratio is equal to $p'$, as depicted in the diagram, then according to the adjustment rule in (39), $z$ will drop until point A is reached. This is the new equilibrium, which is stable. In fact, (26) and (40) show that there is a correspondence between local stability and price-output response: an equilibrium is locally stable if and only if the price-output response is normal. This correspondence can also be illustrated in Figure 1.[7] In the basic model analyzed in Wong (2000b), stability of an equilibrium requires that $\Phi$ be positive. However, in the present model, a positive $\Phi$ is no longer a necessary or sufficient condition.

## 3.3 An Increase in the Capital Endowment

We now make use of the adjustment rule introduced to analyze factor-output responses. Let us focus on the effects of an increase in the capital endowment, as the effects of an increase in labor endowments can be analyzed in the same way. If we keep $z$ constant, the relationship between $p^s$ and $K$ is given by a shift of the supply price schedule as a result of an increase in the capital endowment. In particular, the relationship is given by the following condition

$$\hat{p}^s = -\frac{\sigma}{\mu}\hat{K}. \tag{41}$$

Four cases, depending on how an increase in $K$ may affect the price level, and how the price level affects the output, are shown in panels (a) to (d) of Figure 2. For example, panels (a) and (b) shows the cases of normal price-output response, while panels (c) and (d) illustrate perverse price-output response. Furthermore, panels (a) and (c) of Figure 2 represent the cases in which the supply price schedule shifts down due to an increase in $K$. This happens when $\sigma/\mu > 0$. Similarly, if $\sigma/\mu < 0$, an increase in $K$ will shift the supply price schedule up, as shown in panels (b) and (d).

Since $\sigma$, $\mu$, and $\Phi$ can be positive or negative, how output may respond to changes in the supply price and the capital endowment is not clear. In the following table, we consider all possibilities.

Table 1 shows the types of price-output response, capital-output response, local and global stability, depending on the signs of the three variables, $\Phi$, $\sigma$, and $\mu$. Column (5) shows the price-output response, which may be *normal* (if $\mu/\Phi > 0$) or perverse (if $\mu/\Phi < 0$). We showed earlier that the production equilibrium is locally stable if the price-output response is normal. Column (6) gives the capital-price response. We

[7] This is what Samuelson (1949) called the Correspondence Principle. For more details, see Wong (2000b). See Samuelson (1971) for the Global Correspondence Principle.

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|---|---|---|---|---|---|---|---|---|
| | $\Phi$ | $\mu$ | $\sigma$ | *p-o* | *k-p* | *k-o* | global | panel |
| 1 | + | + | + | *n* | *n* | *n* | *n* | (a) |
| 2 | + | + | − | *n* | p | p | p | (b) |
| 3 | + | − | + | p | p | *n* | p | (d) |
| 4 | + | − | − | p | *n* | p | *n* | (c) |
| 5 | − | + | + | p | *n* | p | *n* | (c) |
| 6 | − | + | − | p | p | *n* | p | (d) |
| 7 | − | − | + | *n* | p | p | p | (b) |
| 8 | − | − | − | *n* | *n* | *n* | *n* | (a) |

*n = normal response;* p = perverse response

Table 1: Output Responses to Prices and Capital Endowments

say that it is normal if an increase in the capital endowment will cause a drop in the supply price at any given output ratio, i.e., if $\sigma/\mu > 0$; otherwise, it is perverse if $\sigma/\mu < 0$. The local capital-output responses are given in column (7). Because sector 1 is capital intensive, we say that the capital-output is normal if an increase in the capital endowment induces locally an increase in $z$. A normal response is achieved if $\sigma/\Phi > 0$, or the response is perverse if $\sigma/\Phi < 0$. Column (8) shows the results if finite changes are allowed. The last column refers to the panels of Figure 2.

Columns (7) and (8) are what our current focus is. A normal local capital-output response indicates that the Rybcyznski effect holds.[8] As shown in Table 1, the Rybcyznski effect is not guaranteed. In the present basic model, it is argued that if finite changes are allowed, the Rybcyznski effect will hold globally even if it is not locally. We now examine whether such results are still true. Column (8) of Table 1 shows that they are no longer true: In fact, they are true in four of the eight cases only: cases 1, 4, 5, and 8. Alternatively, we say that they are true only in panels (a) and (c) of Figure 2.

Let us examine these cases more carefully. Suppose that the exogenously given price ratio is $p^1$, with the initial equilibrium at point A or B, at which the price line at $\mathrm{p}^1$ cuts the initial (solid) supply price schedule. In panel (a) of Figure 2, which describes cases 1 and 8 in Table 1, the supply schedule is positively sloped, and an increase in the capital stock shifts the schedule down, lowering the supply price at

[8]This means that a small increase in capital will induce a local increase in the output of good 1 relative to good 2.

the same output ratio (point F), while the equilibrium point shifts from A to A′, implying an increase in the output ratio. This means that the local capital-output response is normal. In fact, the capital-output response is also true in a global sense. This is because a drop in the supply price (to a level represented by F) will, by the adjustment rule (39), encourage an increase in the output of good 1, i.e., a rise in $z$. In panel (c), the local capital-output response is perverse (a drop in the output ratio from B to B′), but the global capital-output response is normal. The reason is that the drop in the supply price ratio will induce an increase in the output ratio, shifting the equilibrium point B to the right, not to the left, until a stable equilibrium is reached.

Panels (b) and (d) of Figure 2 represent two opposite cases. In panel (d), an increase in the capital stock shifts the supply price schedule up, and the equilibrium point to the right from B to B′. Thus locally the capital-output response is normal. However, an upward shift of the supply price schedule implies a drop in the output ratio until a stable equilibrium with a lower output ratio is reached. Panel (b), however, shows a case in which the capital-output response is not normal both locally and globally.

Table 1 and Figure 2 reveal the following results:

**Proposition 1** *A normal local capital-output response is neither necessary nor sufficient for a normal global capital-output response. A capital-output response is normal locally if* $\sigma/\Phi > 0$*. A capital-output response is normal globally if the capital-price response is normal locally, irrespective to the sign of* $\Phi$*.*

The second part of the proposition can be illustrated by cases 1, 4, 5, and 8 in Table 1, or panels (a) and (c). In particular, in Figure 2, the capital-price is normal when an increase in capital shifts the supply price schedule downward, no matter whether the latter is positively or negatively sloped. The reason is clear: If an increase in the capital stock causes a drop in the supply price, causing an increase in the output ratio, resulting in the Rybcyznski effect at least in a global sense. Note that a normal global capital-output response is independent of the sign of $\Phi$.

We showed in Lemma 2 the conditions under which $\mu$ and $\sigma$ are positive. Table 1 shows the implications if they are positive. The result is given below:

**Proposition 2** *Given conditions A and B, the capital-output response is always normal in a global sense.*

### 3.4 An Increase in the Size of the Economy

The effects of an increase in the size of the economy on the output and the autarkic price ratio can be derived from (31) and (32). For example, (32) shows the effect of an increase in the size of the economy on the output under a given supply price ratio. The response in a local sense is normal if $(\sigma + \zeta)/\Phi > 0$.

A comparison of this case with the case of an increase in the capital endowment shows that the previous analysis can be applied here. For example, a table similar to Table 1 can be constructed, with column 4 representing $\sigma + \zeta$. Whether a local output response is normal, and whether a global change is normal, can also be derived in the same way. We thus have

**Proposition 3** *A normal local size-output response is neither necessary nor sufficient for a normal global size-output response. A size-output response is normal locally if* $(\sigma + \zeta)/\Phi > 0$. *A size-output response is normal globally if the size-price is normal, irrespective to the sign of* $\Phi$. *Given conditions A and B, the size-output response is always normal in a global sense.*

Note that, as explained earlier, if $\varepsilon_{11} + \varepsilon_{12} = \varepsilon_{21} + \varepsilon_{22}$, there is no scale effect so that a uniform increase in the size of the economy will not affect the autarkic price ratio (with homothetic preferences).

# 4 Factor Price Effects

Keeping good 2 as the numeraire, we now examine how a change in the supply price ratio $p^s$ may affect factor prices. In this section, we assume that factor endowments are fixed. Suppose that there is an increase in the supply price of good 1 while the supply price of good 2 (the numeraire) is fixed, i.e., $\hat{p}_1^s > 0$ while $\hat{p}_2^s = 0$. Rearrange the terms in (14) to give

$$\widehat{\tilde{p}}_1^s = \varepsilon_{11}\hat{Q}_1 + \varepsilon_{12}\hat{Q}_2 + \hat{p}_1^s \quad (42)$$

$$\widehat{\tilde{p}}_2^s = \varepsilon_{21}\hat{Q}_1 + \varepsilon_{22}\hat{Q}_2. \quad (43)$$

Equations (42) and (43) show that even though the relative price of good 2 is fixed, a change in the relative price of good 1 could change the virtual prices of both goods due to a change in the outputs of the sectors. Substitute the changes in outputs given in (24) and (25), setting the changes in $p_2^s$, $K$, and $L$ to zero, into the above two equations to give

$$\widehat{\tilde{p}}_1^s = \delta_1 \hat{p}_1^s \tag{44}$$
$$\widehat{\tilde{p}}_2^s = \delta_2 \hat{p}_1^s, \tag{45}$$

where

$$\delta_1 = [\varepsilon_{11}(\eta_{11}\alpha_{22} - \eta_{21}\alpha_{12}) + \varepsilon_{12}(\eta_{21}\alpha_{11} - \eta_{11}\alpha_{12}) + 1]$$
$$\delta_2 = [\varepsilon_{21}(\eta_{11}\alpha_{22} - \eta_{21}\alpha_{12}) + \varepsilon_{22}(\eta_{21}\alpha_{11} - \eta_{11}\alpha_{12})].$$

Equations (44) and (45) show explicitly how a change in the real prices of the goods could affect their virtual prices. This result has the following implications on the corresponding changes in factor prices.

**Lemma 3**. If own-sector externalities are non-negative while cross-sector externalities are non-positive ($\varepsilon_{ii} \geq 0$ and $\varepsilon_{ij} \leq 0$, $i, j = 1, 2$; $i \neq j$), and if $\alpha_{ij} \geq 0$ for $i, j = 1, 2$, with at least one inequality, then $\delta_1 \geq 1$, $\delta_2 \leq 0$.

Lemma 3 follows immediately from the definitions of $\delta_1$ and $\delta_2$. Note that if $\alpha_{ij} = 0$ for all $i, j = 1, 2$, then $\delta_1 = 1$ and $\delta_2 = 0$. From (44) and (45), Lemma 3 implies that given the conditions stated an increase in the relative price of good 1 will raise the virtual relative price of good 1 but lower (or not raise) the virtual relative price of good 2.

Define the unit cost function of sector $i$ in the virtual system by $c_i(w, r)$, $i = 1, 2$, which is linearly homogeneous, differentiable, and concave. With positive outputs of both goods, the cost-minimization conditions are

$$c_i(w, r) = \tilde{p}_i^s. \tag{46}$$

Differentiate both sides of (46), rearrange terms, and make use of (42) and (43) to yield

$$\begin{bmatrix} \vartheta_{1L} & \vartheta_{1K} \\ \vartheta_{2L} & \vartheta_{2K} \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \end{bmatrix} = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \hat{p}_1^s, \tag{47}$$

where $\vartheta_{ij}$ is the elasticity of unit cost function of sector $i$ with respect to the price of factor $j$. The determinant of the matrix in (47) is equal to $D = \vartheta_{1L}\vartheta_{2K} - \vartheta_{2L}\vartheta_{1K} < 0$, where the sign is due to the assumed factor intensity ranking.[9] The equation is then solved for the changes in factor prices

$$\hat{w} = \frac{\delta_1 \vartheta_{2K} - \delta_2 \vartheta_{1K}}{D} \hat{p}_1^s \tag{48}$$

$$\hat{r} = \frac{\delta_2 \vartheta_{1L} - \delta_1 \vartheta_{2L}}{D} \hat{p}_1^s. \tag{49}$$

Conditions (48) and (49) immediately give the following proposition:

**Proposition 4** *The Stolper-Samuelson Theorem in the presence of external economies of scale is valid if condition A holds and if all $\alpha_{ij} > 0$, $i, j = 1, 2$.*

It should be noted that in the above proposition, when given condition A and the assumption that all $\alpha_{ij} > 0$, $\hat{w}/\hat{p}_1^s < 0$ and $\hat{r}/\hat{p}_1^s > 1$. To see the later inequalities, note that in the virtual system, $\hat{r}/\widehat{\tilde{p}}_1^s > 1$, and by Lemma 1 $\widehat{\tilde{p}}_1^s/\hat{p}_1^s > 1$.

# 5 Autarkic Equilibrium

To derive the autarkic equilibrium, we assume that the preferences of the home economy is described by a social utility function, which is homothetic, increasing, differentiable, and quasi-concave. Homotheticity of the preferences means that the demand price ratio, with good 2 as the numeraire, can be expressed as a function of the consumption ratio, $p^d = \gamma(z)$, where the derivative $\gamma'(z)$ is negative. Let the price elasticity of demand be denoted by $\delta \equiv -\hat{z}/\hat{p}^d$.

## 5.1 Equilibrium Conditions

The autarkic equilibrium is described by an output ratio that equalize the supply price and the demand price, i.e.,

$$p^s = p^d = p^a, \tag{50}$$

where the superscript "$a$" denotes the autarkic value of a variable. Conditions (50) and (26) can be combined together to give

$$\hat{p}^a = -\theta \left[ \sigma \hat{K} + \zeta \hat{L} \right], \tag{51}$$

[9] We have used Shephard's lemma that $\partial c_i / \partial w = L_i / \tilde{Q}_i$ and $\partial c_i / \partial r = K_i / \tilde{Q}_i$.

where $\theta \equiv 1/(\delta\Phi + \mu)$. Condition (51) shows how the autarkic price ratio is dependent on the factor endowments. It is clear from the condition that the factor endowment effects depend on the magnitudes and signs of variables $\sigma$, $\zeta$, and $\theta$. For example, suppose that the economy has an increase in its capital stock. The resulting change in the autarkic price ratio is

$$\hat{p}^a = -\theta\sigma\hat{K}. \tag{52}$$

Thus, a small increase in the capital stock will lower the autarkic price ratio if sign($\theta$) = sign($\sigma$). We say that in this case the response of the autarkic price ratio is normal. On the other hand, if $\sigma > 0$ under the conditions mentioned in Lemma 2, then, as argued in Wong (2000b), an increase in the capital stock will lower in a global sense the autarkic price ratio, whether or not $\theta$ is positive. Similar conclusion can be reached for an increase in the labor endowment, or a uniform increase in the size of the economy.

Thus we have

**Proposition 5** *A locally normal response of the autarkic price ratio to an increase in the endowment of either capital or labor, or to a uniform increase in the size of the economy, is neither necessary nor sufficient for a globally normal response of the autarkic price ratio to an increase in the capital stock. Given conditions A and B, the response of the autarkic price ratio to an increase in the factor endowment is globally normal.*

## 5.2 Optimal Policy

Externality in the present model represents a distortion. This means that a market equilibrium in general is not optimal in terms of the welfare of the economy. From the description of the model given in Section 2, it is clear that externality is due to the assumption that the firms ignore the indirect effect of an increase in the employment of a factor on function $h_i(Q_1, Q_2)$. As a result, there is a divergence between the private marginal product and social marginal product of a factor.[10]

The optimal policy considered here is a set of taxes/subsidies imposed on the employment of factors. We argue that the optimal policy on sector $i$ consists of an

[10] For more discussion about private marginal product and social marginal product, see Wong (2000a).

ad valorem employment subsidy $s_i$ plus a lumpsum subsidy of $\phi_i$ on factor $j$, where

$$s_i = \frac{\varepsilon_{ii}(1-\varepsilon_{kk})+\varepsilon_{ik}\varepsilon_{ki}}{\Theta} \tag{53}$$

$$\phi_i = -p_i\left(\frac{h_k F_i h_{ik} F_{kj}}{\Theta}\right), \tag{54}$$

where $i$, $k = 1$, 2, $i \neq k$, $j = K$, $L$, and the variables are evaluated at the optimal point. Note that a negative subsidy is a tax.

To see how this policy works, let us consider the income of labor. The effective wage rate consists of three parts: the payment by a firm, the employment subsidy from the government, and the lumpsum subsidy. The firms take the subsidies as given, and keep on ignoring the externality effects, i.e., they pay the factors they employ according to their private marginal products. The total income of a worker, which is regarded as the effective wage rate, is equal to

$$w = p_i(1+s_i)h_i F_{iL} + \phi_i. \tag{55}$$

To find out what this wage rate is, we note that

$$1+s_i = \frac{1-\varepsilon_{kk}}{\Theta}. \tag{56}$$

Substituting (56) into (55) and using (53) and (54), we have

$$w = p_i Q_{iL} \tag{57}$$

for $i = 1$, 2. Condition (57) means that the wage rate is equal to value of social marginal product of labor. A similar condition can be derived for the rental rate. Thus in the presence of the present policy, both factors are employed optimally.

# 6 International Trade

We now consider open economies and analyze foreign trade. In order to have meaningful comparison of the two countries, we assume that the autarkic equilibrium of each country is unique. As it is done in Wong (2000b), denote the export supply of good 1 of home by $E_1(p; K, L)$, where $p$ is the domestic price ratio of good 1. Note that both $K$ and $L$ are treated as exogenous. Invert the export supply function $E_1 = E_1(p; K, L)$ to give $p = \rho(E_1)$, where for simplicity the factor endowments are ignored in the function. The export functions can be used to derive the offer curve of an economy.

We now introduce the foreign country and examine the implications of international trade. Following Wong (2000b), we assume that both countries have identical technologies and preferences, unless otherwise stated. Their factor endowments, however, may be different. International externality is not assumed. Variables of the foreign countries are distinguished by an asterisk while the home variables do not have an asterisk.

Free trade is allowed between the countries, assuming that neither government imposes the optimality policy described in the previous section. We want to find out whether the fundamental trade theorems are still valid in the present framework.

The major difference between the present model and the basic model is that the comparative-static variables $\mu$, $\sigma$, and $\zeta$ may not be positive in the present model. Thus for each of the fundamental trade theorems, we want to raise two major questions. What happens if these variables are positive? What happens if some or all of them are not positive?

We first consider the Law of Comparative Advantage.

## 6.1 Comparative Advantage

Following Wong (2000b), we assume the following adjustment rule for a trading point:

$$\dot{E}_1 = \gamma(p^* - p) = \gamma(\rho^*(E_1) - \rho(E_1)) = \psi(E_1), \tag{58}$$

where $E_1$ is home's export of good 1 at the given trade point and $\gamma$ is a positive constant. Condition (58) means that home will have an incentive to export more of good 1 to foreign if the relative price of good 1 is higher in foreign than in home. For a stable trade equilibrium, we require that $\mathrm{d}\psi/\mathrm{d}E_1$ be negative. If this condition is satisfied, the equilibrium is said to be locally stable in a Marshallian sense.

Suppose that home has a comparative advantage in good 1 in the sense that $p^a < p^{*a}$. The two countries are initially under autarky, and free trade between the countries is now allowed. Would they trade, and what would each export?

Using condition (58), home will have an incentive to export good 1. In fact, home will export good 1 until the first trade equilibrium is reached. Thus we have

**Proposition 6** ***(Law of Comparative Advantage in the Presence of External Economies of Scale)****. Assuming the Marshallian adjustment rule, under free trade, each country exports the good in which it has a comparative advantage.*

This proposition is the same as the one in Wong (2000b), which means that the law stays as the model becomes more general with externality in both sectors and with cross-sector externality. The intuition is clear: this law does not depend on the signs of the comparative-static variables.

Of course, it should be noted that the law stated above is weaker than the Law of Comparative Advantage in the neoclassical framework.

## 6.2 The Heckscher-Ohlin Theorem

Wong (2000b) shows that in the basic model of externality, a certain form of the Heckscher-Ohlin Theorem exists. The result depends on the fact that in the model $\mu$, $\sigma$, and $\sigma + \zeta > 0$ while $\zeta < 0$. In particular, it was shown that home has a comparative advantage in good 1 if it has more capital or is bigger uniformly, and has a comparative in good 2 if it is abundant absolutely in labor. This result does not depend on the sign of $\theta$ at the autarkic point because comparing the factor endowments of the countries, which are much different from each other, is the same as considering global changes in comparative static analysis.

In the present model, all these comparative static variables may not have the "right" sign; for example, we may have $\mu < 0$ and $\sigma > 0$. What can we say about the comparative advantages of the countries and their patterns of trade?

This question can be answered by using Table 1, which shows the effects of an increase in the capital stock in different cases, depending on the signs of the comparative static variables. As explained before, it is more meaningful to consider global changes as given in column (8) of the table. By making use of the table, we can show the effects of an increase in the capital stock on the autarkic price ratio. In Figure 3, we repeat the four panels in Figure 2, except that we add the demand price schedule, $p^d$, which is negatively sloped. The solid supply price schedule is the initial schedule. Assuming a Marshallian autarkic equilibrium, slope of schedule $p^d$ is algebraically less than that of the supply schedule. In each of the panels, the two schedules cut once at point A. As explained, we assume a unique autarkic equilibrium. An increase in the capital stock will shift the supply price schedule to the broken schedule $p^{s\prime}$. Using the adjustment rule introduced earlier and in Wong (2000b), the autarkic equilibrium shifts to point A′. Depending on the change in the autarkic price ratio, we can distinguish two cases:

**Case (A)**: a decrease in the autarkic price ratio. This case is the one in either panel (a) or panel (c). This corresponds to rows 1, 4, 5, and 8 of Table 1, i.e., when $\mu$ and $\sigma$ have the same sign.

**Case (B)**: an increase in the autarkic price ratio. This case is shown in either panel (b) or panel (d), and corresponds to rows 2, 3, 6, and 7 — $\mu$ and $\sigma$ having different signs.

The table and the analysis can be applied to the other two variables, $\sigma + \zeta$ and $\zeta$. In the basic model, these comparative static variables are of the "right" sign, i.e., $\mu$, $\sigma$, $\sigma + \zeta > 0$ while $\zeta < 0$. One important result is that a modified version of the Heckscher-Ohlin Theorem is valid. In the present model, these variables are of the "right" sign under certain conditions. Thus we have,

**Proposition 7** ***(Heckscher-Ohlin Theorem in the Presence of External Economies of Scale).*** *When given conditions A and B, a country exports the capital-intensive good if it is relatively and absolutely abundant in capital, or if it is uniformly bigger.*

## 6.3 Factor Price Equalization

We now turn to the factor prices in the two countries under free trade. We keep the usual assumptions for the Theorem of Factor Price Equalization, including (a) equalized commodity prices (due to free trade and no transport costs) and (b) diversification in production.

Note that the virtual system behaves like the neoclassical framework, meaning that a necessary condition for factor price equalization is that the virtual commodity prices be the same in both countries. Recall that the virtual commodity prices are related to the (real) commodity prices as follows:

$$\tilde{p}_i = h_i(Q_1, Q_2)p_i. \tag{59}$$

Assuming that $h_{ik} \neq 0$ in the general case, the two equations represented by (59) imply that in order to have the same virtual commodity prices in the countries, the countries must have the same output levels of the two goods. However, since the countries have identical technologies, and since they are facing the same commodity prices, they need to have the same factor endowments. As a result, the only possibility is that the two countries are identical not only in technologies and preferences, but also in factor endowments. In this case, when the countries allow free trade, no trade is an equilibrium, but the no-trade equilibrium can be either stable or unstable. If the equilibrium is unstable, a shock will move the countries from the equilibrium to a free-trade equilibrium, with one of the countries exporting good 1. However, the country that exports (imports) good 1 must produce more (less) good 1 than that in the other countries. Thus we have

**Proposition 8** *When the countries trade, generally factor prices are not equalized under free trade.*

It should be noted that the Factor Price Equalization Theorem in the neoclassical framework does not require identical preferences across countries. This is also true for the basic model of externality introduced in Wong (2000b). In the above analysis, we begin with the assumption that the countries have identical technologies and also preferences, and show that the theorem in general is not valid. This result, which is negative in a sense, holds whether the countries have identical preferences.

# 7 Gains from Trade

Externality is a form of distortion, which means that a market equilibrium in the absence of any government intervention is suboptimal. It is therefore not surprising to find that an economy with externality may have a lower welfare under free trade than under autarky.

There has been a lot of work that derives sufficient conditions for a gainful trade in the presence of external economies of scale.[11] It has been pointed out that external economies of scale creates distortion on the production side of the economy while the consumption side is free from distortion. This means that the consumption gain from trade of the economy is non-negative while the production gain from trade may be negative.[12] Therefore a sufficient condition for a gainful trade is that the production gain is positive (or non-negative if the consumption gain is positive). The literature is characterized by work on determining conditions under which the production gain is non-negative.

Keeping on using subscript "$a$" to denote autarkic values of variables, and using no subscripts for the variables' values under free trade, the production gain is defined as:

$$PG = \sum_{i=1}^{2} p_i Q_i - \sum_{i=1}^{2} p_i Q_i^a. \tag{60}$$

[11]See Wong (1995, Chapter 9) for a recent survey and discussion.

[12]The consumption gain is positive if there is a change in the commodity price ratios and if there are consumption substitution possibilities.

Substitute the production function in (1) into (60), we get

$$\begin{aligned} PG &= \sum_{i=1}^{2} p_i h_i(Q_1, Q_2)\tilde{Q}_i - \sum_{i=1}^{2} p_i h_i(Q_1^a, Q_2^a)\tilde{Q}_i^a \\ &= \sum_{i=1}^{2} \tilde{p}_i \tilde{Q}_i - \sum_{i=1}^{2} p_i h_i^a \tilde{Q}_i^a. \end{aligned} \tag{61}$$

where $h_i^a = h_i(Q_1^a, Q_2^a)$. Because the virtual system behaves like a neoclassical framework, its production gain measured in terms of virtual prices and outputs is non-negative, i.e.,

$$\sum_{i=1}^{2} \tilde{p}_i \tilde{Q}_i \geq \sum_{i=1}^{2} \tilde{p}_i \tilde{Q}_i^a,$$

or

$$\sum_{i=1}^{2} p_i Q_i \geq \sum_{i=1}^{2} p_i h_i(Q_1, Q_2)\tilde{Q}_i^a. \tag{62}$$

Comparing (61) and (62), we can see that a sufficient condition for a non-negative production gain is the following condition

$$\sum_{i=1}^{2} p_i h_i(Q_1, Q_2)\tilde{Q}_i^a \geq \sum_{i=1}^{2} p_i h_i(Q_1^a, Q_2^a)\tilde{Q}_i^a. \tag{63}$$

Various conditions analogous to (63) have been derived and applied in the literature. In the present model, when will condition (63) hold? One sufficient condition is that

$$h_i(Q_1, Q_2) \geq h_i(Q_1^a, Q_2^a), \tag{64}$$

for $i = 1, 2$. Another sufficient condition is to make use of function $h_i(.,.)$ and examine how outputs of the sectors should change in order to guarantee condition (64). In the present two-sector model, one of the sectors must expand while the other sector shrinks as the economy shifts from the autarkic equilibrium to a free-trade equilibrium.[13] Consider the following condition:

[13] Free trade leads to a change in the production point if there is a change in the relative price and production substitution is possible.

**Condition G**. For $i, k = 1, 2$, $h_{kk}$ and $h_{ik}$ are either (a) zero or (b) the same sign as $(Q_k - Q_k^a)$.

To interpret what condition G means, imagine that sector 1 expands, i.e., $Q_1 - Q_1^a > 0$. Then we require that the sector is subject to increasing returns and it has a positive external effect on sector 2.

**Lemma 4**. Given condition G, condition (64) is satisfied.

Lemma 4 is intuitive. In order to satisfy (64), if a sector expands, then it must have a positive externality on itself and also on the other sector, or if a sector shrinks, it must have a negative externality on itself and on the other sector. Using Lemma 4 and the above analysis, we immediately have the following proposition:

**Proposition 9** *A sufficient condition for a gainful free trade is condition G.*

Condition G is a strong one because it has restrictions on not just own-sector externality on both sectors but also on cross-sector externality. Note that in the special case in which cross-sector externality is absent, then condition G reduces to the famous Kemp-Negishi condition (Kemp and Negishi, 1970): an economy gains from trade if the sector that is subject to increasing returns expands while the sector that is subject to decreasing returns shrinks. In the presence of cross-sector externality, the Kemp-Negishi condition is not sufficient for a gainful trade.[14]

# 8 Concluding Remarks

This paper examines the implications of externality in a general model, in which both own-sector externality and cross-externality may be present. This model reduces to the basic model in Wong (2000b, 2000c) as a special case.

It is of course not surprising that some of the results derived in Wong (2000b) may not hold here. For example, four of the five trade theorems may not hold. These theorems are the Rybczynski Theorem, the Stolper-Samuelson Theorem, the Heckscher-Ohlin Theorem, and the Factor Price Equalization Theorem. The modified Law of Comparative Advantage, however, remains valid.[15] The reason is that this law

[14]See Wong (1995, Chapter 9) for extension of the Kemp-Negishi results and a discussion of relevant work by other people.

[15]Strictly speaking, the law proved in Wong (2000b) and the present paper is only half of what is in the Law of Comparative Advantage derived in the neoclassical framework. The other half, that the free-trade world price ratio is bounded by the autarkic price ratios in the countries, is not necessarily valid.

describes the relationship between autarkic price ratios and patterns of trade, and is not affected by externality, or even the technologies in general, in the countries.[16]

As we showed, in general the results are ambiguous. We did find some sufficient conditions under which stronger results are valid.

[16] As a matter of fact, the Law of Comparative Advantage does not require identical technology and/or preferences in the countries.

## References


[1] Ide, Toyonari and Akira Takayama (1991), "Variable Returns to Scale, Paradoxes, and Global Correspondence in the Theory of International Trade," in Akira Takayama, Michihiro Ohyama, and Hiroshi Ohta (eds.), *Trade, Policy, and International Adjustments*, San Diego: Academic Press, Inc.: 108–154.

[2] Kemp, Murray C. and Takashi Negishi (1970), "Variable Returns to Scale, Commodity Taxes, Factor Market Distortions and Their Implications for Trade Gains," *Swedish Journal of Economics*, 72 (1): 1–11.

[3] Marshall, Alfred (1879), *Pure Theory of Foreign Trade*, reprinted by London School of Economics and Political Science, 1930.

[4] Marshall, Alfred (1890), *Principles of Economics*, 8th edition, London: Macmillan, 1920.

[5] Samuelson, Paul A. (1947), *Foundation of Economic Analysis*, Cambridge, Mass.: Harvard University Press.

[6] Samuelson, Paul A. (1971), "On the Trail of Conventional Beliefs about the Transfer Problem," in Jagdish N. Bhagwati *et al.* (eds.), *Trade, Balance of Payments and Growth: Papers in International Economics in Honor of Charles P. Kindleberger*, Amsterdam: North-Holland, 327–351.

[7] Tawada, Makoto (1989), *Production Structure and International Trade*, Berlin: Springer-Verlag.

[8] Wong, Kar-yiu (1995), *International Trade in Goods and Factor Mobility*, Cambridge, Mass.: MIT Press.

[9] Wong, Kar-yiu (2000a), "Externality in the theory of International Trade: Some Basic Concepts," *mimeo*, University of Washington.

[10] Wong, Kar-yiu (2000b), "Fundamental Trade Theorems under External Economies of Scale," *mimeo*, University of Washington.

[11] Wong, Kar-yiu (2000c), "International Trade and Factor Mobility under External Economies of Scale," *mimeo.*, University of Washington.