A Differential Game Theoretic Analysis of International Trade in Renewable Resources[1]

by

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Abstract

We use a Stackelberg differential game to model international trade in renewable resources between a monopsonistic buyer and a monopolistic seller. The buyer uses unit and *ad valorem* tariffs to indirectly encourage conservation of the renewable resource under study. First, we show that the efficacy of these trade policy instruments in furthering conservation depends essentially on whether harvesting costs are stock dependent or independent. When harvesting costs are stock independent, the optimal open loop tariffs are time consistent. In contrast, when harvesting costs are stock dependent, the optimal open loop tariffs are time inconsistent. Second, we point out that because the simultaneous use of both tariffs does not render one tariff extraneous, it makes sense for the buyer to use both tariffs concurrently. Third, we show that when the buyer uses both tariffs simultaneously, she can force the monopolistic seller to behave competitively. Finally, we discuss the implications of these and other findings for renewable resource conservation in general

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1. Introduction

Can trade policies be used to further the conservation of renewable resources? As noted by Barbier *et al.* (1994) and Burgess (1994), this question has assumed great significance in contemporary times. To see why trade policies might be relevant in the context of the conservation of renewable resources, note that wild resources such as fish, timber (from forests), ivory (from elephants), horns (from rhinoceroses), and non-timber products such as rattan, honey, and resins are all commonly traded between nations. However, today, there is considerable apprehension about the deteriorating stock levels of most important renewable resources.[4] Given this state of affairs, the purpose of this paper is to analyze the effects of trade policy on the conservation of renewable natural resources.

However, before we move to the details of the paper itself, let us first describe the contributions of the pertinent literature on international trade in renewable resources. Barbier and Schulz (1997) demonstrate that trade interventions may increase or decrease the equilibrium value of the species stock in a developing country that trades in this species with other nations. On the basis of this finding, they conclude that "ambiguous stock effects make trade interventions a poor policy instrument for securing biodiversity conservation" (Barbier and Schulz, 1997, pp. 160-161). Schulz (1997) shows that the impacts of trade sanctions depend not only on the bioeconomic interactions between the species but also on the management system in the targeted nation. Therefore, the threat

[4] For more on this see, Clark (1973), Jablonski (1991), and Pimm *et al.* (1995).

of trade sanctions will not necessarily result in lower harvesting and higher stocks of marine mammals.

Brander and Taylor (1998) use a two-country, two-good model of trade in renewable resources to show that not only is the basic “gains from trade” idea weakened by the presence of open access renewable resources but that tariffs imposed by a resource importing nation always benefit the resource exporter. Analyzing a two country model of the outcomes of unilateral fishery management, Emami and Johnston (2000) argue that the trade induced losses that arise from not managing this fishery can be mitigated by imposing import tariffs on the resource good. Maestad (2001) has examined the effects that timber trade limitations have on tropical deforestation. He shows that depending on the manner in which trade limitations affect the log prices of alternate tree qualities, trade limitations may decrease or increase timber logging.

The question of the efficacy of trade policies in furthering the conservation of renewable resources has, in all likelihood, been discussed most extensively in the context of the ivory trade between developing countries in Africa and East Asian and other western nations. In an early contribution, Barbier *et al.* (1990) argued against a ban on trade in ivory and suggested an alternate strategy. This strategy would allow limited trade in ivory and the aim of this strategy would be to create sufficient incentives for the sustainable management of African elephant populations. In the aftermath of the Convention on International Trade in Endangered Species (CITES) ban on ivory trade, Bulte and van Kooten (1999) studied the desirability of allowing some ivory trade. Bulte and van Kooten (1999) caution against lifting the trade ban. In particular, they point out that allowing some trade in ivory would encourage illegal poaching and that this could drive the African elephant to extinction. In a more recent paper, Heltberg (2001) uses a numerical model to reason that the ivory trade ban is likely to reduce poaching.

Even though these studies have certainly furthered our understanding of many facets of international trade in renewable resources, none of these studies have analyzed the connections between renewable resource harvesting costs and trade policy. Specifically, how does the stock dependence or the independence of harvesting costs affect the efficacy of trade policy? Although researchers thus far have *not* analyzed this question, as we shall see, the form of the harvesting cost function has significant implications for the efficacy of trade policy in furthering the conservation of renewable resources.

The rest of this paper is organized as follows: Section 2 provides a detailed description of the Stackelberg differential game model of international trade in a renewable resource between a single buyer (the monopsonist) and a single seller (the monopolist). Section 3 studies the effects of trade policy when the renewable resource harvesting cost function is stock independent. Section 4 does the same for the case in which the harvesting cost function is stock dependent. Finally, section 5 concludes and offers suggestions for future research on the nexuses between international trade and the conservation of renewable resources.

2. The Stackelberg Differential Game

Our model is adapted from Karp (1984) and Batabyal (1996). There is a single buyer (the monopsonist and the leader) of the renewable resource and this buyer purchases the resource from a single seller (the monopolist and the follower). If the renewable resource is the black rhinoceros horn, then the reader should think of our model as a description of the interaction between a monopolistic seller in an African country such as Zimbabwe and a monopsonistic buyer in a country such as Korea.[5] Denote the stock of the renewable resource at time t by $x(t)$.[6] The buyer's utility

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[5] As noted by Milliken *et al.* (1993), traditional Korean medicine has sixteen rhino horn prescriptions. This, in large part, accounts for the great demand for the rhino horn in Korea. For interesting accounts of international trade in the horn of the black rhinoceros, see Milliken (1993) and Brown and Layton (2001).

[6] In what follows, we shall frequently suppress the time dependence of the various variables. Nevertheless, the reader should note that all the variables that we work with depend on time.

from consuming the resource at harvest level $h(t)$ is given by the concave and differentiable utility function $u(h)$. We assume that the domestic market in the buyer's country is competitive so that $u'(h) = p(h)$, the price that consumers in the importing nation pay for this resource. The government of the importing nation has access to two trade policy instruments, namely, a unit tariff denoted by $n(t)$ and an *ad valorem* tariff denoted by $a(t)$ where $v = 1/(1+a)$. When the government in the importing nation uses the unit tariff $n(t)$, the price received by the monopolistic seller (exporter) is $p(h) - n(t)$. Similarly, when this government uses the *ad valorem* tariff $a(t)$, the price received by the exporter is $v(t)p(h)$.

The buyer's payoff in the finite horizon games that we analyze in this paper is the discounted stream of the difference between the utility of consuming the resource at level $h(t)$ and the payment to the monopolistic seller, from time $t = 0$ to $t = T$. Therefore, if we denote the interest rate by r, then the buyer's payoff is

$$J_b = \int_0^T e^{-rt}[u(h) - \{p(h) - n\}h]dt, \tag{1}$$

when she uses a unit tariff. Similarly, when she uses the *ad valorem* tariff, her payoff is

$$J_b = \int_0^T e^{-rt}[u(h) - vp(h)h]dt. \tag{2}$$

The monopolistic seller maximizes profit and we assume that this seller gets no utility from consuming the resource under study. A major objective of this paper is to demonstrate the dependence of optimal trade policy on the form of the cost of harvesting the renewable resource x.

To this end, we analyze two kinds of harvest cost functions. The first kind of cost function, which we analyze in section 3, is stock *independent* and it depends only on the harvest level. Let us denote this thrice differentiable cost function by $c(h),$ where h is harvest, $c'(h) \geq 0$ and $c''(h) \geq 0.$ The second kind of cost function, which we analyze in section 4, *depends* on the stock *and* on the harvest level. We denote this cost function by $c(x)h,$ where $c'(x) \leq 0$ and $c''(x) \geq 0.$ All else being equal, the stock independent cost function is more relevant when the amount harvested is small in relation to the total size of the resource stock. In contrast, the stock dependent cost function is more relevant when the total size of the stock is small to begin with and/or when the amount harvested is a significant proportion of the total stock size. When the buyer uses a unit tariff, the seller's payoff is

$$J_s = \int_0^T e^{-rt}[p(h)h - nh - C(\bullet)]dt \tag{3}$$

and when this buyer uses the *ad valorem* tariff, the seller's payoff is

$$J_s = \int_0^T e^{-rt}[vp(h)h - C(\bullet)]dt, \tag{4}$$

where $C(\bullet)$ in equations (3) and (4) is $c(h)$ and $c(x)h,$ respectively.

The buyer controls the tariffs $n(t)$ and $v(t),$ and the seller controls the harvest $h(t).$ As the leader, the buyer announces a tariff trajectory at the beginning of the game and this trajectory is exogenous to the seller. The buyer and the seller are constrained by the differential equation describing the dynamics of the resource stock. That equation is

$$dx(t)/dt = \dot{x} = kx - h(t), \tag{5}$$

where $0 < k < r < \infty$ and $x(0) = x_0 > 0$ is given. Put differently, the net change in the resource stock over time is the difference between the natural growth kx and the harvest $h(t)$. The reader should note that replacing the linear natural growth function with a general growth function $f(x)$ does not alter the main points that we make in this paper. However, the algebra associated with the various derivations with the general growth function $f(x)$ is significantly more complicated. This is why we conduct the rest of the analysis in this paper with the linear natural growth function.

We ascertain the different harvest trajectories by deriving a differential equation satisfied by the optimal $h(t)$. When we are able to compare the various differential equations without resorting to additional assumptions, we shall do so. However, the reader should note that because of the complexity of the underlying mathematics, in many cases it will not be possible to obtain general results.

Before ending this section, let us consider the benchmark case in which there is free trade. In other words, in this case the buyer is passive and she sets $n(t) = 0$ or $v(t) = 1$. In this benchmark case, the seller solves a standard control problem. When the harvest cost function is stock independent, the optimal harvest rate solves

$$\{2p'(h) + h''p(h) - c''(h)\}\dot{h} - (r - k)\{p(h) + h'p(h) - c'(h)\} = 0, \tag{6}$$

and when the harvest cost function is stock dependent, the optimal harvest rate solves

$$\{2p'(h) + h''p(h)\}\dot{h} - (r - k)\{p(h) + h'p(h) - c(x)\} - k'xc(x) = 0. \tag{7}$$

In what follows, we compare the differential equation satisfied by the optimal harvest level in section 3 with equation (6) and that in section 4 with equation (7).

3. The Stock Independent Harvest Cost Function and Open Loop Tariffs

We now derive the optimal open loop unit and *ad valorem* tariffs for our monopsonistic buyer. The reader should note that although open loop tariffs are generally time inconsistent (see

Karp and Newbery (1993) and Batabyal (1996)), for the case studied in this section, these open loop tariffs *are* time consistent. Before we explain why this is the case, let us first comprehend what would happen were these tariffs to be time inconsistent. If these tariffs were time inconsistent, then at some time $t > 0$ the buyer would want, if she could, to deviate from the tariff trajectory she announced at the beginning of the game (at time $t = 0$) and announce an alternate tariff trajectory. The monopolistic seller in this paper is forward looking. Therefore, he would anticipate the buyer's desire to change the tariff trajectory she announced at the beginning of the game and hence this tariff would fail to accomplish its intended goals. In the class of Stackelberg games analyzed in this paper, the stock independence of the harvest cost function accounts for the time consistency of the optimal solution. To see this plainly, we now derive, in turn, the open loop unit and the *ad valorem* tariffs.

3.1. The Open Loop Unit Tariff

We solve the buyer's problem using a procedure delineated in Simaan and Cruz (1973), Karp (1984), and Batabyal (1996). The essential idea is as follows: The buyer treat's the seller's first order condition as an ordinary constraint and his costate variable as a state variable. These two conditions and apposite boundary conditions convert the underlying differential game into a control problem for the buyer.

When the seller takes the buyer's unit tariff $n(t)$ as given, the first order necessary conditions to his problem are

$$p(h) + h'p(h) - c'(h) - \lambda - n = 0 \tag{8}$$

and

$$\dot{\lambda} = (r - k)\lambda, \tag{9}$$

where $\lambda(t)$ is the costate variable. This costate variable tells us the seller's marginal utility of one more unit of the resource stock at time $t.$ The reader should note that equation (9) describes a jump state constraint. That is, the initial value of $\lambda, \lambda(0),$ is free and the value of this jump state variable at any arbitrary point in time is determined by present and/or future events. In other words, equation (9) is not a fixed initial state constraint for the buyer.[7] Now solving for n from equation (8) and substituting in equation (1), we get

$$J_b = \int_0^T e^{-rt}[u(h) - \{c'(h) + \lambda - h'p(h)\}h]dt. \tag{10}$$

Equation (10) gives the buyer's payoff as the present discounted stream of the difference between the utility of consuming $h(t)$ and the sum of the marginal harvest cost times the harvest $h(t)$ and the term λh, less the slope of the inverse demand function times the harvest $h(t)$. The term λh can be interpreted as the total instantaneous rent paid by the buyer for the resource x.

In order to keep the buyer's problem a one state variable problem, let us eliminate λ from (10) by using (9). Solving equation (9), it is clear that $\lambda(t) = 0$. Substituting this value of λ into (10) we get

$$J_b = \int_0^T e^{-rt}[u(h) - \{c'(h) - h'p(h)\}h]dt. \tag{11}$$

Note that we have now converted the buyer's problem from one of maximizing (1) over n subject to (5) to one of maximizing (11) over h subject to (5). The first order necessary conditions to this problem are

$$p(h) + h^2 p''(h) + 2h'p(h) - h''c(h) - c'(h) - \eta = 0, \tag{12}$$

[7] For more on jump state constraints see Karp and Newbery (1993).

and

$$\dot{\eta} = (r - k)\eta, \tag{13}$$

where $\eta(t)$ is the costate variable. Inspecting (12), we see that the solution to the buyer's problem does *not* depend on the initial stock of the resource x_0. This explains why the optimal solution to the buyer's problem *is* time consistent. Put differently, because it is optimal to set $\lambda(t) = 0,$ the total instantaneous rent paid by the buyer for the resource x does not influence her maximization problem. Consequently, the question of altering the total instantaneous rent paid to the seller over time does not arise.

To find the differential equation satisfied by the optimal $h(t)$ when our buyer uses a unit tariff, differentiate (12) with respect to time and then use (13) to simplify the resulting expression. This yields

$$\begin{aligned} &\{3p'(h) + h^2 p'''(h) + 4h''p(h) - h'''c(h) - 2c''(h)\}\dot{h} - \\ &(r - k)\{p(h) + h^2 p''(h) + 2h'p(h) - h''c(h) - c'(h)\} = 0, \end{aligned} \tag{14}$$

where $p\{h(T)\} + \{h(T)\}^2 p''\{h(T)\} + 2h(T)p'\{h(T)\} = h(T)c''\{h(T)\} + c'\{h(T)\}$ is the boundary condition for $h.$ Comparing equations (14) and (6) it is clear that the optimal harvest level when the buyer uses a unit tariff is not identical to the optimal harvest level when this buyer is passive. When the inverse demand function is linear $\{p(h) = \alpha - \beta h, \alpha > 0, \beta > 0\}$ and the harvest cost function is quadratic $\{c(h) = h^2/2\},$ the differential equation for the optimal harvest with a unit tariff approximates the differential equation for the optimal harvest when the buyer is passive. This notwithstanding, there are no straightforward necessary or sufficient conditions under which (6) and (14) coincide. The differential equation for the optimal unit tariff is found by differentiating (8) with respect to time and then using (9) to simplify the ensuing expression. This gives

$$\dot{n} - (r - k)n = \{2p'(h) + h''p(h) - c''(h)\}\dot{h} - (r - k)\{p(h) + h'p(h) - c'(h)\}, \tag{15}$$

where $\dot{h}$ is given by (14) and $n(T) = p\{h(T)\} + h(T)p'\{h(T)\} - c'\{h(T)\}$ is the boundary condition.

3.2. The Open Loop Ad Valorem Tariff

We now derive the solution for the open loop *ad valorem* tariff when the buyer's objective is (2). To maximize (4) subject to (5), we first form the seller's current value Hamiltonian. The relevant first order necessary conditions are

$$v\{p(h) + h'p(h)\} - c'(h) - \lambda = 0, \tag{16}$$

and (9). Now solve for v from (16), substitute into (2), and then simplify the resulting expression. We get

$$J_b = \int_0^T e^{-rt}[u(h) - \{c'(h) + \lambda\}\phi(h)]dt, \tag{17}$$

where $\phi(h) = h/[1 + \{1/\theta(h)\}]$ and $\theta(h)$ is the price elasticity of demand. With this substitution, the buyer's problem is to maximize (17) over h, subject to (5) and (9). The first order necessary conditions to this problem are

$$p(h) - \{c'(h) + \lambda\}\phi'(h) - c''(h)\phi(h) - \eta_1 = 0, \tag{18}$$

$$\dot{\eta}_1 = (r - k)\eta_1, \tag{19}$$

and

$$\dot{\eta}_2 = \phi(h) + k\eta_2, \tag{20}$$

where η_1 and η_2 are the costate variables associated with constraints (5) and (9).

The differential equation satisfied by the optimal harvest h can be determined by differentiating (18) with respect to time and then simplifying. This gives

$$\{p'(h) - c'(h)\phi''(h) - 2c''(h)\phi'(h) - c'''(h)\phi(h) - \lambda\phi''(h)\}\dot{h} -$$

$$(r - k)\{p(h) - c'(h)\phi'(h) - c''(h)\phi(h) - \lambda\phi'(h)\} = 0, \tag{21}$$

where $p\{h(T)\} = c'\{h(T)\}\phi'\{h(T)\} + c''\{h(T)\}\phi\{h(T)\}$ is the boundary condition. In similar fashion, the differential equation for the optimal *ad valorem* tax is obtained by differentiating (16) with respect to time and then simplifying. We get

$$\{p(h) + h'p(h)\}\dot{v} + \{2v'p(h) + v''hp(h) - c''(h)\}\dot{h} - (r - k)\{vp(h) + v'hp(h) - c'(h)\} = 0, \tag{22}$$

where $\dot{h}$ is given by (21) and the boundary condition for v is $v(T)[p\{h(T)\} + h(T)p'\{h(T)\}] = c'\{h(T)\}$. We now discuss the implications of our findings thus far.

3.3. Discussion

Recently, Batabyal and Beladi (2002) have shown that when a monopsonistic buyer of a renewable resource faces competitive sellers, irrespective of whether she uses a unit or an *ad valorem* tariff, her payoff is unchanged. In other words, the leader's payoff is policy invariant. Comparing equations (17) and (10) we see that this "policy invariance" result does *not* hold when a monopsonistic buyer of a renewable resource trades with a monopolistic seller. Moreover, the Batabyal and Beladi (2002) paper also tells us that when a monopsonistic buyer faces competitive sellers, the optimal open loop unit and *ad valorem* tariffs are equivalent. Now, because the payoff functions (17) and (10) are not identical, it follows that in the case studied in this paper, the two tariffs are *not* equivalent. Put differently, the use of these two trade policy instruments will give rise to dissimilar time profiles of harvests and hence to different terminal levels of the resource stock.

Equation (13) tells us that $\eta(t) = 0, \_t.$ In other words, the buyer's marginal utility of one additional unit of the resource stock is zero. Using this value of the marginal utility in (12) we get an implicit equation for the domestic price of the renewable resource in the importing nation. That equation is

$$p(h) = c'(h) + h''c(h) - h^2 p''(h) - 2h'p(h), \_t. \tag{23}$$

The reader will note that the domestic price of the renewable resource in the importing nation does *not* depend on the initial value of the resource stock x_0. This is yet another implication of the fact that when the harvest cost function is stock independent, the optimal solution to the buyer's problem is time consistent.

Since the optimal unit and *ad valorem* tariffs are not equivalent, it is of considerable interest to determine what happens when our buyer uses both tariffs simultaneously. When the buyer uses both tariffs simultaneously, the seller maximizes

$$J_s = \int_0^T e^{-rt}[vp(h)h - nh - c(h)]dt \tag{24}$$

over h, subject to (5). The first order necessary conditions to this problem include $n = v\{p(h) + h'p(h)\} - c'(h) - \lambda$. Now, substituting this condition into the buyer's payoff function and then using $\lambda(t) = 0$ gives

$$J_b = \int_0^T e^{-rt}[u(h) + vh^2 p'(h) - h'c(h)]dt. \tag{25}$$

Inspecting (25) we see that the buyer's current value Hamiltonian is linear and decreasing in v. Because $v \geq 0$, it is optimal to set $v = 0$ and this last condition tells us that $a(t) = \infty$ and $n(t) = -c'(h)$. In words, when our monopsonistic buyer uses both tariffs simultaneously, it is optimal for her to levy an infinite *ad valorem* tariff $\{a(t) = \infty\}$ and to offer a unit subsidy $\{n(t) = -c'(h)\}$ to the seller. Now substituting $v = 0$ in (25) and comparing the resulting equation with equation (11) in Batabyal and Beladi (2002) we see that when our buyer uses both tariffs simultaneously, she is able to force the monopolistic seller to behave competitively. This means that the harvest rate of the resource is the same whether a competitive seller faces an optimal tariff of either kind or a monopolistic seller faces optimal unit and *ad valorem* tariffs. This last result obtains because the simultaneous use of unit and *ad valorem* tariffs allows our buyer to shift and rotate the inverse demand function. As a result, she is able to confront the monopolistic seller with an infinitely elastic non-stationary function. Finally, note that unlike the result obtained for the competitive case studied in Batabyal and Beladi (2002), the concurrent use of both tariffs does not make one tariff superfluous.

Recall that the objective of the importing nation is to encourage the conservation of the renewable resource. In this regard we note that because the tariffs studied here are time consistent, they will achieve their intended conservation aims, albeit obliquely. Having said this, we should also point out that if an importing nation's goal is to encourage conservation of the renewable resource in the exporting country, then tariffs are not the ideal policy instruments. Why not? This is because tariffs target trade and the direct consequence of the tariff is to discourage domestic consumption in the importing nation. Tariffs do not do anything directly to promote conservation of the renewable resource in the exporting nation. Therefore, from the standpoint of resource conservation, tariffs are blunt policy instruments. We now analyze the Stackelberg differential game between our monopsonistic buyer and the monopolistic seller when the harvest cost function is stock dependent.

4. The Stock Dependent Harvest Cost Function and Open Loop Tariffs

When the harvest cost function is stock dependent, the optimal open loop unit and *ad valorem* tariffs are time *inconsistent*. As explained in section 3, this means that at some time $t > 0$ the buyer will want, if she can, to depart from the tariff trajectory she announced at time $t = 0$ and announce a different tariff trajectory. The monopolistic seller in this paper is forward looking. Therefore, he will anticipate the buyer's desire to alter the tariff trajectory she announced at the beginning of the game and hence this tariff will fail to attain its intended conservation objectives. To grasp this critical feature of the optimal policies clearly, we now derive the optimal open loop unit and *ad valorem* tariffs.

4.1. The Open Loop Unit Tariff

Recall from section 2 that the stock dependent harvest cost function is $c(x)h.$ We follow the section 3 method to solve our buyer's problem. Suppose the monopolistic seller takes the buyer's unit tariff $n(t)$ as given. Then, the first order necessary conditions to his problem are

$$p(h) + h'p(h) - n - c(x) - \lambda = 0 \tag{26}$$

and

$$\dot{\lambda} = (r - k)\lambda + c'(x)h, \tag{27}$$

where $\lambda(t)$ is the costate variable. This costate variable gives us the seller's marginal utility of one more unit of the resource stock at time $t.$ Comparing (27) with (9) it is at once clear that when the harvest cost function is stock dependent, $\lambda(t)\_0.$ In words, the rent on the marginal unit of the resource stock is typically not equal to zero. Now solving for n from equation (26) and then substituting into equation (1), we get

$$J_b = \int_0^T e^{-rt}[u(h) + h^2 p'(h) - \{c(x) + \lambda\}h]dt. \tag{28}$$

Equation (28) tells us that the buyer's payoff is the present discounted stream of the utility of consuming the harvest $h(t)$ and the term $h^2 p'(h)$ less the sum of the cost of harvesting $h(t)$ and the term $\lambda h.$ As in section 3, this last term can be interpreted as the total instantaneous rent paid by the buyer for the resource $x.$

In order to keep the buyer's problem a single state variable problem, we now use (27) to eliminate λ from (28). Integrating equation (27) we get

$$\lambda(t) = e^{(k-r)(T-t)}\lambda(T) - e^{-(k-r)t}\int_t^T e^{(k-r)m}c'(x)hdm. \qquad (29)$$

Substituting this value of $\lambda(t)$ from (29) into (28) we obtain

$$J_b = \int_0^T e^{-rt}[u(h) + h^2 p'(h) - c(x)h - \{ e^{(k-r)(T-t)}\lambda(T) - e^{-(k-r)t}\int_t^T e^{(k-r)m}c'(x)hdm\}h]dt.$$
(30)

Equation (30) tells us that for any trajectory of $h(t),$ the buyer's payoff function is maximized by setting $\lambda(T) = 0.$ By doing this, our buyer drives the monopolistic seller's rent to zero at the conclusion of the game. Now substitute $\lambda(T) = 0$ into (30) and then reverse the order of integration of the last term in (30). This gives

$$J_b = \int_0^T e^{-rt}[u(h) + h^2 p'(h) - c(x)h + c'(x)h\{ e^{kt}x_0 - x\}]dt. \qquad (31)$$

As desired, we now have a single state variable control problem for our buyer. In particular, we have converted the buyer's problem from one of maximizing (1) over n subject to (5) to one of maximizing (31) over h subject to (5). Comparing equations (31) and (11) we see that unlike the case studied in section 3, when the harvest cost function is stock dependent, the initial resource stock x_0 enters the buyer's payoff function. The first order necessary conditions to our monopsonistic buyer's problem are

$$p(h) + 2h'p(h) + h^2 p''(h) - c(x) + c'(x)\{ e^{kt}x_0 - x(t)\} - \eta = 0, \qquad (32)$$

and

$$\dot{\eta} = (r - k)\eta + 2h'c(x) - c''(x)h\{ e^{kt} x_0 - x(t)\}, \tag{33}$$

where $\eta(t)$ is the costate variable. Equation (32) tells us that when the harvest cost function is stock dependent, the solution to the buyer's problem *does* depend on the initial stock of the renewable resource x_0. This has the following implication: at some time $m > 0,$ if the buyer were able to revise the tariff she initially announced at the beginning of the game, then x_0 in (32) would have to be replaced with $x(m)$ and the solution for all $t > m$ would be different. Put differently, the buyer's optimal solution is time *inconsistent*. From (32) and (33) it is clear that the optimal solution is time consistent if and only if the harvest cost function is unrelated to the stock of the resource. Indeed, this is precisely what we demonstrated in our analysis of the stock independent harvest cost function in section 3.

Before deriving a differential equation satisfied by the optimal $h(t),$ a comment on the salience of constraint (27) is in order. When the seller exerts market power, he is subject to the sequence of static constraints implied by the buyer's optimizing behavior. However, these constraints enter the seller's problem as parameters and this is what allows him to solve a standard control problem. In contrast, when the buyer exerts market power, she is constrained by the dynamic optimizing behavior of the seller. This means that the buyer solves a non-standard control problem. As noted in Karp (1984), because the seller's problem is dynamic and the buyer exerts market power, a "rational expectations" constraint, given by equation (27), is introduced into the buyer's problem. The reader will recall that in section 3.1, we called this "rational expectations" constraint a jump state constraint.[8]

[8] For more on these issues, see Karp (1984) and Karp and Newbery (1993).

To determine the differential equation satisfied by the optimal $h(t)$ when our buyer uses a unit tariff, we differentiate (32) with respect to time and then use (33) to simplify the resulting expression. This gives

$$\{3p'(h)+4h''p(h)+h^2p'''(h)\}\dot{h}-(r-k)\{p(h)+2h'p(h)+h^2p''(h)-c(x)+e^{kt}x_0c'(x)-c'(x)x\}+$$

$$c'(x)\{2h-2kx+ke^{kt}x_0\}+k''xc(x)\{x_0e^{kt}-x\}=0, \tag{34}$$

where $p\{h(T)\}+2h(T)p'\{h(T)\}+\{h(T)\}^2p''\{h(T)\}=c\{x(T)\}+c'\{x(T)\}\{x(T)-e^{kT}x_0\}+\eta(T)$ is the boundary condition for h. Inspecting (34), we see that the exogenously given initial condition $x(0)=x_0$ affects the temporal behavior of the optimal harvest $h(t)$. Comparing equations (34) and (7) it is clear that as in section 3, the optimal harvest level when our monopsonistic buyer uses a unit tariff and the harvest cost function is stock dependent is not identical to the optimal harvest level when this buyer is passive. Furthermore, there are no perceptible necessary or sufficient conditions under which equations (7) and (34) coincide. The differential equation for the optimal unit tariff is found by differentiating (26) with respect to time and then using (27) to simplify the resulting expression. This gives

$$\dot{n}-(r-k)n=\{2p'(h)+h''p(h)\}\dot{h}-(r-k)\{p(h)+h'p(h)-c(x)\}-k'xc(x), \tag{35}$$

where $\dot{h}$ is given by (34) and the boundary condition is $n(T)=p\{h(T)\}+h(T)p'\{h(T)\}-c\{x(T)\}$.

4.2. The Open Loop Ad Valorem Tariff

When the seller takes the buyer's *ad valorem* tariff as given, the first order necessary conditions to his control problem are

$$v\{p(h)+h'p(h)\}-c(x)-\lambda=0, \tag{36}$$

and (27). Solving for v from (36), substituting into (2), and then simplifying the resulting expression, we get

$$J_b = \int_0^T e^{-rt}[u(h) - \{c(x) + \lambda\}\phi(h)]dt, \qquad (37)$$

where the functions $\phi(h)$ and $\theta(h)$ are as specified in section 3.2. Comparing (37) and (28) it is clear that these two payoff functions are not identical. Hence, unlike the finding for the competitive sellers case contained in Batabyal and Beladi (2002), the unit and the *ad valorem* tariffs are now not equivalent.

Two methods can be used to pose and analyze our buyer's maximization problem. Using the first method, we eliminate λ from (37) by following the procedure described in section 4.1. The use of this method results in a one state variable control problem for our buyer. The solution to this problem is time inconsistent because the elimination of λ from (37) results in the initial resource stock x_0 entering our buyer's payoff function. Using the second method (see section 3.2) we analyze a two state variable version of the buyer's problem. With this second method, it is difficult to see the time inconsistency of the optimal solution in general. Nevertheless, we use this method because it provides an alternate perspective on the time inconsistency of the optimal solution and because we have already demonstrated how the first method can be used.

Our buyer's problem is to maximize (37) over h subject to (5) and (27). The first order necessary conditions to this problem are

$$u'(h) - \phi'(h)\{c(x) + \lambda\} - \eta_1 + \eta_2 c'(x) = 0, \qquad (38)$$

$$\dot{\eta}_1 - (r - k)\eta_1 = \phi(h)c'(x) - \eta_2 hc''(x), \qquad (39)$$

and

$$\dot{\eta}_2 = \phi(h) + k\eta_2, \qquad (40)$$

where η_1 and η_2 are the costate variables associated with (5) and (27). The difficulty in demonstrating the time inconsistency of the optimal solution ((38)-(40)) arises from the presence of the (generally nonlinear) $\phi(h)$ function in (40). However, when the inverse demand function is isoelastic, i.e., when $p(h) = h^{-b}, b \in (0,1)$, $\phi(h)$ is linear and we can show the time inconsistency of the above optimal solution in a straightforward manner.

When $p(h) = h^{-b}, b \in (0,1), \phi(h) = h/(1-b)$. Now substitute this result in (40) and solve the resulting differential equation. We get $(1-b)e^{-kt}\eta_2(t) = \int_0^t e^{-km}h(m)dm$**.** This solution uses the result that it is optimal to set $\eta_2(0) = 0$.[9] Also, solving (5), we get $x_0 - e^{-kt}x(t) = \int_0^t e^{-km}h(m)dm$. Equating these last two solutions we conclude that

$$\eta_2(t) = \frac{e^{kt}x_0 - x(t)}{1-b}. \tag{41}$$

Now, inspecting (38) we see that the optimal harvest of the renewable resource under study is affected by the costate variable η_2 which gives us the marginal value to the buyer of an increase in the seller's rent. In turn, η_2 is a function of the initial resource stock x_0. Therefore, the optimal harvest is itself a function of the initial resource stock x_0, and following the logic of the argument of section 4.1, this optimal solution too is time inconsistent. Put differently, the solution that results when we use this second method to solve the buyer's problem is time inconsistent because at the beginning of the game our buyer is able to choose the initial level of the seller's rent and hence we have $\eta_2(0) = 0$. However, once this initial rent has been chosen, (27)—the jump state or rational expectations constraint—is a binding constraint on the buyer's problem. Having said this, we note

[9] For more on this result, see Simaan and Cruz (1973), Karp (1984), Karp and Newbery (1993), and Batabyal (1996).

once again that the optimal solution ((38)-(40)) is time consistent if and only if the harvest cost function is unrelated to the stock of the resource.

To find a differential equation for the optimal harvest, differentiate (38) with respect to time and then use (39) and (40) to simplify the resulting expression. This gives

$$[p'(h) - \phi''(h)\{c(x) + \lambda\}]\dot{h} - (r - k)\{p(h) - \phi'(h)c(x) + \eta_2 c'(x)\} +$$
$$c'(x)\{k\eta_2 - kx\phi'(h)\} + k\eta_2 x'' c(x) = 0, \quad (42)$$

where $p\{h(0)\} = \phi'\{h(0)\}\{c(x_0) + \lambda(0)\} + \eta_1(0)$ is the boundary condition. Inspecting (42), we see that η_2 and hence the initial resource stock $x(0) = x_0$ influences the temporal behavior of the optimal harvest $h(t)$. Comparing equations (42) and (7) it is plain that the optimal harvest when our buyer uses an *ad valorem* tariff and the harvest cost function is stock dependent is not identical to the optimal harvest when this buyer is passive. Further, as before, there are no straightforward necessary or sufficient conditions under which (42) and (7) coincide. The differential equation for the optimal *ad valorem* tariff can be found by differentiating (36) with respect to time and then simplifying. We get

$$\{p(h) + h'p(h)\}\dot{v} + v\{2p'(h) + h''p(h)\}\dot{h} - (r - k)\{vp(h) + v'hp(h) - c(x)\} - k'xc(x) = 0 \quad (43)$$

and the boundary condition is $v(T)[p\{h(T)\} + h(T)p'\{h(T)\}] = c\{x(T)\}$. We now discuss the connections between the form of the harvest cost function, the buyer's trade (tariff) policies, and the efficacy of these policies in furthering the conservation of the renewable resource under study.

4.3. Discussion

Batabyal and Beladi (2002) have shown that when a monopsonistic buyer of a renewable resource faces competitive sellers, irrespective of whether she uses a unit or an *ad valorem* tariff, her payoff is unchanged. In section 3, we noted that this result does not hold when the seller (exporter) is a monopolist, the buyer (importer) is a monopsonist, and the harvest cost function is stock independent. Now, comparing equations (37) and (28) we see two things. First, the Batabyal and Beladi (2002) "policy invariance" result also does *not* hold when the exporter is a monopolist, the importer is a monopsonist, and the harvest cost function is stock dependent. Second, because the payoff functions given by (37) and (28) are not identical, the two tariffs themselves are *not* equivalent. Stated differently, the use of these two trade policy instruments will give rise to different time profiles of harvests and hence to different terminal levels of the resource stock.

Upon rearranging terms in (32), we get an implicit equation for the domestic price of the renewable resource in the importing nation. That equation is

$$p(h) = c(x) - c'(x)\{e^{kt}x_0 - x(t)\} - 2h'p(h) - h^2p''(h) + \eta(t) = 0, \_t, \qquad (44)$$

where $\eta(t)$ is the buyer's marginal utility of an additional unit of the resource at time t. Note that because the harvest cost function is stock dependent, unlike the case discussed in section 3.3, the buyer's marginal utility of one more unit of the resource stock $\eta(t)$ is now no longer equal to zero. Also, the initial value of the resource stock now affects the domestic price of the renewable resource in the importing nation. Comparing equations (23) and (44) we see that, in general, it is not possible to determine whether the domestic price is higher with the stock independent or with the stock dependent harvest cost function. Further, observe that x_0 affects the optimal solution when the harvest cost function is stock dependent. In contrast, x_0 does not affect the optimal solution in the case of the stock independent harvest cost function. Also, it is not possible, in general, to determine whether the terminal value of the resource stock $x(T)$ is higher with the stock independent or with the stock dependent harvest cost function.

We now show that when our buyer uses both tariffs concurrently, she can force the monopolistic seller to behave competitively. Since the logic of the argument is very similar to that employed in section 3.3, our discussion of this result will be brief. The first order necessary condition to the seller's maximization problem is $n = v\{p(h) + h'p(h)\} - c'(x) - \lambda$. Now, substituting this condition into the buyer's payoff function, we get

$$J_b = \int_0^T e^{-rt}[u(h) + vh^2 p'(h) - hc(x) - \lambda h]dt. \tag{45}$$

Inspecting (45) we can tell that the buyer's current value Hamiltonian is linear and decreasing in v. Since $v \geq 0$, it is optimal to set $v = 0$ and this last requirement tells us that $a(t) = \infty$ and $n(t) = -\{c(x) + \lambda\}$. In words, when our monopsonistic buyer uses both tariffs simultaneously, it is optimal for her to set an infinite *ad valorem* tariff $[a(t) = \infty]$ and to offer a unit subsidy $[n(t) = -\{c(x) + \lambda\}]$ to the seller. Now substituting $v = 0$ in (45) and comparing the resulting equation with equation (21) in Batabyal and Beladi (2002) we see that when our buyer uses both tariffs simultaneously, she is able to force the monopolistic seller to behave competitively. In other words, the harvest rate of the resource is the same whether a competitive seller faces an optimal tariff of either kind or a monopolistic seller faces optimal unit and *ad valorem* tariffs. As explained in section 3.3, this last result holds because the simultaneous use of unit and *ad valorem* tariffs allows our buyer to shift and rotate the inverse demand function. Finally, note that unlike the competitive case studied in Batabyal and Beladi (2002), the coincidental use of both tariffs does not make one tariff extraneous.

The analysis in this paper has shown that efforts to further renewable resource conservation by means of trade policies are problematic in more ways than one. We now discuss this important point in greater detail. The analysis in Karp and Newbery (1993) and in Batabyal (1998) tells us that given a choice between time consistent and inconsistent policies, an economic agent will generally choose time inconsistent policies because inconsistent policies lead to a higher payoff. In the setting of our paper, this tells us than even when the open loop unit and *ad valorem* tariffs are time inconsistent, and this happens when the harvest cost function is stock dependent, the buyer in the importing nation will prefer to use these inconsistent trade policies rather than follow a time consistent course of action. Nevertheless, inconsistent policies are not believable and hence the tariff trajectory announced by the buyer at the beginning of the game will not be believed by the seller and therefore inconsistent policies will fail to attain their intended resource conservation objectives.

In contrast, when the harvest cost function is stock independent, the optimal open loop tariffs are time consistent and hence plausible from the standpoint of the monopolistic seller. Therefore, in this case, the buyer's trade policies (tariffs) will, albeit indirectly, attain their resource conservation aims. As indicated previously in section 3.3, in an ideal situation, trade policies such as tariffs should *not* be used to promote the conservation of renewable resources. This is because tariffs reduce the domestic consumption of the traded resource in the importing nation and hence get at the conservation issue in an oblique manner. However, if the seller is unwilling and/or unable to take measures in his own nation to further resource conservation, perhaps because of the lack of appropriate property rights, then tariffs are one imperfect instrument with which the seller can be encouraged to take the relevant conservation measures.

Although tariffs are an imperfect way of promoting conservation, the analysis in this paper has shown that they may not function as desired when they are most needed. Why not? Because the believability of the optimal tariffs depends on the form of the harvest cost function and this form is *not* controllable by the buyer. The extant literature on international trade in renewable resources has *not* recognized this essential point. The reader should note that the stock dependent cost function is the more appropriate cost function for threatened renewable resources such as the African black rhinoceros. Renewable resources like the African black rhinoceros are threatened, *inter alia*, because satisfactory *domestic* measures designed to prevent overexploitation have not been taken in the relevant nations. In the case of the African black rhinoceros, the pertinent nations include Namibia, South Africa, and Zimbabwe. It is for these threatened resources, where the proper domestic conservation measures have not been taken, that imperfect supra-national measures such as trade policies are most useful. Regrettably, the analysis in this paper tells us that trade policies are likely to be ineffectual (because they are not believable) precisely when they are most needed (when the harvest cost function is stock dependent).

5. Conclusions

In this paper we conducted a Stackelberg differential game theoretic analysis of international trade in renewable resources between a monopolistic seller (the follower) and a monopsonistic buyer (the leader). Trade policies, i.e., unit and *ad valorem* tariffs are used by the government in the importing nation to obliquely further the conservation of the renewable resource under study. Unlike the finding contained in Batabyal and Beladi (2002), the optimal open loop unit and *ad valorem* tariffs are *not* equivalent. Consequently, when the buyer uses both tariffs concurrently, she is able to effectively dispense with the market power of the seller by forcing him to behave competitively. The efficaciousness of trade policies in furthering resource conservation is contingent upon the form of the harvest cost function. Our analysis shows that because of believability problems, trade policies such as tariffs are likely to be ineffectual in furthering the conservation of *threatened* renewable resources.

The analysis in this paper can be extended in a number of different directions. We now propose two possible extensions. First, given the time inconsistency of the optimal solution when the harvest cost function is stock dependent, it would be useful to compare and contrast the properties of time inconsistent and time consistent tariffs in a differential game model of international trade in renewable resources. Second, it would be interesting to study the trade game between a monopolistic seller and a fringe of competitive buyers and one dominant buyer. An examination of these aspects of the problem will allow richer analyses of the nexuses between international trade, trade policies, and the conservation of renewable resources.

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