Did Output Recover from the Asian Crisis?

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and

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Abstract

This paper investigates the extent to which output in the Asian countries has recovered after the crisis. A regime switching approach that introduces two state variables is used to decompose recessions into permanent and transitory components. We found evidence of permanent output losses in all countries.

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1. Introduction

While the recent Asian financial crisis of 1997 generated a plethora of research that analyzed the causes of the crisis, less attention has been paid to the aftermath. How long do crises last and to what extent does output recover? Although there is copious evidence that the economies suffer from a recession right after a crisis, the literature has not examined whether the recession following the crisis permanently lowers the level of output. In other words, this paper analyzes if the output reduction after the crisis is a mere "pluck" down from the trend level, which eventually is reversed as output reverts to trend (i.e., recession temporarily lowers output), or alternatively, whether the level of output tends to go down permanently.

In order to answer this question, this paper employs a regime switching approach. We decompose the recessions into permanent and temporary components by introducing different state variables that control recovery and recessions for the two components, and allow asymmetric adjustment in the temporary component to model the temporary "pluck" down from trend. Section 2 discusses these concepts in the context of their origin in the US business cycle literature. In Section 3, the model used for the empirical analysis is specified. Section 4 discusses the data and procedure and presents the results. Concluding remarks are made in Section 5.

2. Theory and Literature Review

There has been a considerable amount of research devoted to examining the properties of business cycles in the United States. Two main focuses of the literature have been to incorporate the idea of comovement across economic times series using the dynamic linear factor models innovated by Stock and Watson (1989, 1991, 1993) and to probe the idea of asymmetry through regime switching as pioneered by Hamilton (1989).

The latter idea has spurred a considerable debate on the nature of US business cycle fluctuations. Two general types of parametric time-series models have been proposed, which have vastly different implications for the welfare effects of recessions.

The first type of model owes to Hamilton (1989), in which the stochastic trend in output undergoes regime switching between positive and negative growth states. Since the regime switch occurs in the growth rate of the permanent component, a negative state results in an output loss that is permanent.

The second type of model assumes that regime switching occurs in a common temporary component. This idea has its roots in the work of Friedman (1964, 1993), in which recessions can be characterized as a temporary "pluck" down of output. After this large negative transitory shock dissipates, output returns to trend in a high growth recovery phase. Since this type of recession represents a temporary deviation from trend, followed by a full recovery to trend, the output loss is temporary.

The analysis in this paper draws on these concepts and debates about the US business cycle. Parallel to the common factor US business cycle literature, the crisis-induced recessions in the Asian countries involved a simultaneous decline in several economic variables. Moreover, the main interest of this paper is to study the nature of the recessions. Most of the US business cycle literature that investigates asymmetry considers only regime switching in either a temporary or a permanent component. This stems from the typical use of univariate analysis. Two exceptions are Kim and Murray (1998) and Kim and Piger (2000), which investigate the comovement of several economic series and asymmetry in both temporary and permanent common factors. Kim and Piger, although specifying that output contains both permanent and temporary components, use only one state variable to control both components. This forces each recession to contain both temporary and permanent explanations. As in Kim and Murray, we use a model that has two separate state variables for the temporary and permanent factors. This allows us to identify whether the recessions due to the Asian crisis involved regime switches in the temporary or permanent components of output. However, Kim and Murray use a series of variables intended to capture comovement with industrial production and focus on constructing a coincident indicator. We follow Kim and Piger in using output, investment, and consumption, which theory predicts should share a common stochastic trend.

3. Econometric Model

This section presents the specification of the dynamic two factor model. The logs of each series can be decomposed into a deterministic component, DT_i, a permanent component, P_{it}, and a transitory component, T_{it}.

$$\overline{Y}_{it} = DT_i + P_{it} + T_{it}$$

$$P_{it} = \gamma_i n_t + \varsigma_{it}$$

$$T_{it} = \lambda_i x_t + \omega_{it}$$

where $\overline{Y}$ = [output, investment, consumption], n is the common permanent component, x is the common temporary component, and ζ and ω are the independent idiosyncratic permanent and temporary components, respectively. The model can be written in differenced deviations from means as follows:

$$\Delta y_{it} = \gamma_i \Delta n_t + \lambda_i \Delta x_t + z_{it}$$

where $z_{it} = \Delta \varsigma_{it} + \Delta \omega_{it}$ is a stationary composite of the idiosyncratic components and γ_i and λ_i are the factor loadings on the common permanent and common transitory components, respectively.

The growth rate of the common permanent component is stationary and is approximated by a second order autoregressive process. Note that a stationary growth rate implies that the level is nonstationary, in accordance with the definition of a stochastic trend. In addition, there is a constant, β, that depends on the permanent state, S_{1t}:

$$\Delta n_t = \beta_{S1t} + \phi_1 \Delta n_{t-1} + \phi_2 \Delta n_{t-2} + v_t\ , \quad v_t \sim i.i.d.\, N(0,1)$$

The state-dependent constant introduces asymmetry along the lines of Hamilton (1989).

$$\beta_{S1t} = \beta_0 + \beta_1 S_{1t}\ ;\ S_{1t} = \{0,1\}$$

During an expansion phase (S_{1t}=0) the stochastic trend grows with the drift rate β_0. If β_1 is negative, the trend shifts to a lower growth state when S_{1t}=0, and shifts to a recession phase if $\beta_0+\beta_1<0$.

The common temporary component is stationary in its levels and is approximated by a second order autoregressive process. To incorporate Friedman's type of asymmetry, we allow the temporary component of each series to undergo regime switching in response to a second state variable, S_{2t}.

$$x_t = \tau S_{2t} + \phi_{11} x_{t-1} + \phi_{12} x_{t-2} + u_t\ , \quad u_t \sim i.i.d.\, N(0,1)$$

In state S_{2t}=0, the intercept is zero. If $\tau_i<0$, then the economic series is "plucked" down when S_{2t}=1. When the state returns to normal, S_{2t}=0, the economy reverts back to trend.

Finally, each series has its own stationary idiosyncratic component, again approximated by an AR(2).

$$z_{it} = \psi_{i1} z_{it-1} + \psi_{i2} z_{it-2} + e_{it}\ ,\quad e_{it} \sim i.i.d.\,N(0,1)$$

$$E(v_r u_s e_{it}) = 0\ , \forall\, i, r, s, t$$

Both state variables are assumed to be independent first order Markov switching processes with transition probabilities given by:

$$\Pr[S_{1t} = 0 \mid S_{1t-1} = 0] = q_1\ ,\ \Pr[S_{1t} = 1 \mid S_{1t-1} = 1] = p_1$$

and

$$\Pr[S_{2t} = 0 \mid S_{2t-1} = 0] = q_2\ ,\ \Pr[S_{2t} = 1 \mid S_{2t-1} = 1] = p_2$$

4. Econometric Analysis and Results

Quarterly data from six Asian countries[1] (Hong Kong, Indonesia, Korea, the Philippines, Malaysia, and Singapore) have been taken for the logs of GDP, private consumption, and gross fixed capital formation in constant prices, and the data have been seasonally adjusted. The data sources are described in Appendix 1. Augmented Dickey-Fuller and Phillips-Perron tests provide strong evidence that each of these series contain unit roots (see Table 1). Standard theoretical models of capital accumulation in an intertemporal optimizing framework show that output, investment, and consumption share a common stochastic trend. The permanent income hypothesis would identify consumption with the trend, but we do not impose that restriction here in order to allow for possible liquidity constraints that would make at least a fraction of the population consume out of current income.

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[1] Quarterly data for Thailand was only available for three years, so it was dropped from consideration.

The model outlined in the previous section can be written in state space form (Appendix 2), which allows the application of a Kalman filter. The regime switch is estimated by Kim's (1994) approximate maximum likelihood algorithm, which is a computationally efficient method of estimating Markov switching in both the observation and transition equation.

The maximum likelihood parameter estimates are shown in Table 2. All of the factor loadings for output, investment, and consumption are positive on the permanent component with the exception of consumption for Indonesia and investment for Singapore. There is some evidence of binding liquidity constraints since for a few countries the elasticity of consumption is greater than output, indicating that individuals are not fully capable of smoothing their consumption. The elasticities to the temporary shock are negative for Hong Kong, Korea, and Malaysia, consistent with Friedman's plucking model. However, the elasticities are positive for the other three countries. The parameter estimates can be used to calculate the expected duration of the expansionary and contractionary phases. For example, the expected durations for Hong Kong are 12 quarters for the expansion phase of the permanent component and 4 quarters for the contraction phase of the permanent component.

Expected Durations (Quarters) of State Affecting the Permanent Component

	Hong Kong	Indonesia	Korea	Malaysia	Philippines	Singapore
Expansion	12	26	70	15	3	7
Contraction	4	2	3	2	2	2

Figures 1 and 2 show the probabilities that the permanent and temporary common components, respectively, undergo a regime switch. It is evident from Figure 1 that the crisis induced a permanent recession in all of the countries. The probability of being in the recessionary state 2 reaches or approaches one in all of the countries at the time of the Asian crisis. On the other hand, Figure 2 indicates that there is not much evidence of a Friedman-style temporary loss in any of the countries, except for Korea. Figure 3 illustrates the common permanent component for each of the countries, while Figure 4 illustrates the common temporary component. The common permanent components for all countries are shown jointly in Figure 5, with a vertical line indicating the start of the Asian crisis in the third quarter of 1997. In Hong Kong, Korea, and Malaysia, the permanent component reaches a peak in 3Q 1997, and immediately declines into a recession. The Philippines and Indonesia decline after a lag of one or two quarters. For Indonesia, this is consistent with the finding of contagion in Cerra and Saxena (2000).

5. Conclusions

The chief objective of this paper has been to investigate whether output losses associated with the Asian crisis have been permanent or temporary. This was accomplished through a two common factor model with regime switching in each of the factors. Real GDP, gross fixed capital formation, and consumption were used to identify the common transitory and stochastic trends.

The results show that output loss has been permanent in all. Output in most of the countries therefore appear to behave according to Hamilton's model, in which the growth rate of output is negative during a recession and the level of output is permanently lowered.

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Table 1: Unit Root Test

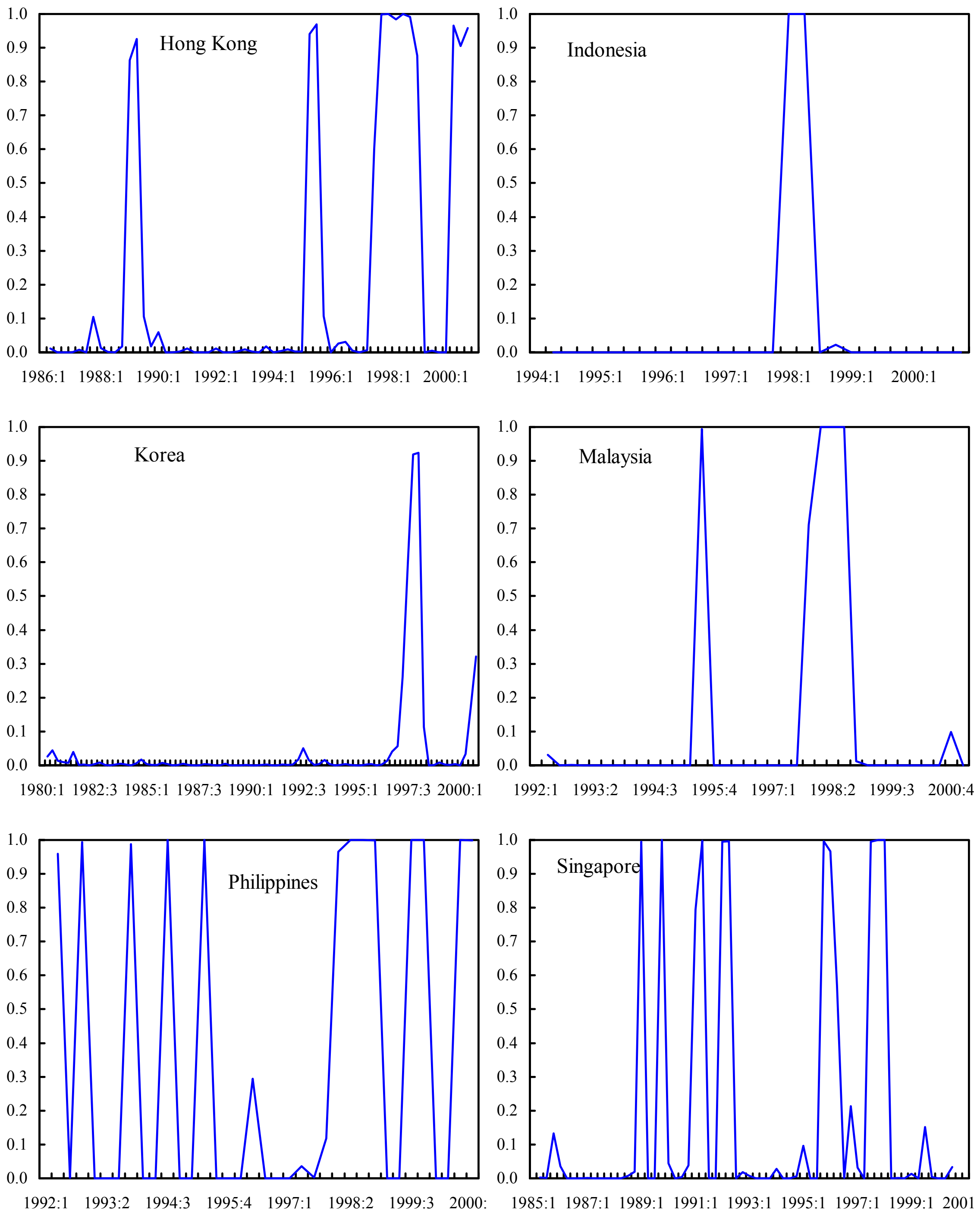
Country	No. of Obs.	Variable	k	ADF	PP
HK	60	LRGDP	5	-1.34 *	-1.84 *
		LRINV	4	-1.10 *	-1.71 *
		LRPCON	4	-1.66 *	-2.07 *
IDN	28	LRGDP	0	-2.14 *	-2.20 *
		LRINV	1	-1.32 *	-1.44 *
		LRPCON	0	-2.04 *	-2.10 *
KOR	84	LRGDP	6	-1.96 *	-1.39 *
		LRINV	5	-1.94 *	-1.77 *
		LRPCON	5	-1.49 *	-1.28 *
MYS	37	LRGDP	4	-1.85 *	-2.26 *
		LRINV	0	-2.07 *	-2.08 *
		LRPCON	4	-2.27 *	-0.80 *
PHL	36	LRGDP	4	-1.21 *	-1.81 *
		LRINV	4	-2.09 *	-1.95 *
		LRPCON	4	-1.62 *	-2.17 *
SGP	65	LRGDP	4	-2.02 *	-0.58 *
		LRINV	4	-0.60 *	0.18 *
		LRPCON	4	-2.47 *	-0.92 *

Note: Variables are as defined above in Appendix 1. The value of k corresponds to the highest-order lag for which the corresponding t-statistic in the regression is significant. Asterisk * denotes non-rejection of null hypothesis of a unit root at 1% significance level. Critical values are from MacKinnon. These are the results from Unit Root testing in levels. However, all non-stationary series were stationary in first differences.

Table 2. Maximum Likelihood Estimates

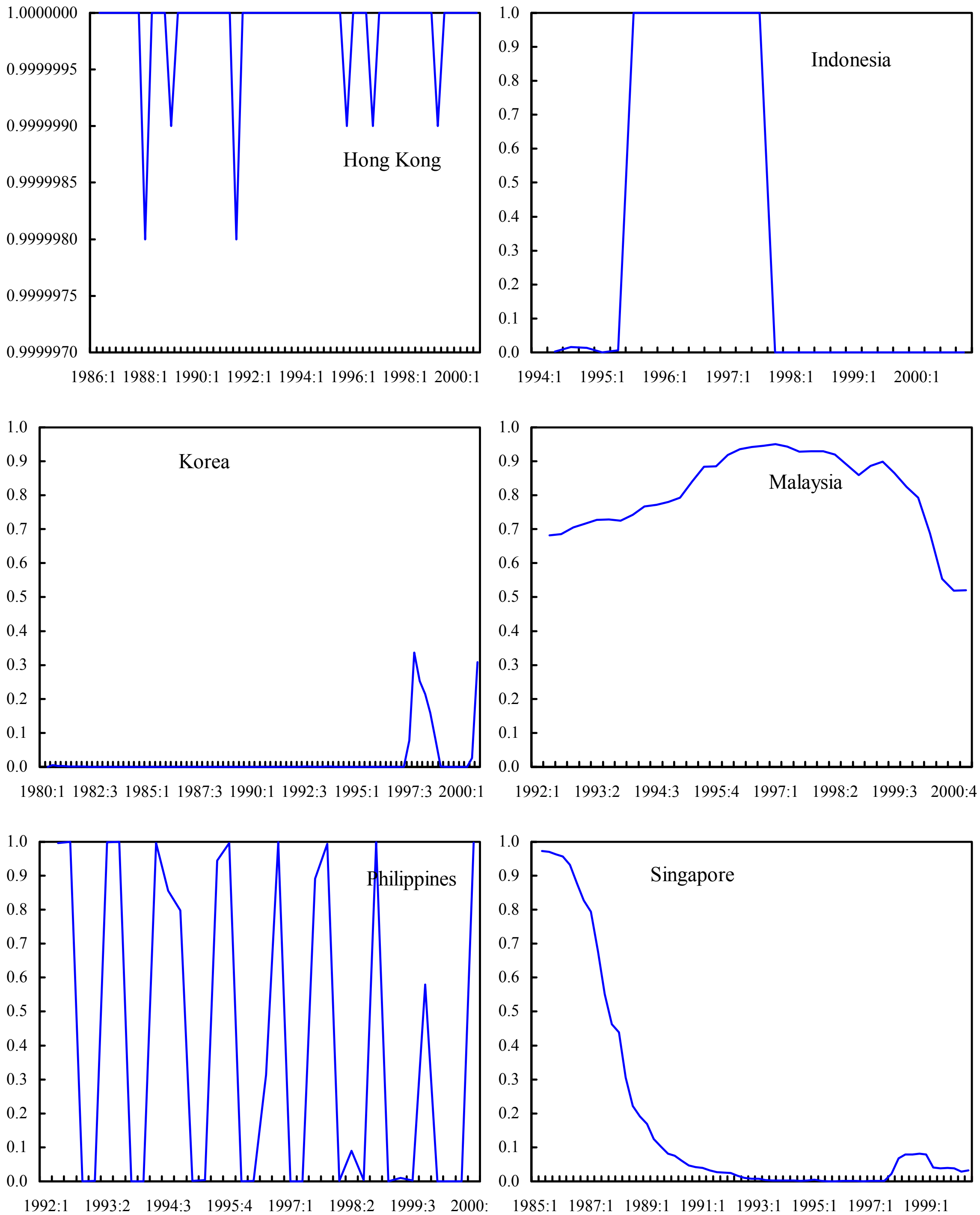
Parameters	Hong Kong	Indonesia	Korea	Malaysia	Philippines	Singapore
q1	0.915	0.961	0.986	0.933	0.658	0.856
	(0.044)	(0.038)	(0.014)	(0.047)	(0.100)	(0.058)
p1	0.733	0.483	0.674	0.550	0.437	0.436
	(0.130)	(0.356)	(0.284)	(0.229)	(0.146)	(0.147)
q2	0.279	0.944	0.989	0.914	0.590	0.981
	(3.959)	(0.065)	(0.012)	(0.231)	(0.111)	(0.023)
p2	1.000	0.861	0.786	0.955	0.440	0.967
	(0.000)	(0.109)	(0.178)	(0.189)	(0.140)	(0.047)
ϕ1	0.102	-0.138	0.846	-0.295	-0.140	-0.519
	(0.118)	(0.140)	(0.180)	(0.150)	(0.073)	(0.104)
ϕ2	0.075	0.059	-0.179	0.010	-0.005	-0.067
	(0.104)	(0.128)	(0.076)	(0.125)	(0.005)	(0.027)
ϕ11	-0.194	0.314	0.040	0.094	0.410	0.822
	(0.153)	(0.149)	(0.160)	(0.293)	(0.097)	(0.147)
ϕ12	-0.009	0.163	0.000	-0.002	0.228	-0.018
	(0.015)	(0.126)	(0.003)	(0.014)	(0.102)	(0.172)
ψ11	1.638	-1.612	-0.289	-0.859	0.220	-0.433
	(0.183)	(0.250)	(0.161)	(12.170)	(0.564)	(5.118)
ψ12	-0.670	-0.650	-0.021	-0.184	-0.012	0.051
	(0.150)	(0.202)	(0.023)	(5.666)	(0.062)	(1.621)
ψ21	-0.264	0.139	-0.094	-1.313	-1.633	-1.843
	(0.130)	(0.263)	(0.130)	(10.117)	(0.182)	(0.120)
ψ22	-0.017	0.004	-0.002	-0.429	-0.667	-0.849
	(0.017)	(0.389)	(0.006)	(6.551)	(0.149)	(0.111)
ψ31	-1.032	-1.174	-1.652	-0.404	-0.409	-0.386
	(6.729)	(27.495)	(0.163)	(0.174)	(0.199)	(0.136)
ψ32	-0.266	-0.340	-0.682	-0.041	-0.042	-0.037
	(3.472)	(19.452)	(0.135)	(0.035)	(0.041)	(0.026)
γ1	0.584	0.466	0.037	0.553	0.143	0.190
	(0.067)	(0.068)	(0.020)	(0.078)	(0.038)	(0.090)
γ2	0.233	0.404	0.062	0.547	0.321	-0.304
	(0.064)	(0.093)	(0.036)	(0.068)	(0.048)	(0.073)
γ3	0.334	-0.111	0.060	0.425	0.152	0.054
	(0.048)	(0.066)	(0.033)	(0.082)	(0.047)	(0.075)
λ1	0.010	0.082	0.517	-0.550	-0.238	-0.852
	(0.051)	(0.054)	(0.066)	(0.138)	(0.108)	(0.086)
λ2	0.156	0.127	0.397	0.156	0.076	-0.652
	(0.081)	(0.080)	(0.067)	(0.104)	(0.036)	(0.076)
λ3	0.515	0.509	0.645	-0.049	-0.139	-0.390
	(0.060)	(0.079)	(0.060)	(0.117)	(0.067)	(0.101)
σ1	0.059	0.067	0.509	0.000	0.288	0.003
	(0.031)	(0.057)	(0.044)	(0.011)	(0.156)	(0.530)
σ2	0.831	0.544	0.667	0.000	0.067	0.025
	(0.077)	(0.089)	(0.055)	(0.014)	(0.042)	(0.020)
σ3	0.001	0.000	0.051	0.662	0.654	0.840
	(0.174)	(0.021)	(0.030)	(0.078)	(0.083)	(0.075)
β0	0.691	0.483	0.729	0.543	2.010	3.630
	(0.206)	(0.214)	(0.484)	(0.208)	(0.378)	(0.445)
β1	-3.595	-6.569	-19.668	-4.630	-7.486	-6.514
	(0.714)	(1.305)	(11.310)	(0.956)	(1.281)	(0.955)
τ1	-1.929	4.975	-4.328	-0.726	5.912	1.155
	(1.486)	(1.018)	(1.221)	(0.892)	(2.715)	(0.489)
loglikelihood	-95.828	-30.150	-107.125	-50.994	-52.797	-113.083
sample period	1986:1-2000:4	1994:1-2000:4	1980:1-2000:4	1992:1-2001:1	1992:1-2000:4	1985:1-2000:3

Figure 1. Probability of Permanent Recession



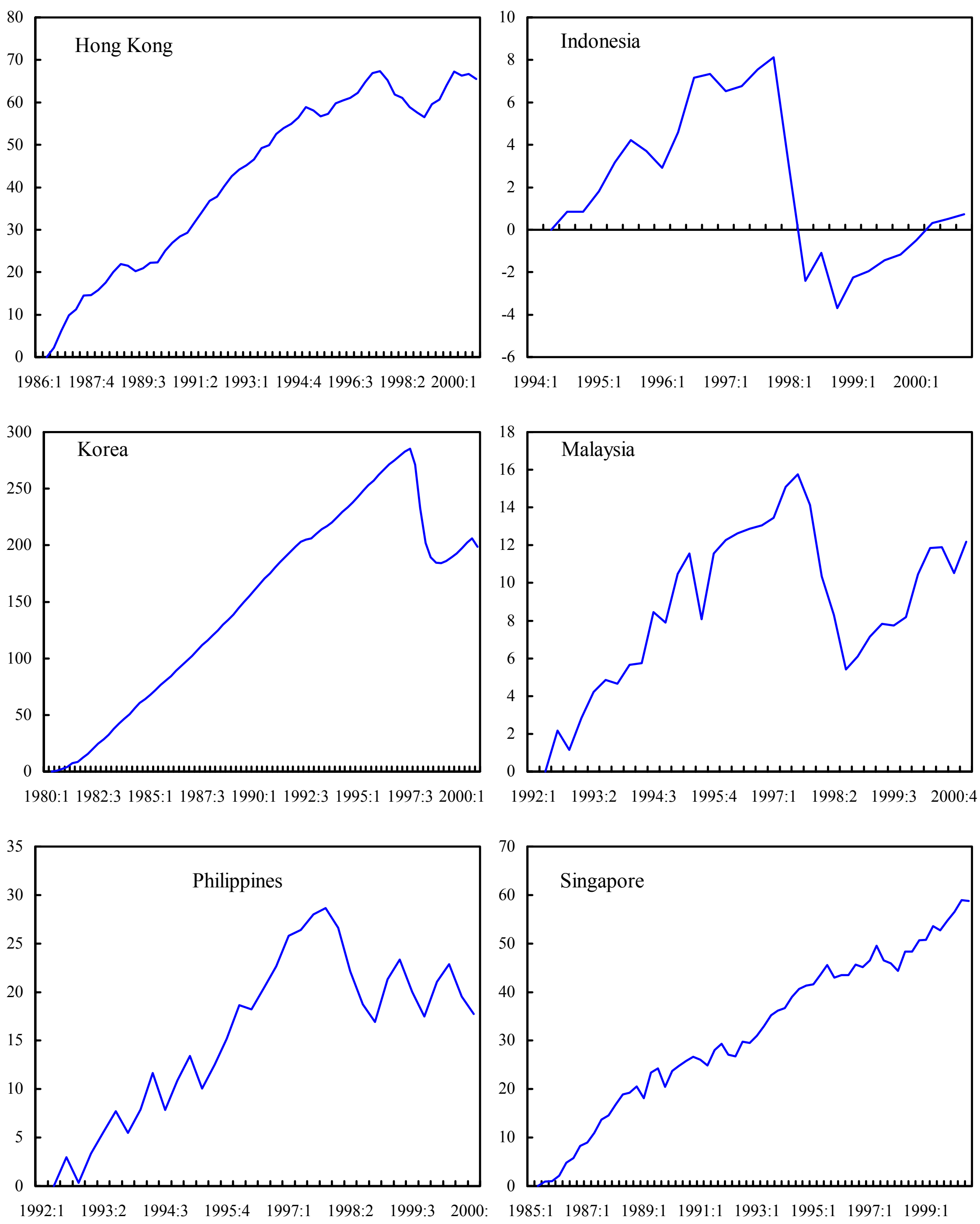
Source: Authors' calculations.

Figure 2. Probability of Temporary Recession



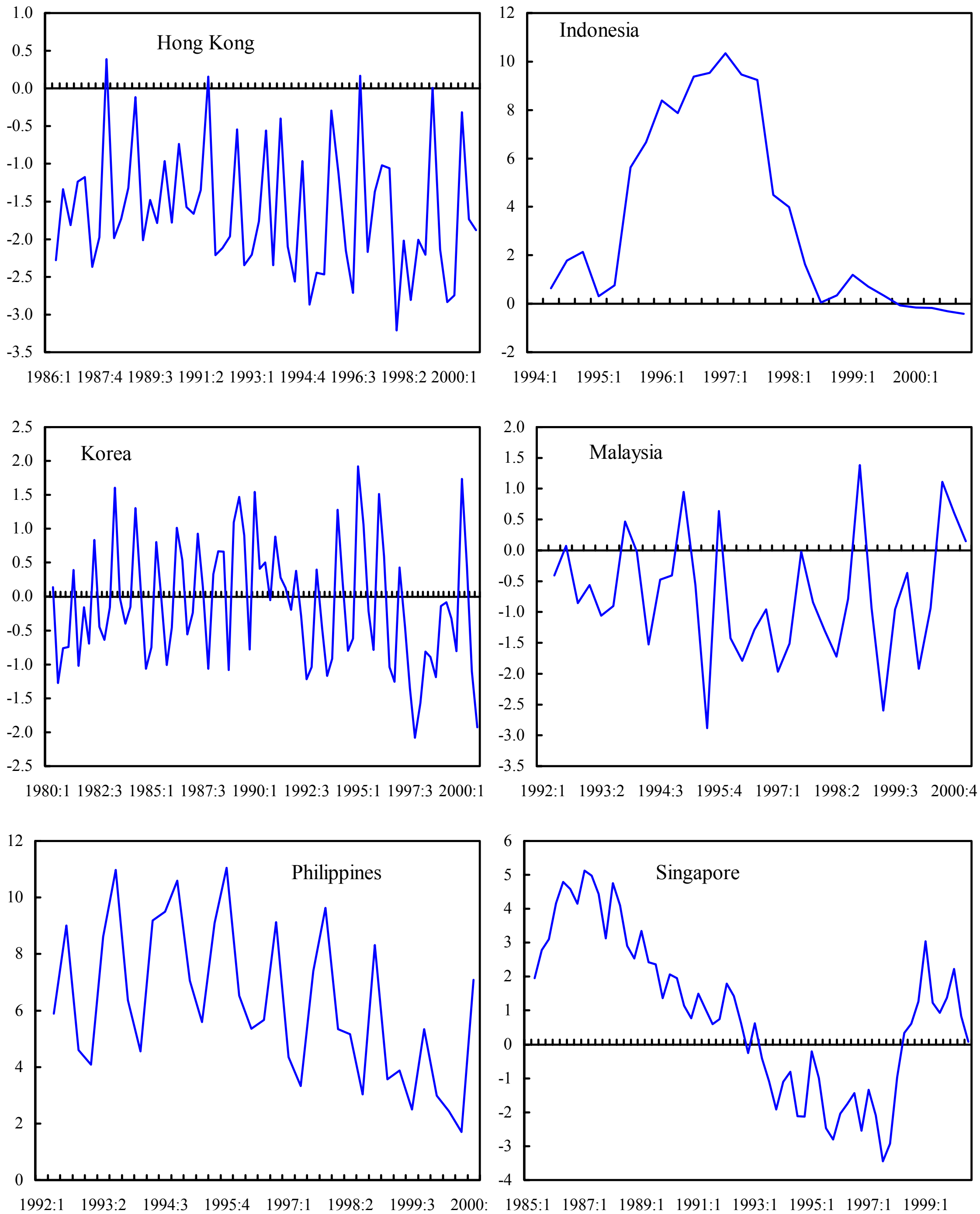
Source: Authors' Calculations

Figure 3. Common Permanent Component



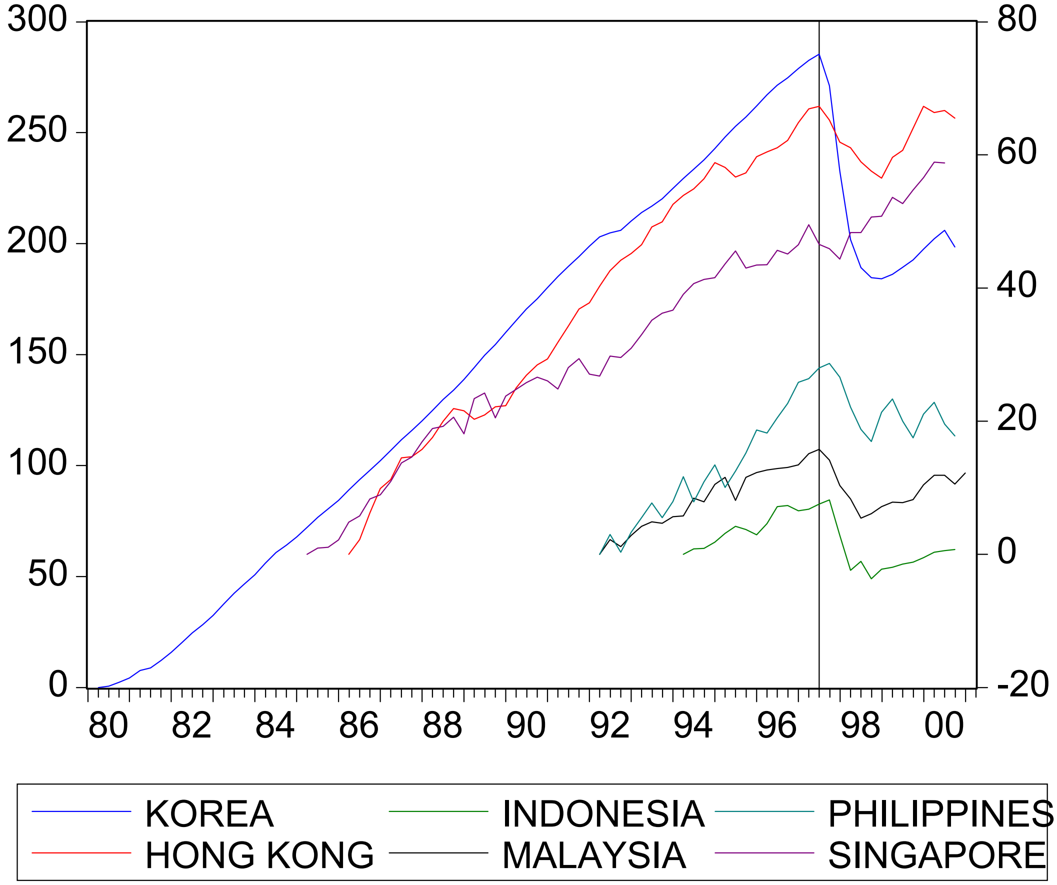
Source: Authors' Calculations

Figure 4. Common Temporary Component



Source: Authors' Calculations

Figure 5. Permanent Components



Appendix 1: Data Sources

VARIABLE	COUNTRY	SAMPLE	SOURCE
Real Gross Domestic Product (RGDP)	Hong Kong (HK)	1986:1-2000:4	WEFA
	Indonesia (IDN)	1994:1-2000:4	Buletin Statistik Bulanan (Monthly Statistical Bulletin), Indikator Ekonomi
	Korea (KOR)	1980:1-2000:4	WEFA
	Malaysia (MYS)	1992:1-2001:1	Sharan Perangkann Bulanan (Monthly Statistical Abstract), Department of Statistics
	Philippines (PHL)	1992:1-2000:4	WEFA
	Singapore (SGP)	1985:1-2000:3	WEFA
Real Gross Fixed Capital Formation (RINV)	Hong Kong (HK)	1986:1-2000:4	WEFA
	Indonesia (IDN)	1994:1-2000:4	Buletin Statistik Bulanan (Monthly Statistical Bulletin), Indikator Ekonomi
	Korea (KOR)	1980:1-2000:4	WEFA
	Malaysia (MYS)	1992:1-2001:1	Sharan Perangkann Bulanan (Monthly Statistical Abstract), Department of Statistics
	Philippines (PHL)	1992:1-2000:4	WEFA
	Singapore (SGP)	1985:1-2000:3	WEFA
Real Personal Consumption (RPCON)	Hong Kong (HK)	1986:1-2000:4	WEFA
	Indonesia (IDN)	1994:1-2000:4	Buletin Statistik Bulanan (Monthly Statistical Bulletin), Indikator Ekonomi
	Korea (KOR)	1980:1-2000:4	WEFA
	Malaysia (MYS)	1992:1-2001:1	Sharan Perangkann Bulanan (Monthly Statistical Abstract), Department of Statistics
	Philippines (PHL)	1992:1-2000:4	WEFA
	Singapore (SGP)	1985:1-2000:3	WEFA

Appendix 2: State Space Representation
This section presents the state space representation of the model discussed in Section 2.

Observation Equation:

$$\begin{bmatrix} \Delta y_t \\ \Delta i_t \\ \Delta c_t \end{bmatrix} = \begin{bmatrix} \gamma_1 & 0 & \lambda_1 & -\lambda_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \gamma_2 & 0 & \lambda_2 & -\lambda_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ \gamma_3 & 0 & \lambda_3 & -\lambda_3 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} \Delta n_t \\ \Delta n_{t-1} \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \end{bmatrix}$$

Transition Equation:

$$\begin{bmatrix} \Delta n_t \\ \Delta n_{t-1} \\ x_t \\ x_{t-1} \\ z_{1t} \\ z_{1t-1} \\ z_{2t} \\ z_{2t-1} \\ z_{3t} \\ z_{3t-1} \end{bmatrix} = \begin{bmatrix} \beta_0 + \beta_1 S_{1t} \\ 0 \\ \tau S_{2t} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi_{11} & \phi_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi_{11} & \psi_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \psi_{21} & \psi_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \psi_{31} & \psi_{32} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} \Delta n_{t-1} \\ \Delta n_{t-2} \\ x_{t-1} \\ x_{t-2} \\ z_{1t-1} \\ z_{1t-2} \\ z_{2t-1} \\ z_{2t-2} \\ z_{3t-1} \\ z_{3t-2} \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix}$$

Covariance Matrix of the Disturbance Vector:

$$E\left(\begin{bmatrix} v_t \\ 0 \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix} * \begin{bmatrix} v_t \\ 0 \\ u_t \\ 0 \\ e_{1t} \\ 0 \\ e_{2t} \\ 0 \\ e_{3t} \\ 0 \end{bmatrix}' \right) = \begin{bmatrix} \sigma_v^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{e1}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{e2}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{e3}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$