Collateral Damage and Economic Recovery*

Charles Ka Yui Leung and Nan-Kuang Chen†

This version: April 2001, Very Preliminary

Abstract

This study is motivated by observations that the economic recovery after a financial crisis is somewhat slow. We concentrate on the impact of debt deflation on the households, and more importantly, how does the shrinkage of household net worth hinder the recovery of the economy. The idea is that households are "locked in" by "nominal contracts", such as mortgages, when the value of their properties is dropping. We builds a simple dynamic general equilibrium to examine the validity of this commonly cited cause of a slow recovery. Simulstionresults indicate that ... *(To be added)*

Key words: collateral damage, economic recovery

JEL classification: D91, E30, E50, R21

*Acknowledgement: This paper is prepared for the Asian Crisis III Conference. The authors are grateful to ... for comments and Chinese University of Hong Kong Direct Grant, Hong Kong RGC Earmark Grant for financial support. The usual disclaimer applies.

†Correspondence. Leung: Department of Economics, Chinese University of Hong Kong, Shatin, Hong Kong, (Phone) 2609-7049, 2609-7158; (Fax) 2603-5805; (E mail) charlesl@cuhk.edu.hk. Chen: Department of Economics, National Taiwan University, 21 Hsu Chow Road, Taipei 10020, Taiwan, (Email) nankuang@ccms.ntu.edu.tw.

1 Introduction

This study is motivated by observations that the economic recovery after a financial crisis is somewhat slow. Japan is a classic example. A decade after the bubble burst, Japan is still having difficult to reclimb to the higher growth era.[1] It has been suggested that the highly leveraged credit market plays an important role. This paper takes this position seriously and builds a simple dynamic general equilibrium to examine the validity of this commonly cited cause of a slow recovery.

Much literature has devoted to the understanding of debt deflation on the borrowing capacity and thus investment of firms and corporations, which is broadly referred to the balance sheet effect. *(citations to be added)* In this paper we concentrate on the impact of debt deflation on the households, and more importantly, how does the shrinkage of household net worth hinder the recovery of the economy. The idea is that households are "locked in" by "nominal contracts", such as mortgages, when the value of their properties is dropping. As a negative shock sets in, households are earning lower income while obligating the same (nominal) mortgage payments. More importantly, the slump of real estate prices may render the liabilities of mortgage payments in excess of market value of the property, leaving the households with negative net worth. This furthermore restricts households' consumption, investment and financing capability. Even more, households may be voluntary to cease on mortgage payments, or may be forced to default. When more houses are seized by their financiers and auctioned off in the market, this further depresses housing prices. Therefore, recession is prolonged and recovery is delayed.

A sketch of the model is following. Consider an overlapping generations model with three-period-lived households/producers. There are two goods – a non-durable consumption good and a durable residential property. At the first period of a household's life, she works, purchases a housing with a down payment, and invests. In the middle-aged period, she pays off the mortgage payment (durable consumption), if affordable. When old, the household sells off the property and consumes.

When a negative exogenous shock hits, the old generation simply sells off their properties at whatever the value is, consumes, and exits from the economy. The young may be

[1] For instance, see Krugman (1998) and the reference therein.

able to purchase housing at a lower price, but she also earns a lower income and investment returns, and lower expected future housing value. The behavior of the middle-aged is crucial. If they expect that the property value next period is going to be very low, they may choose to default on mortgage payment (In case there is credit constraint, they may be forced to default). As more houses are auctioned off by their financiers, the property value is further depressed.

This paper tries to explain why a high "real" growth may not reflect the true state of the economy when there is a deflation, particularly, debt deflation.[2] We guess that may be economists are accustomed to think of economic performance in real terms. The paper tries to argue that different measures of economic performance may be desirable under different economic environments. In the paper when the dynamic paths can not be explicitly characterized, simulations will be called for to understand the impulse response of an exogenous shock. *(More)*

A paper that is close to ours is Schneider and Tornell (2000). They distinguish two sectors, Tradable (T) and Non-tradable (N) sectors. A benefit of distinguish these two sectors is that the real exchange rate is the ratio of prices of the two sectors. Since foreign bonds are denominated in T goods, a real depreciated (or relative price change) causes a twin crisis. They show that, after the shock, the N-sector recovers slower than the T-sector due to the balance sheet effect.

The organization for the rest of the paper is as follows. Section 2 outlines a benchmark model in which credit markets are perfect. Section 3 examines the effects of a productivity shock and an interest rate shock. The model is simulated to see the quantitative properties of a exogenous shock. Section 4 extends the basic model to an imperfect credit market version. Section 5 concludes.

[2] A recent observation can be found in Kent and Lowe (1998). They compare the two asset price busts in the early 1970s and late 1980s in Australia. They find that the percentages of asset prices drop in real terms were approximately the same for both cases. However, since the inflation rate in the late 1980s was much lower than that in the 1970s, the percentage of price drop in nominal terms of the former was three times larger than the latter, and thus the subsequent recession was far more severe and persistent in the 1980s.

2 A Baseline Model

Time is discrete and the horizon is infinite. There is a small open economy which takes the world interest rate R_t as given. This artificial economy is populated by a sequence of three-period lived, overlapping generations. The population of each cohort is constant and is normalized to unity. There are two goods – a consumption good that is not durable, and the durable residential property. Agents are endowed with one unit of labor when they are young. We adopt the formulation from Greenwood and Hercowitz (1991) The total stock of residential property is assumed fixed in supply, denoted as $2H$. (to be added)

At time t, $t = 0, 1, 2, ...$, the agent maximizes life-time utility (Problem P)

$$\max .\beta \frac{\left(C_{t+1}^{m}\right)^{1-\sigma}}{1-\sigma} + \beta^2 \frac{\left(C_{t+2}^{o}\right)^{1-\sigma}}{1-\sigma} + \gamma \frac{\left(h_t^y\right)^{1-\sigma}}{1-\sigma} + \gamma\beta \frac{\left(h_{t+1}^{m}\right)^{1-\sigma}}{1-\sigma}$$

$$\text{s.t. } W_t l_t \geq \psi Q_t h_t^y + k_{t+1}^m, \tag{1}$$

$$R_{t+1} k_{t+1}^m \geq C_{t+1}^m + \left[(1-\psi) Q_t h_t^y R_{t+1} + \psi Q_{t+1} h_{t+1}^m - Q_{t+1}(1-\delta) h_t^y\right] + k_{t+2}^o, \tag{2}$$

$$R_{t+2} k_{t+2}^o + Q_{t+2}(1-\delta) h_{t+1}^m \geq C_{t+2}^o + (1-\psi) Q_{t+1} h_{t+1}^m R_{t+2}, \tag{3}$$

where (1), (2) and (3) are the budget constraints for the first, second and third period of their lives respectively. Equation (1) says that the only source of income in the first period of life comes from the wage bill, and it will finance both the downpayment $\psi Q_t h_t^y$ for h_t^y units of housing, and the saving k_{t+1}^m. In the second period of life, as equation (2) describes, the agent will receive interest income $R_{t+1} k_{t+1}^m$, and spend it over non-durable consumption C_{t+1}^m, paying the full mortgage debt $(1-\psi) Q_t h_t^y R_{t+1}$, the downpayment for new housing units $\psi Q_{t+1} h_{t+1}^m$ net of the revenue from selling the old housing $-Q_{t+1}(1-\delta) h_t^y$, and the remaining part will be saved k_{t+2}^o. Equation (3) formulate the budget constraint for the last period of life. The agent recevie income from selling the housing $Q_{t+2}(1-\delta) h_{t+1}^m$ as well as from saving $R_{t+2}k^o_{t+2}$, in order to finance the expenditure on non-durable consumption C^o_{t+2} and also the remaining mortgage debt $(1-\psi)Q_{t+1}h^m_{t+1}R_{t+2}$. Neither debt nor bequest will be left. Some implicit assumptions have been made in these equations. First, the international capital market is "efficient": the agents can borrow from the international capital market for domestic mortgage lending. Second, in (2) and (3), is the assumption that a fraction δ of the housing units will depreciate and the owners will provide repair and maintainance so that the physical unit to be sold out is still h^y_t at unit price Q_{t+1} at time $t+1$ (h^m_{t+1} at unit price Q_{t+2} at time $t+2$) whereas the actually amount that the owners will receive, after paying the repair and maintainance, is only $Q_{t+1}(1-\delta)h^y_t$ $(Q_{t+1}(1-\delta)h^m_t)$. For simplicity, we further assume that the labor is supplied inelastically and by normalization, thus $l_t = 1$, $\forall t$. It is assumed that the utility function is well behaved, $\beta, \gamma, \sigma > 0$. Notice that the agent pays only a fraction ψ, $0 \le \psi \le 1$, of the total value of the housing unit in the fraction when he/she purchases the unit in the first period. Yet when the agent sells it of in the third period, the agent will get full payment from the bank of the buyer within the same period. Implicitly, a costless intermediation technology is assumed. We also assume that the consumption, and investment on housing in all periods are non-negative, $\left(C^m_{t+1}, C^o_{t+2}, h^y_t, h^m_{t+1}\right) \ge 0$. However, we allow for k^m_{t+1}, k^o_{t+2} to be negative. The intrepretation is clear. If k^o_{t+2} (k^m_{t+1}) is negative, it means that the middle-aged (old aged) agents are borrowing from the capital and since there is no risk involved, the interest rate for such borrowing is equalized to the rate of return from capital investment. For future reference, we use the a shorthand to represent all these variables, $\Theta_t = \left\{C^m_{t+1},\ C^o_{t+2},\ k^m_{t+1},\ k^o_{t+2},\ h^y_t, h^m_{t+1}\right\}$. For simplicity, this paper will take the usual small open economy assumption that the interest rate is constant over time, $R_t = R$. In the appendix, it is shown that Θ_t can all be solved as functions of parameters and prices, $\mathbb{P}_t \equiv \{W_t, Q_t, Q_{t+1}, Q_{t+2}\}$.

The production side is very simple. There is an aggregate production function which combines capital and labor and convert it into output,

$$Y_t = A_t (K_t)^{\alpha} (L_t)^{1-\alpha}, \tag{4}$$

where $0 < \alpha < 1$, and A_t is the technological coefficient or "productivity" at period t. Alternatively, we can interpret the artificial economy as a small open economy, which produces only to export. In that case, the term A also reflects the real exchange rate. The input market is competitive, and therefore the input prices are simply the marginal product of the corresponding inputs,

$$R = \partial Y_t / \partial K_t,\ W_t = \partial Y_t / \partial L_t. \tag{5}$$

At the equilibrium, several market clearing conditions are to be met. First, labor supplied by young is fully utilized. Second, the capital in production are jointly supplied by domestic middle, domestic old agents and investment (from or to) foreign countries (k_t^f). Third, the output are either consumed or invested. Formally, it means that

$$L_t = l_t = 1, \tag{6}$$

$$K_t = k_t^m + k_t^o + k_t^f, \tag{7}$$

$$Y_t = C_t^m + C_t^o + k_{t+1}^m + k_{t+1}^o. \tag{8}$$

Implicitly in (7), we assume that investment from foreign countries (if $k_t^f > 0$) will contribute to goods production totally symmetric to domestic investors, and their existence ensure that the return of capital in the domestic market is equal to the world capital market. When domestic agents invest in foreign countries, i.e. $k_t^f < 0$, we assume that they earn exactly the same return as they would in the domestic market. Finally, the housing market should also clears, which means that the amount of houses being sold and being bought are the same, and equals to the total stock,

$$h_t^y + h_t^m = h_{t-1}^y + h_{t-2}^m,\ \forall t. \tag{9}$$

Here is our solution strategy. We first characterize the steady state. And then we examine whether the existence and uniqueness of steady state. We will then study the impact on the economy if there is an temporary unexpected shock in technology or policy.

Clearly, if $A_t = A$, i.e. when there is no fluctuation in productivity, at the steady state, all choice variables and price variables are invariant of time, $\Theta_t = \Theta^* = \{C^{m*},\ C^{o*},$ $k^{m*},\ k^{o*},\ h^{y*},\ h^{m*}\}$ and $\mathbb{P}_t = \mathbb{P}^* = (W^*,\ Q^*,\ Q^*,\ Q^*)$.

3 Shocks

In this section, we consider different shocks to the system and examine how would the economy responds. Assume that the economy is originally at the steady state. Then some shocks hits the economy unexpectedly. We will focus on how the aggregate variables change. The first shock being study is a temporary productivity shock.

3.1 Temporary Productivity Shock

Consider the following scenerio. At the *beginning of* time $T-1$, the economy is informed that the productivity A at Time T will go down a lower level $\underline{A}$, $\underline{A} < A$, and the productivity will be back to the "normal level" (or the steady state level) A in subsequent periods. Since the foreign interest rate R is unchanged, the capital stock must adjust. In particular, The capital stock and hence the wage at time T will be lower than the steady state level,

$$K_T = \left(\frac{\alpha \underline{A}}{R}\right)^{1/(1-\alpha)} < \left(\frac{\alpha A}{R}\right)^{1/(1-\alpha)} = K^*, \tag{10}$$

$$W_T = (\underline{A})^{1/(1-\alpha)}(1-\alpha)(\alpha/R)^{\alpha/(1-\alpha)} < W^*. \tag{11}$$

By definition, the capital stock at time T is pre-determined by the investment in earlier period. In particular, we know that

$$K_T = k_T^m + k_T^o + k_T^f,$$

where $k_T^m = W_{T-1} - \psi Q_{T-1} h_{T-1}$ and $k_T^o = R k_{T-1}^m - C_{T-1}^m - \left[(1-\psi)R_{T-1}Q^* h^{y*} + \psi Q_{T-1} h_{T-1}^m\right]$ $+ Q_{T-1}(1-\delta)h_{T-2}^{y*}$, since $Q_{T-2} = Q^*$, $h_{T-2}^y = h^{y*}$. From period $T+1$, however, the productivity and hence the wage will return to the "normal level,"

$$A_{T+i} = A, \;\; K_{T+i} = K^*, \tag{12}$$

$i = 1, 2, \ldots$ It implies that

$$k_{T+i}^o + k_{T+i}^m + k_{T+i}^f = K_{T+i} = K^*. \tag{13}$$

To completely describe the dynamics, it is necessary to describe the consumption and investment choices of different cohorts. For those who were born at $T-3$, they become old-aged at time $T-1$, and simply consume when they get from selling housing stock and capital income,

$$\begin{aligned} C^o_{T-1} &= Rk^o_{T-1} + Q_{T-1}(1-\delta)h^m_{T-2} - (1-\psi)RQ_{T-2}h^m_{T-2} \\ &= Rk^{o*} + h^{m*}\left[(1-\delta)Q_{T-1} - (1-\psi)RQ^*\right], \end{aligned}$$

as all the investment made in previous period cannot be changed. The consumption of the old agents are changed only through the change of the re-sale value of their houses.

For those who were born at $T-2$, they have purchased housing stock at period $T-2$ and become middle-aged at time $T-1$. Therefore, they face a "truncated" Problem P:

$$\max.\beta\left[\frac{\left(C^m_{T-1}\right)^{1-\sigma}}{1-\sigma} + \beta\frac{\left(C^o_T\right)^{1-\sigma}}{1-\sigma} + \gamma\frac{\left(h^m_{T-1}\right)^{1-\sigma}}{1-\sigma}\right]$$

$$\text{s.t. } Rk^{m*} \geq C^m_{T-1} + \left[(1-\psi)Q^*h^{y*}R + \psi Q_{T-1}h^m_{T-1} - Q_{T-1}(1-\delta)h^{y*}\right] + k^o_T, \tag{14}$$

$$Rk^o_T + Q_T(1-\delta)h^m_{T-1} \geq C^o_T + (1-\psi)Q_{T-1}h^m_{T-1}R, \tag{15}$$

taking k^m_{T-1} and h^y_{T-2} as "initial conditions," and the fact that the variables at time $T-2$ is at $Q_{T-2} = Q^*$, $k^m_{T-1} = k^{m*}$, $h^y_{T-2} = h^{y*}$. While the details of the analysis is provided in the appendix, some observations can be made heuristically. Notice that the expenditure $(1-\psi)Q^*h^{y*}R$ is precommitted. And if the anticipated productivity shock leads to a drop of housing price in the current period, it means that the income from re-sale $Q_{T-1}(1-\delta)h^{y*}$ will decrease. It might lead to a drop in current consumption, future residential expenditure and investment, which would affect future non-durable consumption as well.

For those who were born at time $T-1$, they are aware that the wage willbe depressed at time T, but their own wages will not be affected. On the other hand, they need to take the housing price deviations (from the steady state) in time $T-1$ and following periods into consideration. They also solve Problem P. For those who were born at time T, they also solve Problem P, except that the steady state wage rate W^* is not replaced by a lower wage level, W_T. And for those who were later than that, they also solve Problem P, taking into consideration of the change in housing prices.

Since we are unable to solve the transition path analytically, we will follow the approach of Cooley and Ohanian (1997), Ohanian (1997) to solve the dynamics numerically.[3]

(Simulation results: To be added)

3.2 Temporary Interest Rate Shock

In this section, we will consider the interest rate shock and examine how would the economy responds. Assume that the economy is originally at the steady state. A temporary interest rate shock hits the economy unexpectedly. The aggregate variables will hence change. Formally, the economy is already at the steady state before period T. At time $T-1$, the economy is informed that the interest rate R goes up a higher level $\overline{R}$, $\overline{R} > R$ at time T, and the interest rate will be back to the normal level R in subsequent periods. Since the productivity A is unchanged, the capital stock must adjust. In particular, The capital stock and hence the wage at time T will be lower than the steady state level,

$$K_T = \left(\frac{\alpha A}{\overline{R}}\right)^{1/(1-\alpha)} < \left(\frac{\alpha A}{R}\right)^{1/(1-\alpha)} = K^*, \tag{16}$$

$$W_T = (1-\alpha)\alpha^{\alpha/(1-\alpha)}(A)^{1/(1-\alpha)}(\overline{R})^{-\alpha/(1-\alpha)} < W^*. \tag{17}$$

By definition, the capital stock at time T is pre-determined by the investment in earlier period. In particular, we know that

$$K_T = k_T^m + k_T^o + k_T^f,$$

where $k_T^m = W^* - \psi Q_{T-1} h_{T-1}^y$ and $k_T^o = R k_{T-1}^m - C_{T-1}^m - \left[(1-\psi) R Q^* h^{y*} + \psi Q_{T-1} h_{T-1}^m\right]$ $+ Q_{T-1}(1-\delta) h_{T-2}^{y*}$, since $W_{T-1} = W^*$, $Q_{T-2} = Q^*$, $h_{T-2}^y = h^{y*}$. To completely describe

[3] A merit of this approach is that it can solve a pretty board class of dynamic model and minimize the accurary loss due to approximation. The parameter values imposed will be based on the survey paper by Cooley and Prescott (1995). The details will be explained in the appendix.

the transition of the artifical economy, it is necessary to describe the consumption and investment choices of different cohorts. For those who were born at $T-3$, they become old-aged at time $T-1$, and simply consume when they get from selling housing stock and capital income,

$$\begin{aligned} C^o_{T-1} &= Rk^o_{T-1} + Q_{T-1}(1-\delta)h^m_{T-2} - (1-\psi)RQ_{T-2}h^m_{T-2} \\ &= Rk^{o*} + h^{m*}\left[(1-\delta)Q_{T-1} - (1-\psi)RQ^*\right], \end{aligned}$$

The consumption of the old agents are changed only through the change of the re-sale value of their houses.

For those who were born at $T-2$, they have purchased housing stock at period $T-2$ and become middle-aged at time $T-1$. Therefore, they face a "truncated" Problem P:

$$\max .\beta\left[\frac{\left(C^m_{T-1}\right)^{1-\sigma}}{1-\sigma} + \beta\frac{\left(C^o_T\right)^{1-\sigma}}{1-\sigma} + \gamma\frac{\left(h^m_{T-1}\right)^{1-\sigma}}{1-\sigma}\right]$$

$$\text{s.t. } Rk^{m*} \geq C^m_{T-1} + \left[(1-\psi)Q^*h^{y*}R + \psi Q_{T-1}h^m_{T-1} - Q_{T-1}(1-\delta)h^{y*}\right] + k^o_T, \tag{18}$$

$$\overline{R}k^o_T + Q_T(1-\delta)h^m_{T-1} \geq C^o_T + (1-\psi)Q_{T-1}h^m_{T-1}\overline{R}, \tag{19}$$

taking k^m_{T-1} and h^y_{T-2} as "initial conditions," and the fact that the variables at time $T-2$ is at $Q_{T-2} = Q^*$, $k^m_{T-1} = k^{m*}$, $h^y_{T-2} = h^{y*}$. While the details of the analysis is provided in the appendix, some observations can be made heuristically. Notice that the expenditure $(1-\psi)Q^*h^{y*}R$ is precommitted. And if the anticipated productivity shock leads to a drop of housing price in the current period, it means that the income from re-sale $Q_{T-1}(1-\delta)h^{y*}$ will decrease. It might lead to a drop in current consumption, future residential expenditure and investment, which would affect future non-durable consumption as well.

For those who were born at time $T-1$ and T, they are aware that the interest rate will rise at time T. They also need to take the housing price deviations (from the steady state) in time $T-1$ and following periods into consideration. They solve Problem P. For those who were born at time $T+1$ or later, they also solve Problem P, except that the interest rate restores to its steady state level R, rather than $\overline{R}$.

(Simulation results: To be added)

4 Extension: Capital Market Imperfection

In the earlier section, we have assumed that the representative agent is subject to the collateral constraint and yet they are allowed to freely borrow for consumption and investment at the world interest rate. Also, the agent is not allowed to adjust the amount of the residential investment they made in the frist period of life. In this section, we will realx these assumptions and examine how the aggregate implications will change. To achieve this, the model will be modified as follows. First, we appeal to the imperfect enforceability of fianncial contracts considered by Kiyotaki and Moore (1997). The credit constraints of a household when young and middle-aged respectively are given by

$$\begin{aligned} b_{t+1}^m &\leq Q_{t+1}^e h_t, \\ b_{t+2}^o &\leq Q_{t+2}^e h_t, \end{aligned} \tag{20}$$

where Q_{t+1}^e is the expected date $t+1$ housing price given information at date t. The representative household then maximizes his life-time utility

$$\max .\beta\frac{\left(C_{t+1}^m\right)^{1-\sigma}}{1-\sigma}+\beta^2\frac{\left(C_{t+2}^o\right)^{1-\sigma}}{1-\sigma}+\beta\frac{(h_t)^{1-\sigma}}{1-\sigma}+\gamma\beta\frac{(h_{t+1})^{1-\sigma}}{1-\sigma}$$

$$\text{s.t. } W_t l_t + b_{t+1}^m \geq \psi Q_t h_t + k_{t+1}^m, \tag{21}$$

$$R_{t+1}k_{t+1}^m + b_{t+2}^o \geq R_{t+1}b_{t+1}^m + C_{t+1}^m + (1-\psi)R_{t+1}Q_t h_t + k_{t+2}^o, \tag{22}$$

$$R_{t+2}k_{t+2}^o + (1-\delta)Q_{t+2}h_t \geq C_{t+2}^o + R_{t+2}b_{t+2}^o, \tag{23}$$

and b_{t+1}^m, b_{t+2}^o, k_{t+1}^m, $k_{t+2}^o \geq 0$.

5 Conclusion

(To be added)

References

[1] Capie, Forrest and Geoffrey E. Wood (1997), "Introduction: Asset Prices and the Real Economy," in Forrest Capie and Geoffrey E. Wood, eds., **Asset Prices and the Real Economy**, New York: St. Martin's Press.

[2] Caplin, Andrew, Charles Freeman and Joseph Tracy (1997), "Collateral Damage: Refinancing Constraints and Regional Recessions," *Journal of Money, Credit & Banking*, 29(4), 496-516.

[3] Carlstrom, C. T. and T. S. Fuerst (1997), "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis," *American Economic Review*, 87, 893-910.

[4] Cooley, Thomas and Lee Ohanian (1997), "Postwar British Economic Growth and the Legacy of Keynes," *Journal of Political Economy*; 105(3), 439-72.

[5] Cooley, Thomas and Edward Prescott (1995), "Economic growth and business cycles," in Thomas Cooley ed., **Frontiers of Business Cycle Research**, Princeton: Princeton University Press.

[6] Greenwood, J. and Z. Hercowitz (1991), "The Allocation of Capital and Time over the Business Cycle," *Journal of Political Economy*, 99, 1188-1214.

[7] Higgins, M. and C. Osler (1997), "Asset Market Hangovers and Economic Growth: the OECD During 1984-93," *Oxford Review of Economic Policy*, Vol.13, 110-134.

[8] Kent, C. and P. Lowe (1998), "Property-price Cycles and Monetary Policy," in **The Role of Asset Prices in the Formulation of Monetary Policy**, BIS, 239-263.

[9] Ludvigson, Sydney and Charles Steindel (1999), "How Important Is the Stock Market Effect on Consumption?" *Economic Policy Review, Federal Reserve Bank of New York*, 5(2), 29-52.

[10] Ohanian, Lee (1997), "The Macroeconomic Effects of War Finance in the United States: World War II and the Korean War," *American Economic Review*; 87(1), 23-40.

[11] Peek, J. and Eric S. Rosengren (1992), "Crunching the Recovery: Bank Capital and the Role of Bank Credit," in **Real Estate and the Credit Crunch**, eds. by Lynn E. Brown and Eric S. Rosengren, FRB of Boston, Conference Series No.36.

[12] Renaud, Bertrand (1997), "The 1985 to 1994 Global Real Estate Cycle: An Overview," *Journal of Real Estate Literature*, 5(1), 13-44.

[13] Schneider, Martin and Aaron Tornell (2000), "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *NBER Working Paper 8060*.

Appendix

(**NOT** for publication; available upon request)

A Calibration

The rest of the calibration procedure is standard and the parameter values are summarized in the following table:

Parameter	Numerical Value	Source
δ	10%	
A	1	(normalization)
α	0.33	Cooley and Prescott (1995)
β	0.7, 0.8, 0.9	
(σ, γ)	(2.0,0.5), (0.5,2.0)	(various)
R	1.04	

The absolute level of A is important probably well known. Here, we follow the literature to normalize it as unity. We choose $\alpha = 0.33$ so that the labor share of aggregate income is 1/3, and it is within the range of the calculation of Cooley and Prescott (1995). We do not know the value of β and therefore we simply experiment diff values of them. There is no concensus about the depreciation rate of housing capital, δ and we choose 10%, which is in line with most of the studies. R is set at 1.04 to reflect 4% real interest rate a year. We find that the values of σ and γ cannot be separately identified. Our target for calibration is that (1) the relative value of housing capital relative to the business capital, $Q^*(h^{y*} + h^{m*})/K$ is in between unity and ten (this statistics seems to vary across countries), and (2) the ratio of aggregate consumption relative to the aggregate output $(C^{m*} + C^{o*})/Y$ is close to 60%. We also learn from Cooley and Prescott (1995) that the reasonable range of σ is that $1/2 \leq \sigma \leq 10$, and from Greenwood and Hercowitz (1991) that the reasonable range for γ is that $1/2 \leq \gamma \leq 2$. We have experienced many different combinations of (σ, γ) and find that the two combinations which gives the most reasonable outcomes are (2.0,0.5), (0.5,2.0).

As it is well known, there is no general procedures to compute transitional dynamics. The computation procedures here follow Cooley and Ohanian (1997), Ohanian (1997). First, we assume that the economy is at the steady state (period T). Then the shock comes and we assume that after T^* periods the economy will be restored to the steady state (period $T+T^*$). We write down all the first order conditions and market clearing in between $T-1$ and $T+T^*$. And then we solve them as a system of simultaneous equations. We can then adjust T^* to examine the sensitivity of the results.

B Proofs

B.1 Solve for $(C^m_{t+1}, C^o_{t+2}, k^m_{t+1}, k^o_{t+2}, h^y_t, l_t)$ in terms of $\mathbb{P}_t$ (baseline case)

We intend to solve for the optimal choices for an individual, taking prices, $\mathbb{P}_t = (W_t, Q_t, Q_{t+1}, Q_{t+2})$, as given. Let λ_{1t}, λ_{2t}, λ_{3t} be the Lagrange multipliers for the constraints (1), (2) and (3) respectively. Furthermore, we assume that $C^m_{t+1} \geq 0$, $C^o_{t+2} \geq 0$, $h_t \geq 0$, $k^m_{t+1} \neq 0$, $k^o_{t+2} \neq 0$ respectively. Imposing the constant interest rate condition, the first order conditions are

$$\begin{aligned}
\beta(C^m_{t+1})^{-\sigma} &= \lambda_{2t}, \\
\beta^2(C^o_{t+2})^{-\sigma} &= \lambda_{3t}, \\
\gamma(h^y_t)^{-\sigma} + \lambda_{2t}(1-\delta)Q_{t+1} &= Q_t\left[\psi\lambda_{1t} + (1-\psi)\lambda_{2t}R\right], \\
\gamma\beta(h^m_{t+1})^{-\sigma} + \lambda_{3t}(1-\delta)Q_{t+2} &= Q_{t+1}\left[\psi\lambda_{2t} + (1-\psi)\lambda_{3t}R\right], \\
\lambda_{2t}R &= \lambda_{1t}, \\
\lambda_{3t}R &= \lambda_{2t}.
\end{aligned}$$

The system of equations can be reduced to

$$C^o_{t+2} = (\beta R)^{1/\sigma} C^m_{t+1}, \tag{24}$$

$$h^y_t = \left(C^m_{t+1}\right)\left[\frac{\gamma}{\beta\left(RQ_t - (1-\delta)Q_{t+1}\right)}\right]^{1/\sigma}, \tag{25}$$

$$h^m_{t+1} = \left(C^o_{t+2}\right)\left[\frac{\gamma}{\beta\left(RQ_{t+1}-(1-\delta)Q_{t+2}\right)}\right]^{1/\sigma}, \tag{26}$$

Now combining (1), (2) and (3), we get

$$R^2W_t = RC^m_{t+1} + C^o_{t+2} + Rh^y_t\left[Q_tR-(1-\delta)Q_{t+1}\right] + h^m_t\left[Q_{t+1}R-(1-\delta)Q_{t+2}\right]. \tag{27}$$

Substituting (24), (25) and (26) into it, we get

$$C^m_{t+1} = \frac{R^2W_t}{\left[R+(\beta R)^{1/\sigma}\right]+\Psi(Q_t,Q_{t+1},Q_{t+2})}, \tag{28}$$

where $\Psi(Q_t,Q_{t+1},Q_{t+2}) = (\gamma/\beta)^{1/\sigma}\Big\{R\left[RQ_t-(1-\delta)Q_{t+1}\right]^{-(1-\sigma)/\sigma}$
$+(\beta R)^{1/\sigma}\left[RQ_{t+1}-(1-\delta)Q_{t+2}\right]^{-(1-\sigma)/\sigma}\Big\}$.

Notice that C^m_{t+1} is now only in terms of parameters and prices. Substituting (28) back to (24), (25), and (26), we can solve for C^o_{t+2}, h^y_t, h^m_{t+1} respectively. We can deduce k^m_{t+1} and k^o_{t+2} by these results with the help of (2) and (3) and the market clearing conditions.

Starting with the factor markets, we know that $R = \partial Y_t/\partial K_t$, $W_t = \partial Y_t/\partial L_t$, which means that

$$K_t = \left(\frac{\alpha A_t}{R}\right)^{1/(1-\alpha)}, \tag{29}$$

and

$$W_t = (1-\alpha)\left(\frac{\alpha}{R}\right)^{\alpha/(1-\alpha)}(A_t)^{1/(1-\alpha)}. \tag{30}$$

Recall that at the equilibrium, the investments are pinned down by the budget constraints. Changing the time indices of (1) and (2), it delivers

$$k^m_t = W_{t-1} - \psi Q_{t-1}h^y_{t-1}, \tag{31}$$

$$k^o_t = Rk^m_{t-1} - C^m_{t-1} - \left[(1-\psi)R_{t-1}Q_{t-2}h^y_{t-2} + \psi Q_{t-1}h^m_{t-1}\right] + Q_{t-1}(1-\delta)h^y_{t-2}. \tag{32}$$

Notice that combining (31) and (32) we get $k^o_t = RW_{t-2} - C^m_{t-1} + \left[Q_{t-1}(1-\delta) - R_{t-1}Q_{t-2}\right]h^y_{t-2} - \psi Q_{t-1}h^m_{t-1}$.

Now the capital market equilibrium condition (7) imposes that

$$K_t = k_t^m + k_t^o + k_t^f,$$

where K_t is given by (29), and k_t^m by (31), k_t^o by (32).

B.2 Steady State Characterization (baseline case)

At the steady state, $\Theta_t = \Theta^* = (C^{m*}, C^{o*}, k^{m*}, k^{o*}, h^{y*}, h^{m*})$, $\mathbb{P}_t = \mathbb{P}^* = (W^*, Q^*, Q^*, Q^*)$, where by (30),

$$W^* = (1-\alpha)\left(\frac{\alpha}{R}\right)^{\alpha/(1-\alpha)} (A)^{1/(1-\alpha)}, \tag{33}$$

which is determined by world interest rate and the productivity level. And by (29),

$$K^* = \left(\frac{\alpha A}{R}\right)^{1/(1-\alpha)}. \tag{34}$$

We assume the solution to be interior. In other words, we proceed with the assumption that $\Theta^* \neq (0,0,0,0,0)$. In particular, $\Psi(Q_t, Q_{t+1}, Q_{t+2}) = (\gamma/\beta)^{1/\sigma}\left\{R\left[RQ_t - (1-\delta)Q_{t+1}\right]^{-(1-\sigma)/\sigma} + (\beta R)^{1/\sigma}\left[RQ_{t+1} - (1-\delta)Q_{t+2}\right]^{-(1-\sigma)/\sigma}\right\} = (\gamma/\beta)^{1/\sigma}\left[Q^*\left(R-(1-\delta)\right)\right]^{-(1-\sigma)/\sigma}\left[R + (\beta R)^{1/\sigma}\right]$, and (28) is reduced to

$$(C^{m*}) = \frac{R^2 W^*}{\left[R + (\beta R)^{1/\sigma}\right]\left\{1 + (\gamma/\beta)^{1/\sigma}\left[Q^*\left(R-(1-\delta)\right)\right]^{-(1-\sigma)/\sigma}\right\}}. \tag{35}$$

And (24), (25) and (26), become

$$C^{o*} = (\beta R)^{1/\sigma} C^{m*}, \tag{36}$$

$$h^{y*} = (C^{m*})\left[\frac{\gamma}{\beta Q^*\left(R-(1-\delta)\right)}\right]^{1/\sigma}, \tag{37}$$

$$h^{m*} = (C^{o*})\left[\frac{\gamma}{\beta Q^*\left(R-(1-\delta)\right)}\right]^{1/\sigma}. \tag{38}$$

In other words,

$$h^{y*}/h^{m*} = C^{m*}/C^{o*} = (\beta R)^{-1/\sigma},$$

by (36). Also, notice that (9) is automatically satisfied in the steady state.

Now we need to compute the steady state capital holdings. We know that at the steady state, there shall be no capital inflow, $k^{f*} = 0$. To compute k^{m*}, we combine (35), (38) with (31) and get

$$k^{m*} = W^* - \psi (Q^*)^{-(1-\sigma)/\sigma} \left[\frac{\gamma}{\beta (R - (1-\delta))}\right]^{1/\sigma} (C^{m*}), \tag{39}$$

where C^{m*} is determined by (35). To compute k^{o*}, we combine (35), (36), (38) with (3) and get

$$k^{o*} = RW^* - C^{m*} - Q^* h^{y*} \left\{(R - (1-\delta)) + \psi (\beta R)^{1/\sigma}\right\}, \tag{40}$$

where C^{m*} is determined by (35). And combine (39), (40), with (34) and (7) will determine the equilibrium housing price Q^*,

$$K^* = k^{m*} + k^{o*}.$$

B.3 A negative productivity shock

In this subsection, we will derive the decision rules for agents facing an unexpected shock in productivity.

Consider the case where the productivity shock will arrive at time T and known at time $T-1$. For those who were born at time $T-2$, they have already purchased some housing unit at time $T-2$, h^y_{T-2}, and they have consumed the service derived from it. Therefore, they face a "truncated" Problem P:

$$\max . \beta \left[\frac{\left(C^m_{T-1}\right)^{1-\sigma}}{1-\sigma} + \beta \frac{\left(C^o_T\right)^{1-\sigma}}{1-\sigma} + \gamma \frac{\left(h^m_{T-1}\right)^{1-\sigma}}{1-\sigma}\right]$$

$$\text{s.t. } Rk^{m*} \geq C^m_{T-1} + \left[(1-\psi) Q^* h^{y*} R + \psi Q_{T-1} h^m_{T-1} - Q_{T-1}(1-\delta) h^{y*}\right] + k^o_T,$$

$$Rk^o_T + Q_T(1-\delta)h^m_{T-1} \geq C^o_T + (1-\psi)Q_{T-1}h^m_{T-1}R,$$

taking k^m_{T-1} and h^y_{T-2} as "initial conditions," and the fact that the variables at time $T-2$ is at $Q_{T-2} = Q^*$, $k^m_{T-1} = k^{m*}$, $h^y_{T-2} = h^{y*}$. The first order conditions are easy to derive,

$$(C^o_T) = (\beta R)^{1/\sigma} \left(C^m_{T-1}\right),$$

$$h^m_{T-1} = (C^o_T) \left[\frac{\gamma}{\beta \left(RQ_{T-1} - (1-\delta)Q_T\right)}\right]^{1/\sigma}.$$

Combining these first order conditions with the two budget constraints delivers the formula for C^m_{T-1},

$$C^m_{T-1} = \left\{R^2 k^{m*} - R^2(1-\psi)Q^* h^{y*} + RQ_{T-1}(1-\delta)h^{y*}\right\} / \Psi_2 \left(Q_{T-1}, Q_T\right)$$

where $\Psi_2 \left(Q_{T-1}, Q_T\right) = R + (\beta R)^{1/\sigma} + (\gamma R)^{1/\sigma} \left(RQ_{T-1} - (1-\delta)Q_T\right)^{-(1-\sigma)/\sigma}$. Then we can recover C^o_T and h^m_{T-1} by the first order conditions. Finally, we can use the budget constraint to recover the saving for the old-aged period, $k^o_T = Rk^{m*} - C^m_{T-1} - \{(1-\psi)Q^* h^{y*} R + \psi Q_{T-1} h^m_{T-1} - Q_{T-1}(1-\delta)h^{y*}\}$.

For those who were born at time $T-1$, they are aware that the wage will be depressed at time T, but their own wages will not be affected. On the other hand, they need to take the housing price deviations (from the steady state) in time $T-1$ and following periods into consideration. They also solve Problem P. For those who were born at time T, they also solve Problem P, except that the steady state wage rate W^* is not replaced by a lower wage level, W_T. And for those who were later than that, they also solve Problem P, taking into consideration of the change in housing prices. Formally, for those who are born at $T-1$, we have

$$C^o_{T+1} = (\beta R)^{1/\sigma} C^m_T, \tag{41}$$

$$h^y_{T-1} = (C^m_T) \left[\frac{\gamma}{\beta \left(RQ_{T-1} - (1-\delta)Q_T\right)}\right]^{1/\sigma}, \tag{42}$$

$$h_T^m = \left(C_{T+1}^o\right)\left[\frac{\gamma}{\beta\left(RQ_T - (1-\delta)Q_{T+1}\right)}\right]^{1/\sigma}, \tag{43}$$

where

$$C_T^m = \frac{R^2 W^*}{\left[R + (\beta R)^{1/\sigma}\right] + \Psi(Q_{T-1}, Q_T, Q_{T+1})}, \tag{44}$$

where $\Psi(Q_{T-1}, Q_T, Q_{T+1}) = (\gamma/\beta)^{1/\sigma}\left\{R\left[RQ_{T-1} - (1-\delta)Q_T\right]^{-(1-\sigma)/\sigma}\right.$
$\left. + (\beta R)^{1/\sigma}\left[RQ_T - (1-\delta)Q_{T+1}\right]^{-(1-\sigma)/\sigma}\right\}$. And (9) still applies (i.e. the housing market at time $T-1$ still needs to be clear), $h_{T-1}^y + h_{T-1}^m = h_{T-2}^y + h_{T-3}^m$, or

$$h_{T-1}^y + h_{T-1}^m = h^{y*} + h^{m*}.$$

For those who are born at T, the conditions are analogous,

$$C_{T+2}^o = (\beta R)^{1/\sigma} C_{T+1}^m, \tag{45}$$

$$h_T^y = \left(C_{T+1}^m\right)\left[\frac{\gamma}{\beta\left(RQ_T - (1-\delta)Q_{T+1}\right)}\right]^{1/\sigma}, \tag{46}$$

$$h_{T+1}^m = \left(C_{T+2}^o\right)\left[\frac{\gamma}{\beta\left(RQ_{T+1} - (1-\delta)Q_{T+2}\right)}\right]^{1/\sigma}, \tag{47}$$

where

$$C_{T+1}^m = \frac{R^2 W_T}{\left[R + (\beta R)^{1/\sigma}\right] + \Psi(Q_T, Q_{T+1}, Q_{T+2})}, \tag{48}$$

where W_T is defined by (11), $\Psi(Q_T, Q_{T+1}, Q_{T+2}) = (\gamma/\beta)^{1/\sigma}\left\{R\left[RQ_T - (1-\delta)Q_{T+1}\right]^{-(1-\sigma)/\sigma}\right.$
$\left. + (\beta R)^{1/\sigma}\left[RQ_{T+1} - (1-\delta)Q_{T+2}\right]^{-(1-\sigma)/\sigma}\right\}$. And (9) still applies (i.e. the housing market at time T still needs to be clear), $h_T^y + h_T^m = h_{T-1}^y + h_{T-2}^m$, or

$$h_T^y + h_T^m = h_{T-1}^y + h^{m*}.$$

For those who are born at $T+1$, the conditions are analogous,

$$C_{T+3}^o = (\beta R)^{1/\sigma} C_{T+2}^m, \tag{49}$$

$$h^y_{T+1} = \left(C^m_{T+2}\right)\left[\frac{\gamma}{\beta\left(RQ_{T+1}-(1-\delta)Q_{T+2}\right)}\right]^{1/\sigma}, \tag{50}$$

$$h^m_{T+2} = \left(C^o_{T+3}\right)\left[\frac{\gamma}{\beta\left(RQ_{T+2}-(1-\delta)Q_{T+3}\right)}\right]^{1/\sigma}, \tag{51}$$

where

$$C^m_{T+2} = \frac{R^2W^*}{\left[R+(\beta R)^{1/\sigma}\right]+\Psi(Q_{T+1},Q_{T+2},Q_{T+3})}, \tag{52}$$

where W^* is the steady state wage, $\Psi(Q_{T+1},Q_{T+2},Q_{T+3}) = (\gamma/\beta)^{1/\sigma}\Big\{R\left[RQ_{T+1}-(1-\delta)Q_{T+2}\right]^{-(1-\sigma)/\sigma}$ $+(\beta R)^{1/\sigma}\left[RQ_{T+2}-(1-\delta)Q_{T+3}\right]^{-(1-\sigma)/\sigma}\Big\}$. And (9) still applies (i.e. the housing market at time $T+1$ still needs to be clear),

$$h^y_{T+1}+h^m_{T+1} = h^y_T + h^m_{T-1}.$$

The case for agents born at later periods are similar and skipped due to the space constraint.

B.4 An interest rate shock

In this subsection, we will derive the decision rules for agents facing an unexpected (upward) shock in the world interest rate.

Consider the case where the interest shock will arrive at time T and known at time $T-1$. For those who were born at time $T-2$, they have already purchased some housing unit at time $T-2$, h^y_{T-2}, and they have consumed the service derived from it. Therefore, they face a "truncated" Problem P:

$$\max.\beta\left[\frac{\left(C^m_{T-1}\right)^{1-\sigma}}{1-\sigma}+\beta\frac{\left(C^o_T\right)^{1-\sigma}}{1-\sigma}+\gamma\frac{\left(h^m_{T-1}\right)^{1-\sigma}}{1-\sigma}\right]$$

$$\text{s.t. } Rk^{m*} \geq C^m_{T-1} + \left[(1-\psi)Q^*h^{y*}R+\psi Q_{T-1}h^m_{T-1}-Q_{T-1}(1-\delta)h^{y*}\right]+k^o_T,$$

$$\overline{R}k^o_T + Q_T(1-\delta)h^m_{T-1} \geq C^o_T + (1-\psi)Q_{T-1}h^m_{T-1}\overline{R},$$

taking k^m_{T-1} and h^y_{T-2} as "initial conditions," and the fact that the variables at time $T-2$ is at $Q_{T-2} = Q^*$, $k^m_{T-1} = k^{m*}$, $h^y_{T-2} = h^{y*}$. The first order conditions are easy to derive,

$$(C^o_T) = \left(\beta \overline{R}\right)^{1/\sigma} \left(C^m_{T-1}\right),$$

$$h^m_{T-1} = (C^o_T) \left[\frac{\gamma}{\beta\left(\overline{R}Q_{T-1} - (1-\delta)Q_T\right)}\right]^{1/\sigma}.$$

Combining these first order conditions with the two budget constraints delivers the formula for C^m_{T-1},

$$C^m_{T-1} = \left\{R\overline{R}k^{m*} - R\overline{R}(1-\psi)Q^*h^{y*} + \overline{R}Q_{T-1}(1-\delta)h^{y*}\right\} / \Psi_2\left(Q_{T-1}, Q_T\right)$$

where $\Psi_2\left(Q_{T-1}, Q_T\right) = \overline{R} + \left(\beta\overline{R}\right)^{1/\sigma} + \left(\gamma\overline{R}\right)^{1/\sigma}\left(\overline{R}Q_{T-1} - (1-\delta)Q_T\right)^{-(1-\sigma)/\sigma}$. Then we can recover C^o_T and h^m_{T-1} by the first order conditions. Finally, we can use the budget constraint to recover the saving for the old-aged period, $k^o_T = Rk^{m*} - C^m_{T-1} - \{(1-\psi)Q^*h^{y*}R + \psi Q_{T-1}h^m_{T-1} - Q_{T-1}(1-\delta)h^{y*}\}$.

For those who were born at time $T-1$ and T, they are aware that the interest rate will rise at time T. They also need to take the housing price deviations (from the steady state) in time $T-1$ and following periods into consideration. They solve Problem P. For those who were born at time $T+1$ or later, they also solve Problem P, except that the interest rate restores to its steady state level R, rather than $\overline{R}$. Formally, for those who are born at $T-1$, we have

$$C^o_{T+1} = (\beta R)^{1/\sigma} C^m_T, \tag{53}$$

$$h^y_{T-1} = (C^m_T) \left[\frac{\gamma}{\beta\left(\overline{R}Q_{T-1} - (1-\delta)Q_T\right)}\right]^{1/\sigma}, \tag{54}$$

$$h^m_T = \left(C^o_{T+1}\right) \left[\frac{\gamma}{\beta\left(RQ_T - (1-\delta)Q_{T+1}\right)}\right]^{1/\sigma}, \tag{55}$$

where

$$C^m_T = \frac{R\overline{R}W^*}{\left[R + (\beta R)^{1/\sigma}\right] + \Psi(Q_{T-1}, Q_T, Q_{T+1})}, \tag{56}$$

where $\Psi(Q_{T-1}, Q_T, Q_{T+1}) = (\gamma/\beta)^{1/\sigma} \Big\{ R \left[\overline{R} Q_{T-1} - (1-\delta) Q_T\right]^{-(1-\sigma)/\sigma}$
$+ (\beta R)^{1/\sigma} \left[R Q_T - (1-\delta) Q_{T+1}\right]^{-(1-\sigma)/\sigma} \Big\}$. And (9) still applies (i.e. the housing market at time $T-1$ still needs to be clear), $h^y_{T-1} + h^m_{T-1} = h^y_{T-2} + h^m_{T-3}$, or

$$h^y_{T-1} + h^m_{T-1} = h^{y*} + h^{m*}.$$

For those who are born at T, the conditions are analogous,

$$C^o_{T+2} = (\beta R)^{1/\sigma} C^m_{T+1}, \tag{57}$$

$$h^y_T = \left(C^m_{T+1}\right) \left[\frac{\gamma}{\beta \left(R Q_T - (1-\delta) Q_{T+1}\right)}\right]^{1/\sigma}, \tag{58}$$

$$h^m_{T+1} = \left(C^o_{T+2}\right) \left[\frac{\gamma}{\beta \left(R Q_{T+1} - (1-\delta) Q_{T+2}\right)}\right]^{1/\sigma}, \tag{59}$$

where

$$C^m_{T+1} = \frac{R^2 W^*}{\left[R + (\beta R)^{1/\sigma}\right] + \Psi(Q_T, Q_{T+1}, Q_{T+2})}, \tag{60}$$

where $\Psi(Q_T, Q_{T+1}, Q_{T+2}) = (\gamma/\beta)^{1/\sigma} \Big\{ R \left[R Q_T - (1-\delta) Q_{T+1}\right]^{-(1-\sigma)/\sigma}$
$+ (\beta R)^{1/\sigma} \left[R Q_{T+1} - (1-\delta) Q_{T+2}\right]^{-(1-\sigma)/\sigma} \Big\}$. And (9) still applies (i.e. the housing market at time T still needs to be clear), $h^y_T + h^m_T = h^y_{T-1} + h^m_{T-2}$, or

$$h^y_T + h^m_T = h^y_{T-1} + h^{m*}.$$

For those who are born at $T+1$, the conditions are analogous,

$$C^o_{T+3} = (\beta R)^{1/\sigma} C^m_{T+2}, \tag{61}$$

$$h^y_{T+1} = \left(C^m_{T+2}\right) \left[\frac{\gamma}{\beta \left(R Q_{T+1} - (1-\delta) Q_{T+2}\right)}\right]^{1/\sigma}, \tag{62}$$

$$h^m_{T+2} = \left(C^o_{T+3}\right) \left[\frac{\gamma}{\beta \left(R Q_{T+2} - (1-\delta) Q_{T+3}\right)}\right]^{1/\sigma}, \tag{63}$$

where

$$C^m_{T+2} = \frac{R^2 W^*}{\left[R + (\beta R)^{1/\sigma}\right] + \Psi(Q_{T+1}, Q_{T+2}, Q_{T+3})}, \tag{64}$$

where W^* is the steady state wage, $\Psi(Q_{T+1}, Q_{T+2}, Q_{T+3}) = (\gamma/\beta)^{1/\sigma} \Big\{ R \left[RQ_{T+1} - (1-\delta)Q_{T+2}\right]^{-(1-\sigma)/\sigma}$ $+ (\beta R)^{1/\sigma} \left[RQ_{T+2} - (1-\delta)Q_{T+3}\right]^{-(1-\sigma)/\sigma} \Big\}$. And (9) still applies (i.e. the housing market at time $T+1$ still needs to be clear),

$$h^y_{T+1} + h^m_{T+1} = h^y_T + h^m_{T-1}.$$

The case for agents born at later periods are similar and skipped due to the space constraint.