Bank runs and noisy signals

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We show that even with noisy signals on the quality of a bank's assets multiple equilibria exist in models of banking. We argue that the conditions under which this happens arise naturally in models of banking.
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1 Introduction

We investigate the conditions under which multiple equilibria due to coordination failure can be ruled out when individual agents have noisy signals about the fundamentals of an economy. Morris and Shin (1998) study a model of currency crisis where, with common knowledge of fundamentals, self-fulfilling currency attacks lead to multiple equilibria. They show that even a small amount of noise about fundamentals yields a unique equilibrium[1]. The insights generated by their paper extends to other market situations where multiple equilibria may exist due to coordination failures. These insights present a major challenge to any attempts that seek to explain either currency attacks (as in Obstfeld (1986)) or bank runs (as in Diamond-Dybvig (1983)) using models with self-fulfilling multiple equilibria. Indeed, Jeanne and Masson (2000), using a model with multiple equilibria to study the experience of the French franc from 1987-1994, explicitly acknowledge the criticism of their approach implied by the uniqueness result in Morris and Shin (1998).

Our purpose here is to address this challenge to models with self-fulfilling multiple equilibria by studying the robustness of the uniqueness result with noisy signals about fundamentals.

We change the original context of Morris and Shin (1998) paper from currency crisis to banking. We can justify this as follows. First, ever since Diamond and Dybvig's seminal (1983) paper, the phenomena of bank runs arising out of coordination failure has been extensively studied[2]. realising several authors have applied the methodology developed by Morris and Shin (1998) to models of banking (see for example, Goldstein and Pauzner (1999), Morris and Shin (2000)). Like these papers here, we also study a model of banking where the banking fundamentals affect both the returns to the bank's asset and the minimum size of the bank. This guarantees that multiple equilibria exist in our model due to a coordination failure among depositors when these banking fundamentals are common knowledge.

However, unlike these authors, we obtain very different results. We show,

[1]For further applications of this uniqueness result, see Corsetti, Morris and Shin (1999) who use it to study the vulnerability of a country to speculative attacks in the presence of large traders, Morris and Shin (1999a) who use it provide a theory of the onset of currency crisis and Morris and Shin (1999b) who use this result to study the way debt pricing can incorporate the risk due to coordination failure.

[2]See for instance Freixas and Rochet (1998) and the references contained therein.

using our model, that even with noisy signals on fundamentals, multiple equilibria may persist. To obtain our results, we do away with a key assumption in Morris and Shin (1998). Applied to our model of banking this assumption implies that with complete information, the space of fundamentals can be partitioned into three non-empty subsets, an unstable region where the bank always fails, a middle region in which there is multiple equilibria and a stable region where the bank always survives. Assuming all three regions are non-empty, a small noise in the observation of the true value of the fundamentals, implies that it can never be common knowledge that fundamentals are in the middle region. It is this feature that is crucial for the uniqueness result in Morris and Shin (1998) to go through. In fact, Morris and Shin (2000), following Goldstein and Pazner (1999), develop a model of banking where all the three regions of fundamentals are non-empty and show the existence of a unique equilibrium with noisy signals on fundamentals.

In our analysis we do away with the assumption that all these regions are non-empty and show that, in contrast, multiple equilibria exist even with noisy signals on the fundamentals of banking. We study two different cases where this assumption no longer holds. In the first case, we assume that both the stable and the unstable regions are empty. In this case, even with noisy signals on fundamentals, it remains common knowledge that these fundamentals are in the middle region. From this it follows that both bank runs and bank survival continue to remain equilibrium outcomes. This case serves as a useful benchmark for our main result. In order to prove this result, we investigate the less restrictive case when only the stable region is assumed to be empty. In this case, even with noisy signals on fundamentals, it remains common knowledge that the fundamentals lie in either the unstable or in the middle region. Under this assumption, we show that when the fundamentals fall below a certain critical value, the bank always fails, but above that same critical value, both self-fulfilling bank runs and bank survival remain equilibrium outcomes.

Is the case with an empty stable region economically relevant? The case for an empty stable region as shown in section 3 can be justified in two ways. First, we examine the optimal decision of banks when faced with a potential collapse and derive endogenously the assumption of an empty stable region. When the proportion of early withdrawal is high, a bank may decide to defend itself by liquidating some of its long-term assets or borrowing from the outside third party to pay for withdrawal demand. Under these circumstances, we show that banks will decide to collapse regardless of fundamentals when (a) the credit rationing in the market for bank loans is severe, (b) the cost of bank loans is high, or (c) there is a high cost of early liquidation. In all these cases, the stable region is empty.

Second, a minimum size of the bank which is bounded away from zero can also be justified if the bank's asset portfolio has illiquid assets. By reinterpreting fundamentals to represent an exogenous liquidity shock, we show that our model of banking can be derived as a reduced form version of Diamond and Dybvig (1983)'s model of banking where the proportion of illiquid assets is derived endogenously.

In the final section of the paper, we discuss some empirical and policy issues that arise from our analysis. One implication of Morris and Shin (1998) (see also Morris and Shin (1999a)), is that the onset of a crisis should be anticipated as the fundamentals evolve to approach the critical value needed to trigger a speculative attack. On the other hand, our main result implies that a financial crisis should be largely unanticipated by markets. In fact, a wide array of empirical papers that supports the view that episodes of financial and currency crisis are largely unanticipated. A related issue is on the role of policy interventions that prevent a crisis. Our main result here suggests a role for policy interventions that coordinate the expectations of traders on the "right" equilibrium. In contrast to Morris and Shin (1998), our results suggest that suspension of convertibility, restrictions on capital flows, lenders of last resort can be rationalized as policy interventions that prevent runs due to self-fulfilling expectations.

In the next section we present the model of banking and characterize its equilibria with and without noisy signals on fundamentals. Section 4 discuss empirical and policy issues. The last section concludes.

2 Banking with noisy signals

There are three time periods, $t = 0, 1, 2$. In each period there is a single perishable good x_t. There is a continuum of identical depositors, indexed by i, of Lebesgue measure 1, each endowed with one unit of the perishable good at time period $t = 0$. Depositors preferences are identical and are summarized by the utility function $u(x_0, x_1, x_2) = x_1 + x_2$.

In addition, there is a bank endowed with a non-convex technology that converts the inputs of the perishable good at $t = 0$ to outputs of the perishable good at $t = 1$ or $t = 2$. Depositors choose whether to invest their endowment of the perishable with the bank at $t = 0$.

Let θ represent the fundamentals of banking. As we discuss in section 3, we can interpret θ as either representing the quality of a bank's assets or an exogenous liquidity shock. We assume that θ is uniformly distributed over $\left[\underline{\theta}, \overline{\theta}\right]$.

Both the returns and the minimum size of the bank depend on θ. In order to simplify the analysis, we take this dependence on θ as exogenously determined in this section of the paper. We postpone an endogenous derivation of the dependence of the minimum size on θ to section 3.

One unit of the consumption good invested in $t = 0$ yields either one unit of the good in $t = 1$ or $r(\theta)$, units of the consumption good in period $t = 2$, if the proportion of deposits with the bank is at least as great as $a(\theta)$. Otherwise the bank fails at $t = 1$ and yields nothing at $t = 2$. Here, Further, at each θ, $a(\theta)$ is a minimum size requirement: if the proportion of deposits falls below $a(\theta)$, the bank is no longer viable at $t = 1$.

Given the specification of preferences, without loss of generality we may assume that a depositor will choose to invest in the bank at $t = 0$. This is because no depositor has access to the technology in period $t = 0$ and derives no utility from consuming the commodity in period $t = 0$. However, by allowing depositors to withdraw their deposits before the asset matures, we are able to capture the maturity mismatch with demand deposit contracts.

We study the equilibrium outcomes in two different versions of this model.

In **Version 1**, as in Diamond and Dybvig (1983), the fundamentals of banking are common knowledge. The sequence of events is as follows. In $t = 0$, depositors invest in the bank before they observe θ. In period $t = 1$, the true value of θ is realized according to the uniform distribution over $\left[\underline{\theta}, \overline{\theta}\right]$. All depositors observe θ. Depositors then decide whether to withdraw or continue their investments in the bank. If they withdraw, they get back the one unit of the commodity. If they continue to invest, they get back $r(\theta)$ at $t = 2$ if the bank survives. Otherwise, they get nothing. As remarked before, without loss of generality we may assume that all depositors will choose to invest in the bank at $t = 0$. This allows us to focus on the game that depositors play at $t = 1$, when θ is observed by all depositors. In this game, a depositor chooses an action from $\{withdraw,\ not\ withdraw\}$. A profile of actions is a Nash equilibrium if no individual depositor has an incentive to deviate.

Let $\boldsymbol{\theta}_{us} = \{\theta | r(\theta) < 1\}$, with $\boldsymbol{\theta}_s = \{\theta | r(\theta) \geq 1\ and\ a(\theta) = 0\}$. Let $\boldsymbol{\theta}_m = \{\theta | r(\theta) \geq 1\ and\ 0 < a(\theta) < 1\}$. The following remark is immediate and is stated without proof.

Remark 1 *In **Version 1** of the model, for each* $\theta \in \boldsymbol{\theta}_{us}$*(resp., for each* $\theta \in \boldsymbol{\theta}_s$*) there is a unique pure strategy Nash equilibrium where all depositors choose to withdraw (resp. choose not to withdraw) while for each* $\theta \in \boldsymbol{\theta}_m$*, there are two pure strategy Nash equilibria, one where all depositors choose* $\{$*not withdraw*$\}$ *and another where all depositors choose* $\{$*withdraw*$\}$*.*

This remark justifies our interpretation of $\boldsymbol{\theta}_{us}$ as the unstable region where the bank always collapses and $\boldsymbol{\theta}_s$ as the stable region where the bank always survives. $\boldsymbol{\theta}_m$ is the middle region where the there are two equilibrium outcomes: one where the bank always survives and another where the bank always fails. This is the region with self-fulfilling multiple equilibria: the first equilibrium is the one where the bank always survives and the second equilibrium is the one where there is a bank run and represents a coordination failure. As we shall show, whether or not these three regions are non-empty is the key to characterizing equilibria with noisy signals on fundamentals.

Next we study **Version 2** where depositors no longer observe the quality of the bank's assets, θ, at $t = 1$ but instead receive a noisy signal about it. The timing of events is exactly as before except that now at $t = 1$, depositors cannot observe θ but instead observe a signal y which is drawn independently and uniformly from $Y(\theta) \subset \left[\underline{\theta}, \overline{\theta}\right]$, where $Y(\theta) = [\theta - \varepsilon, \theta + \varepsilon]$ if $\theta \in [\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon]$, where $\varepsilon > 0$ is the term representing noise in the fundamentals; $Y(\theta) = [\theta, \theta + \varepsilon]$ if $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon)$; $Y(\theta) = [\theta - \varepsilon, \theta]$ if $\theta \in (\overline{\theta} - \varepsilon, \overline{\theta}]$[3]. A strategy for a depositor at $t = 1$ is a function from what he observes to his set of actions which has two components $\{$*withdraw*, *not withdraw*$\}$. A profile of strategies is a Bayesian equilibrium if no individual depositor has an incentive to deviate. Consider the strategy profile where every depositor withdraws if and only if observed signal y is less than $\bar{y}$. Such a strategy profile is called a strategy profile with a switching point $\bar{y}$.

In the remaining part of this section, we study the equilibria of **Version 2** of this model.

Remark 2 *Assume* $r(\theta)$ *is continuous and strictly increasing in* θ *while* $a(\theta)$ *is continuous and strictly decreasing in* θ*,* $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$*. If* $\boldsymbol{\theta}_{us}, \boldsymbol{\theta}_s, \boldsymbol{\theta}_m$ *are all non-empty, then for a suitable choice of* $\varepsilon > 0$ *there exists a unique Bayesian*

[3]This signalling structure is the same as in Morris and Shin (1998). We use this signalling structure to maintain comparability with their results.

equilibrium in strategies with a switching point y^*. In particular, there exists a unique value of θ, denoted by θ^*, such that the bank fails if $\theta < \theta^*$ and survives if $\theta > \theta^*$.

Proof. Follows from Morris and Shin (1998). ∎

This remark shows that when the space of fundamentals can be partitioned into three non-empty subsets, the first region in which the bank always fails, a second region in which there is multiple equilibria and a third region in which the bank always survives, the unique equilibrium result of Morris and Shin (1998) applies to our model as well. When all three regions are non-empty, a small noise in the observation of the true value of the fundamentals, implies that it can never be common knowledge that fundamentals are in $\boldsymbol{\theta}_m$. Morris and Shin's proof then goes through in our case with a suitable relabelling of variables.

In contrast, the next proposition provides a set of conditions under which multiple equilibria persist even with noisy signals on fundamentals.

Proposition 3 *Suppose it is common knowledge that $\boldsymbol{\theta}_m = \left[\underline{\theta}, \overline{\theta}\right]$ while $\boldsymbol{\theta}_s$ and $\boldsymbol{\theta}_{us}$ are empty. Then, multiple equilibria exist and in particular self-fulfilling bank runs exist.*

Proof. If it is common knowledge that $\boldsymbol{\theta}_s$ and $\boldsymbol{\theta}_{us}$ are empty, for $\theta \in [\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon]$, where $\varepsilon > 0$, at each $y \in [\theta - \varepsilon, \theta + \varepsilon]$, it remains common knowledge that $\theta \in \boldsymbol{\theta}_m$. The same remains true when $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon)$ and $\theta \in (\overline{\theta} - \varepsilon, \overline{\theta}]$. Therefore, conditional on θ, at every signal $y \in Y(\theta)$, it is common knowledge that $\theta \in \boldsymbol{\theta}_m$. Therefore, the strategy profile where each depositor chooses $\{$*not withdraw*$\}$ at each y is an equilibrium. Further, the strategy profile where each depositor chooses $\{$*withdraw*$\}$ at each y is an equilibrium. ∎

Proposition 1 serves as a benchmark case because it specifies the key common knowledge restrictions for the existence of multiple equilibrium outcomes. In the benchmark case, the assumption is that $\boldsymbol{\theta}_{us}$, the region where the bank always fails and $\boldsymbol{\theta}_s$, region where the bank always survives, are both empty. Under this assumption, even with noisy signals on fundamentals, it remains common knowledge that the fundamentals are in $\boldsymbol{\theta}_m$. From this it follows that both bank runs and bank survival continue to remain equilibrium outcomes. For $\boldsymbol{\theta}_{us}$ to be empty we need that $r(\theta) \geq 1$. This is restrictive as it assumes that the quality of the asset portfolio of the bank always guarantees a high enough return at $t = 2$.

In the next proposition, we make the weaker assumption that only $\boldsymbol{\theta}_s$ is empty. We show that even with this weaker assumption, multiple equilibria exist.

Before we state the next proposition, we need the following definitions (see also Morris and Shin (1998)). Let $\pi(y)$ be the aggregate proportion of deposits not withdrawn in period 1 when the value of signal is y. Let $s(\theta, \pi)$ be the realized proportion of deposits not withdrawn in period 1 when the state of the fundamentals is θ, given π. Under our assumptions, when $\theta \in [\underline{\theta} + \varepsilon, \overline{\theta} - \varepsilon]$,

$$s(\theta, \pi) = \frac{1}{2\varepsilon} \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(y) dy$$

Further, for $\theta \in [\underline{\theta}+\varepsilon, \overline{\theta}-\varepsilon]$, let $u(y, \pi)$ be the expected payoff of $\{not\ withdraw\}$, given signal y and π. We have

$$\begin{aligned} u(y, \pi) &= \frac{1}{2\varepsilon} \int_{[y-\varepsilon, y+\varepsilon] \cap \{\theta | s(\theta, \pi) \geq a(\theta)\}} r(\theta)\, d\theta, \\ &= 0 \ otherwise \end{aligned}$$

Note that when θ is close to $\underline{\theta}$ or $\overline{\theta}$, in particular, when $\theta \in [\underline{\theta}, \underline{\theta} + \varepsilon)$ and $\theta \in (\overline{\theta} - \varepsilon, \overline{\theta}]$, the limits of the integrations must be adjusted.

Denote a strategy profile with a switching point $\bar{y}$ by $I_{\overline{y}}$ and let $u(\overline{y}, I_{\overline{y}})$ denotes the corresponding payoff.

Proposition 4 *Assume $r(\theta)$ is continuous and strictly increasing in θ while $a(\theta)$ is continuous and strictly decreasing in θ, $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Suppose it is common knowledge that $\theta \in \boldsymbol{\theta}_m \cup \boldsymbol{\theta}_{us}$ while $\boldsymbol{\theta}_s$ is empty, then multiple equilibria exist and in particular self-fulfilling bank runs exist.*

Proof. As it is common knowledge that $\boldsymbol{\theta}_s$ is empty, then at every signal y, it is common knowledge that $\theta \in \boldsymbol{\theta}_m \cup \boldsymbol{\theta}_{us}$. Therefore a strategy profile in which every depositor choose $\{withdraw\}$ regardless of the signal value y is an equilibrium. It remains to show that there exists $\theta^* \in \left[\underline{\theta}, \overline{\theta}\right]$ such that for all $\theta \in (\theta^*, \overline{\theta}]$, in addition, $\{not\ withdraw\}$ is also an equilibrium outcome. First note that if $\pi(y) \geq \pi'(y)$ for all y, then $u(y, \pi) \geq u(y, \pi')$ for all y and

$u(\overline{y}, I_{\overline{y}})$ is continuous and strictly increasing in $\overline{y}$.This follows by arguments analogous to lemma 1 and lemma 2 in Morris and Shin (1998). Further, observe that for a suitable choice of ε, we have $Y(\underline{\theta}) \subset \boldsymbol{\theta}_{us}$ while $Y(\bar{\theta}) \subset \boldsymbol{\theta}_m$. Now, a depositor with a signal $y \in Y(\underline{\theta})$ knows that $\theta \in \boldsymbol{\theta}_{us}$. Therefore, we have $u(\overline{y}, I_{\overline{y}}) < 1$ if $\bar{y} \in Y(\underline{\theta})$. Next, a depositor with a signal $y \in Y(\bar{\theta})$, knows that $\theta \in \boldsymbol{\theta}_m$. For each $\theta \in \boldsymbol{\theta}_m$, each depositor will choose $\{not\ withdraw\}$ if he expects that all other depositors will choose $\{not\ withdraw\}$. Therefore, if every depositor expects every other depositor to choose $\{not\ withdraw\}$, as there is a continuum of depositors, for an individual depositor with a signal $\bar{y} \in Y(\bar{\theta})$, $u(\bar{y}, I_{\overline{y}}) > 1$. Therefore, there exists a unique signal $y^* \in \left[\underline{\theta}, \overline{\theta}\right]$ such that $u(y^*, I_{y^*}) = 1$. Also, $y^* \notin Y(\underline{\theta})$ and $y^* \notin Y(\bar{\theta})$. In words, a depositor with signal y will choose $\{withdraw\}$ if $y \leq y^*$ and $\{not\ withdraw\}$ if $y > y^*$. Now, when the equilibrium strategy is I_{y^*}, $s(\theta, I_{y^*})$ is strictly increasing in $\theta \in (y^* - \varepsilon,\ y^* + \varepsilon)$. Further, when $\theta \in (y^* - \varepsilon,\ y^* + \varepsilon)$, $0 < s(\theta, I_{y^*}) < 1$ and at $\theta = y^* - \varepsilon$, $s(\theta, I_{y^*}) = 0$ while at $\theta = y^* + \varepsilon$, $s(\theta, I_{y^*}) = 1$. Further, for a suitable choice of ε, when $\theta + 2\varepsilon \in \boldsymbol{\theta}_{us}$, $s(\theta, I_{y^*})$ is equal to zero. In addition, $s(\theta, I_{y^*})$ is continuous in θ and by assumption, $a(\theta)$ is continuous and strictly decreasing in θ. Therefore, there is a unique value of θ such that $s(\theta, I_{y^*}) = a(\theta)$. Denote this by θ^*. Therefore, when $\theta < \theta^*$, the bank always fails and when $\theta \geq \theta^*$, the bank can either fail or survive. ■

3 Discussion of the model

So far in our analysis, we have taken as given the assumptions on the emptiness or non-emptiness of the regions $\boldsymbol{\theta}_{us}, \boldsymbol{\theta}_s, \boldsymbol{\theta}_m$. As we have already seen in the previous section, especially proposition 2.2, the assumptions made on the (non)emptiness of the regions $\boldsymbol{\theta}_{us}, \boldsymbol{\theta}_s, \boldsymbol{\theta}_m$ are crucial. In this section, we endogenize the assumption $\boldsymbol{\theta}_s$ is empty.

3.1 The return function and minimum size

In this section, we analyse in more detail the process in which bank failure occurs and show that for some economies the bank may collapse regardless of the value of the fundamentals θ.

To this end, we introduce a more complex return function $r(\theta, \alpha)$ that depends on both the fundamental θ and the proportion of deposit remaining at the end of $t = 1$, denote by α. As before, we assume that $r(\theta, \alpha)$ is continuous, strictly increasing in its first argument θ; in addition, now we asssume that it is continuous and strictly increasing in α as well. remark that under this assumption, the argument presented in the previous section is unaffected. Further, we require that for values of θ close to $\underline{\theta}$, $r(\theta, \alpha) < 1$, regardless of the proportion of deposits remaining. In particular observe that this guarantees that $\boldsymbol{\theta}_{us}$ is non-empty. We also take as given that $\boldsymbol{\theta}_m$ is non-empty.

In what folows it will be convenient to fix the interpretation of θ as representing the quality of the bank's assets. We have in mind a situation where the bank in question is a regional monopoly with both its assets and liabilities specific to that region. The fundamentals θ now represent shocks that are idiosyncratic to the region in which the bank operates itself.

Suppose at $t = 1$, after observing θ, a proportion $1 - \alpha$, $0 \leq \alpha \leq 1$, of depositors decide to withdraw from the bank. The bank will have to liquidate a proportion $1 - \alpha$ of its assets in order to satisfy the withdrawal demand[4]. When $r(\theta, \alpha) \geq 1$ the bank will survive to $t = 2$. When $r(\theta, \alpha) < 1$ the bank will fail at $t = 1$. Here we analyse the action of the bank when faced with a possible collapse. If the return $r(\theta, \alpha) < 1$ the bank has a choice between two actions, labelled A and NA. Action A denotes the choice to defend the bank. Action NA denotes the choice of not doing anything to prevent the bank from collapsing. When the action NA is chosen, the bank collapses at $t = 1$, and a payoff equal to zero is obtained by the bank. If action A is chosen, it can defend itself by borrowing from the outside third party or via early liquidation of its assets.

Denote the amount that the bank decides to borrow at $t = 1$ by γ. If the bank borrows γ to satisfy some of its withdrawal demand, it will only need to liquidate $\delta = 1 - \alpha - \gamma$ of its assets. Therefore at the end of $t = 1$ the amount of deposits remaining with the bank is $\alpha + \gamma$. The net return in this case is given by

$$r(\theta, \alpha + \gamma) - c(\gamma)$$

where $c(\gamma)$ is assumed to be continuous and strictly increasing in γ, and reflecting the costs of borrowing. The per capita return is assumed to be increasing in θ, α and γ.The bank will choose action A if

$$r(\theta, \alpha + \gamma) - c(\gamma) \geq 1$$

[4]This is the case when the salvage value of production at $t = 1$ is the initial investment. Later this assumption is relaxed.

and action NA if

$$r(\theta, \alpha + \gamma) - c(\gamma) < 1$$

We first show that the minimum size requirement $a(\theta)$ is continuous and strictly decreasing in θ. $a(\theta)$ is the value of α which solves $r(\theta, \rho(\alpha + \gamma)) - c(\gamma) = 1$. Denote $R(\theta, \alpha, \gamma) = r(\theta, \rho(\alpha + \gamma)) - c(\gamma)$. Consider $\theta' < \theta''$ such that $R(\theta', a(\theta'), \gamma(\theta', a(\theta'))) = 1$ and $R(\theta'', a(\theta''), \gamma(\theta'', a(\theta''))) = 1$. By assumption, as $R(\theta, \alpha, \gamma)$ is increasing in θ and α, this implies that when $\theta' < \theta'', a(\theta') > a(\theta'')$. Therefore $a(\theta)$ is strictly decreasing in θ. Continuity of $a(\theta)$ follows from continuity of $r(\theta, \rho(\alpha + \gamma))$ and $c(\gamma)$.

Next, we argue that when banks are faced with severe rationing, high cost of borrowing or high cost of early liquidation they may decide to collapse regardless of fundamentals θ. This may be true for banks in emerging market economies with weak financial system. In such economies, $\boldsymbol{\theta}_s$ is empty.

We examine the optimal decision of the bank under three different scenarios.

3.1.1 Credit rationing

Here we show that when there is severe credit rationing in market for bank loans, the stable region $\boldsymbol{\theta}_s$ is empty. In fact, credit rationing is observed in most financial markets and in particular, in the market for bank loans (see for instance, Guttentag and Herring (1987)). We can rationalize credit rationing in our set-up by observing that the market may not be able distinguish between banks that borrow to ward off a liquidity threat and banks that borrow because they are insolvent. This can be justified in our model if the potential creditors of the bank cannot verifiably observe θ. Potential lenders faced with uncertainty may ration the borrower instead of raising the rate to cover a greater potential for loss especially, when as in our case, the solvency of potential borrowers is not observed. Therefore, there can be a limit on the amount the bank can borrow above which the cost of borrowing will be infinite. Denote the limit on borrowing by $\overline{\gamma}$ and let $\gamma_{\theta}(\alpha)$ be the value of γ such that

$$r(\theta, \alpha + \gamma_{\theta}(\alpha)) - c(\gamma_{\theta}(\alpha)) = 1$$

We shall assume that rationing by the markets is severe i.e. $\gamma_{\underline{\theta}}(0) > \overline{\gamma}$. Then, remark that there exists α such that $\gamma_{\theta}(\alpha) > \overline{\gamma}$ and the bank will collapse.

Let $\overline{\alpha}_\theta$ be the value of α such that $\gamma_\theta(\alpha) = \overline{\gamma}$. Therefore, for any given θ, if $\alpha < \overline{\alpha}_\theta$ the bank will collapse. Under our assumption, $\gamma_\theta(\alpha)$ is decreasing in θ. Therefore, $\forall\theta$, there exists $\overline{\alpha} > 0$ such that $\overline{\alpha} \leq \overline{\alpha}_\theta$. This implies that there exists $\alpha < \overline{\alpha}$ such that the bank will collapse regardless of $\theta \Rightarrow \boldsymbol{\theta}_s$ is empty.

3.1.2 The cost of borrowing

Apart from the fact that banks may have limited access to credits, troubled banks, especially when its solvency is not observed, may also be subjected to high rate of interest for loans. Also the costs of borrowing does not only reflect the loan rates but among other things, the transaction cost and more importantly the cost to its reputation. In order to borrow it will have to convince the market that it is solvent. This, in itself, will damage its reputation as being a worthy borrower and therefore, affect the terms on which it will be able to borrow in any future crisis[5].

Consider the case when all depositors wish to withdraw, $\alpha = 0$. The amount of deposits remaining at the end of $t = 1$ will be γ the amount that it borrows from the markets, and the per capita return will be $r(\theta, \gamma) - c(\gamma)$. Again, the bank will choose to collapse, action NA if $r(\theta, \gamma) - c(\gamma) < 1$. Let assume that the cost of borrowing is high such that

$$r(\theta, 1) - c(1) < 1, \forall\theta.$$

Therefore when $\alpha = 0$ and $\gamma = 1$ the bank will always choose to collapse. By continuity of $r(\theta, \alpha + \gamma)$ and $c(\gamma)$ in α and γ, there exist $\widetilde{\alpha}_\theta$ close to 0 and $\widetilde{\gamma}_\theta$ close to 1 such that

$$r(\theta, \alpha + \gamma) - c(\gamma) < 1, \forall\alpha < \widetilde{\alpha}_\theta, \forall\gamma > \widetilde{\gamma}_\theta$$

Therefore, $\forall\theta$, there exists $\widetilde{\alpha} > 0$ such that $\widetilde{\alpha} \leq \widetilde{\alpha}_\theta$. This implies that there exists $\alpha < \overline{\alpha}$ such that the bank will collapse regardless of $\theta \Rightarrow \boldsymbol{\theta}_s$ is empty.

[5]As Bagehot (1873) noted 'Every banker knows that if he has to *prove* that he is worthy of credit, however good maybe his argument, in fact his credit is gone...'. (quoted in Guttentag and Herring (1987)).

3.1.3 The cost of early liquidation

Previously, we assume that the salvage value of production at $t = 1$ is equal to the initial investment. Now, we will relax this assumption and assume that the production technology is such that if interrupted at $t = 1$, its salvage value will be less than the initial investment and the bank will be left with a proportion $\rho(\alpha + \gamma)$, $\rho < 1$. The cost of early liquidation may reflect the fact that the secondary market for bank loans remains thin as loans are not perfectly marketable and there exists a lemon type problem for bank loans. This, in turn, implies that the bank will incur a cost as the value of loans are heavily discounted by potential buyers (for a more detailed discussion on this point, see Guttentag and Herring (1987)).The per capita return is given by

$$r(\theta, \rho(\alpha + \gamma)) - c(\gamma)$$

Consider the case when $\alpha = 0$. The amount of deposits remaining at the end of $t = 1$ is $\rho\gamma$, the per capita return will be $r(\theta, \rho\gamma) - c(\gamma)$. Let assume that the cost of early liquidation is high such that when $\alpha = 0$,

$$r(\theta, 1) - c(1/\rho) < 1, \forall\theta$$

Remark that as long as $\rho > 1$, this assumption is less severe that requiring that $r(\theta, 1) - c(1) < 1, \forall\theta$. It follows that when $\alpha = 0$ the bank will always choose to collapse. By continuity of α and γ, there exist $\widetilde{\alpha}_\theta$ close to 0 and $\widetilde{\gamma}_\theta$ close to $1/\rho$ such that

$$r(\theta, \rho(\alpha + \gamma)) - c(\gamma) < 1, \forall\alpha < \widetilde{\alpha}_\theta, \forall\gamma > \widetilde{\gamma}_\theta$$

Therefore, $\forall\theta$, there exists $\widetilde{\alpha} > 0$ such that $\widetilde{\alpha} \leq \widetilde{\alpha}_\theta$. This implies that there exists $\alpha < \overline{\alpha}$ such that the bank will collapse regardless of $\theta \Rightarrow \boldsymbol{\theta}_s$ is empty.

3.2 Liquidity shocks and minimum size

In this section, we argue that typically a certain proportion of the bank's assets are always illiquid and this feature makes banks inherently vulnerable to bank runs, thus ensuring that θ_s is empty. We reinterpret fundamentals of banking as liquidity shocks and show that the model of banking presented in section 2 can be derived as a reduced form of the monopoly bank model of Diamond and Dybvig (1983) (hereafter D-D) where the proportion of the bank's assets that are illiquid is derived endogenously.

In (D-D), bank can achieve optimal risk sharing to its customers by offering demand deposit contract which transform illiquid assets into liquid liabilities. There are three time periods, $T = 0, 1, 2$ and two type of agents: type 1 (2) agent cares only about consumption in $T = 1$ ($T = 2$). Each agent has a utility function of the form

$$\begin{aligned} U(c_1, c_2; \Theta) &= u(c_1) && \text{if } j \text{ is of type 1 in state } \Theta \\ &= \rho u(c_1 + c_2) && \text{if } j \text{ is of type 2 in state } \Theta \end{aligned}$$

where Θ represents the state private information. At $T = 0$ all agents are identical and will choose to deposit with the bank. At $T = 1$ agents privately learn their type and make a choice between withdrawing and remaining with the bank. At $T = 2$ returns on production are realised and divided between the remaining depositors. In addition, both D-D and our models assume long term production technology with a cost of early liquidation. Demand deposit contract however offers liquid liabilities with a fixed interest rate r_1 at $T = 1$,(r_1 is greater than the value of early liquidation of assets). In D-D r_1 is derived from optimal risk sharing when the proportion of type 1 depositor, $t \in (0, 1)$, is known ex ante and r_1 is strictly decreasing in t[6]. The return on investments at $T = 2$, r_2 if the bank is not bankrupt depends on r_1, the fraction of deposits withdrawn at $T = 1$, f and R which is a constant representing a fixed return from long term production in period 2, $r_2 = R(1 - r_1 f)/(1 - f)$.

The fraction of type 1 depositors, t, in D-D can be interpreted as representing the state of nature θ in our model where $\theta \in \left[\underline{\theta}, \overline{\theta}\right], \underline{\theta} > 0, \overline{\theta} < 1$. Thus r_2 represents $r(\theta)$ in our model and for any given level of deposits withdrawn, $r(\theta)$ is increasing in θ and $r(\theta) > 1$ when the demand deposit contract is chosen optimally.

In D-D, it shown (pages 410-411) that there exists an upper bound of the fraction of deposits withdrawn at $T = 1$ that allows banking to continue to $T = 2$. If the realised fraction of deposits withdrawn at $T = 1$, w, is equal to this upper bound, denoted by $\overline{w} = (R - r_1)/r_1(R - 1)$, the rates of returns for both periods will be the optimally chosen interest rate at $T = 1$, r_1. If it is greater the bank will fail because all type 2 depositors will rush to withdraw

[6]The fact that the return in period 1 is greater than one is inessential since in Diamond and Dybvig (1983) bank runs occur even when the return is 1

their deposits. Since the minimum size requirement is the lowest possible proportion of deposits remaining in period 2 that permits the ongoing of production, the minimum size $a(\theta)$ is equal to $1-\overline{w}$. The upper bound $\overline{w}$ is a function of r_1 and it is increasing in θ, the fraction of type 1 deposits. Thus $a(\theta)$ is decreasing in θ and $a(\theta) > 0$ if $\theta \in (0,1)$. Therefore $\boldsymbol{\theta}_s$ is empty. Note that using the formula described in D-D, $\theta > 0$, implies that $a(\theta) < 1$. Thus it is in fact common knowledge that $\boldsymbol{\theta}_{us}$ is empty as well provided that $r(\theta) > 1$.

Given the specifications of D-D, we have shown that the following properties are true: $\boldsymbol{\theta}_{us}$ and $\boldsymbol{\theta}_s$are empty and for any $\theta \in \left[\underline{\theta}, \overline{\theta}\right], \underline{\theta} > 0, \overline{\theta} < 1$. Therefore, it is common knowledge that $\theta \in \boldsymbol{\theta}_m$ even when there is noisy observation of fundamentals. It follows that multiple equilibria always exist.

However, note that "narrow banks" (Fama (1985), Goodhart (1987), Wallace (1996)), where the bank is required to back demand deposits entirely by safe short-term assets are not vulnerable to runs. Wallace (1996) argues that requiring "narrow banks" may imply that socially optimal risk-sharing will not be implemented by the banking system. Our model rules out "narrow banks". Our main result suggests that there is a discontinuity in the transition from "narrow banks" to banks with a "small" maturity mismatch and a minimum size requirement due to a non-convex banking technology. In this case, multiple equilibria persist.

4 Empirical and policy issues

In our model of banking, a crisis will occur if the fundamentals are weak enough i.e. when $\theta \in [\underline{\theta}, \theta^*]$. It can also occur as a result of a shift of expectation even when the fundamentals are sound i.e. $\theta \in (\theta^*, \bar{\theta}]$. Therefore, our model is able to explain both fundamental based bank runs and panic runs. Indeed for a certain range of fundamentals, multiple equilibria exist even with noisy signals about the fundamentals. This has applications to the study of specific episodes of financial collapse and currency crisis such as the recent Asian financial crisis or the earlier Mexican Peso crisis in 1994. In our model of banking,

As pointed out by Thomas (1999), one implication of Morris and Shin (1998) (see also Morris and Shin (1999a)), is that the onset of a crisis should be anticipated as the fundamentals evolve to approach the critical value needed to trigger a speculative attack On the other hand, our main result, that there is a region of fundamentals with multiple equilibrium outcomes, implies that a financial crisis should be largely unanticipated by markets. In fact, Edwards (1986), using international data on the pricing of bank loans to developing countries, finds that international financial markets had only partially anticipated the debt crisis of 1982. Several empirical studies of other episodes of financial and currency crisis make a similar claim. Examples include Rose and Svensson (1994) on the crisis in currencies in the European exchange rate mechanism, Jeanne (1997) (see also Jeanne and Masson (2000)) on the experience of the French franc from 1987-1994, Flood and Marion (2000) on the crisis in the Mexican Peso in 1994.

Furthermore, the severity and contagion effects of the crisis is certainly unaccounted for purely by economic fundamentals. Several recent papers on the Asian financial crisis collected in Agenor et al (1999))[7] argue that the phenomenon of *contagion,* where a crisis in one economy triggers off a crisis in another institutionally similar economy, can be usefully studied using models of multiple equilibria. Masson (1997) concludes that there is a role for pure contagion which is not linked to macroeconomic fundamentals and can only occur in a situation in which multiple equilibria were possible.

The implications of our model for policies to prevent banking crises are similar to that of other multiple equilibria models of banking. Chang and Velasco (1998) apply the Diamond and Dybvig (1983) model of banking to the study of the Asian financial crisis to show that, with fixed exchange rates, domestic self-fulfilling bank crisis translates into a run on that country's currency (see Sachs, Tornell and Velasco (1996) for a related attempt to explain the Mexico Peso crisis).Our analysis also suggests that the problem of maturity mismatches can have a negative impact when capital markets are liberalized in emerging economies. The problem of maturity mismatches can be aggravated by financial liberalization and capital flows from abroad. Indeed, the nature of demand deposit contracts and of banks assets coupled with potentially large changes in financial flows ensure that a surprised run resulting from refusals to roll over debt can always occur. Rogoff (1999), citing Diamond and Dybvig (1983), makes a similar point. Fischer (1999) takes this argument further to make a case for an international lender of last resort.

[7]In particular see chapters 1, 2, 8, 9 and 10 in Agenor et al (1999). For a dissenting view, however, see the chapter by Morris and Shin (1999a) in the same volume where they study the onset of currency attacks by looking at a dynamic extension of their 1998 paper in which the fundamentals of the economy evolve stochastically over time.

Our model also suggests that policy interventions that facilitate the ability of agents to coordinate on the right equilibrium will prevent bank runs and other forms of financial crisis. Deposit insurance and a guarantee of bail-out in case of trouble by the central banks or international lender of last resort may prevent runs. However, this may cause severe moral hazard problem and encourage large capital inflows which in turn aggravate the problem. Financial liberalisation thus should be implemented with care and proper punishment strategies and appropriate monitoring of bank should be in place to eliminate the risk of moral hazard. It is also argued that institution may suspend convertibility (standstills) in order to prevent runs which are due to self-fulfilling expectation. However, these policy interventions that facilitate coordination on the right equilibrium do not make sense in models with a unique equilibrium since bank runs are driven by fundamentals.

Another contrast is in the analysis of the role of transparency in preventing a financial crisis. In our analysis, public announcements that increase transparency coupled with policy interventions that facilitate coordination, lessens the possibility of bank runs. However, this is may not always be true in models of bank runs with a unique equilibrium. Chan and Chui (2000) in their paper which provide an extension of Morris and Shin (1998) find that in some cases currency attacks are prevented by non-transparency. Thus the role of transparency in unique equilibrium models are somewhat unclear.

In addition, in the event of a potential crisis emerging, our model would argue for the involvement of a third party in providing liquidity support in the case of panics. When the equilibrium is unique and runs reflect the value of the fundamentals then the case for liquidity support is less strong.

5 Conclusion

We show that even with noisy signals on fundamentals, multiple equilibria due to a coordination failure exist in models of banking. We argue that the conditions under which this happens arise naturally in models of banking.

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