# Bailouts, Endogenous Financial Regime, and Market Sentiments

Ngo Van Long and Huilan Tian
McGill University

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**Abstract**

This paper examines the role of strategic interactions among a small number of financial intermediaries (FIs) that know that the current financial regime is subject to change with an endogenously determined probability. The current financial regime offers protection to the FIs against bad outcomes, but there is always the possibility that a new regime will be instituted, when the government perceives that the current regime is becoming too costly. Each financial intermediary's (FI's) optimal investment strategy is shown to depend on its rivals' strategies. Their Nash equilibrium strategies are subject to shocks caused by a variety of factors, such as a change in an FI's perception of the state of the economy. It is shown that a small shock that causes an initially small deviation from an FI's equilibrium strategy can lead to a series of deviations by its rivals, creating a multiplier effect, thereby precipitating a change of financial regime. Thus, a change in market sentiments can lead to outcomes that justify it. This result is in the same vein as the "self-fulfilling prophecy" result in the literature on currency crisis or on bank runs. The main innovation in our paper is that we are dealing with a situation where agents behave strategically, in sharp contrast to existing models of currency or financial crisis in which each agent has no weight.Our model is set in continuous time, and the FIs are assumed to be intertemporal maximizers. (Filename:Asia.tex. This version: 21 December 1998.

**Correspondence:** Ngo Van Long, Department of Economics, McGill University, 855 Sherbrooke St West, Montreal, Qc, Canada H3A 2T7.

Fax: 514-398-4938. Telephone: 514-398-4844, 514-985-4011. Email: innv@musicb.mcgill.ca

# 1 Introduction

Writing on Asia's financial crisis, Krugman (1998) pointed to the following major features: (i) moral hazard and overinvestment, (ii) inflated asset prices, and (iii) disintermediation. The moral hazard element is associated with the fact that the governments of these countries implicitly or explicitly guarantee the liabilities of financial intermediaries (FIs). This guarantee has the (unintended) effect of encouraging the FIs to finance highly risky projects. If the assets in which they invest are not in perfectly elastic supply, asset prices will be artificially inflated because, from each FI's viewpoint, bad outcomes are insured against by the implicit promise of bailouts made by the government. Disintermediation occurs when a series of bailouts triggers an unfavorable change in the government's willingness to continue to maintain the current financial regime, because it has become too costly. The death of the feather-bedding financial regime entails the collapse of asset prices, and the bankrupcy of many FIs and other firms.

The most interesting part of Krugman's analysis is that the change in financial regime is endogenous. As Krugman put it, "throughout Asia's arc of crisis there has indeed been a major change in financial regime. Finance companies have been closed, banks forced to curtail risky lending at best and closed at worst; even if the IMF were not insisting on financial housecleaning as a condition for aid, the days of cheerful implicit guarantees and easy lending for risky investments are clearly over for some time to come. But what provoked this change of regime? Not an exogenous change in economic philosophy: financial intermediaries have been curtailed precisely because they were seen to have lost a lot of money."(Krugman, 1998.)

Krugman's informal model is based on the assumption that financial institutions are large in number and behave as if they have no weight: each FI perceives that its actions will have no noticeable impact on the economy, nor on the probability of change of financial regime. Consequently, there are no strategic interactions among the FIs. However, since in some of the countries in question, the blame is often laid on the so-called "cronies capitalism", it seems to us that it would be more appropriate to develop a model in which the major FIs are few in numbers and behave strategically. Each of the major FIs would take into account both the reactions of their rivals, and the impact of their individual investment strategy on the endogenous probability of regime change.

In this paper we present a model where the FIs behave strategically. Each FI can directly influence the politicians' goodwill of maintaining the current financial regime, and indirectly influence the investment decisions of other FIs. We formulate a dynamic game among the FIs, and examine their interactions. One of our key results is that, unlike the "weightless agent models", where a self-fulfilling crisis can arise only if a significant number of market participants are in a pessimistic mood at the same time, in our model, a crisis can begin with a **single agent** that deviates from his equilibrium strategy. Such a deviation can trigger a series of responses that culminate in a crisis.

We will focus on the characterization of a Markov perfect Nash equilibrium (MPNE) and on the effect of a deviation from it by a single agent[1]. A MPNE is a profile of strategies that specify actions at each point in time as a function of the observed level of a relevant state variable. One may justify the use of this equilibrium concept on the ground that: (i) the FIs cannot precommit themselves to a given time path of investment, (ii) they are suspicious of each other and thus are ready to react at each date to any possible deviation from an expected equilibrium path, (iii) their reactions are based solely on the observed level of some relevant state variable, thus admitting the possibilty that no player can observe the action of each opponent. We show that the MPNE tends to lead to a more unfavourable outcome to all FIs, as compared with a precommitment equilibrium[2] where each player can precommit to a whole time path of action.We also examine the stability of the MPNE strategy profile, and show that if one FI deviates from its equilibrium strategy, then other FIs will follow suit, leading to a self-fulfilling crisis.

[1]See Fudenberg and Tirole (1991), Benchekroun and Long (1998), Dockner et al. (1999) for exposition of this concept.

[2]A precommitment equilibrium is also called an Open-Loop Nash Equilibrium. See Dockner et al. (1998).

## 2 A Review of Models of Speculative Attacks with Non-strategic Behaviour.

In this section we review some representative models of speculative attacks. A common feature of these models is that each agent is so small that he has no weight (no significant impact on the state variable) and does not expect anyone to react to his action. We restrict attention to models where agents are fully rational.[3]

The first generation of models of speculative attacks include the work of Salant and Henderson (1978), Krugman (1979), Flood and Garber (1984a,b), and Obstfeld (1984, 1986)[4]. Salant and Henderson (1978) examine, in a continous time model, the concept of rational speculative attack in the context of a gold market where the price of gold is fixed by the government, until the date at which the stock held by the government becomes zero. The government fixes the gold price at $\bar{P}$, and at that price, the government is willing to sell any amount of gold that private agents demand, regardless of whether they want to hold gold as an asset, or use the gold as an input. There is no uncertainty in the Salant-Henderson model; therefore, strictly speaking, no one is really a"speculator". The word "arbitrageur" would be more appropriate. Agents in that model rationally expect that the fixed price cannot last for ever, because the stock is finite, and gold is assumed to be used up gradually (i.e., it is assumed that gold is used as an input in production processes and cannot be recovered[5]). The authors assume that the flow demand function for gold (as an input) is stationary. The price intercept of this demand curve is denoted by $P^{choke}$, which is assumed to be higher than the price fixed by the government.

If the government did not fix the price of gold, then the equilibrium price of gold would rise at the rate of interest, according to the well-known Hotelling Rule. With the fixed price in place, agents know that an equilibrium condition is that when the government's stock is exhausted, there must remain some stock in private hands, for otherwise the price would jump up

[3] There exist models of speculative behavior where not all agents are fully rational. See, for example, De Long et al. (1990).

[4] These models have subsequentlty been modified to take into account other considerations, such as borrowing from foreign central banks, alternative specifications of exchange rate regimes after a collapse, etc. See Blackburn and Sola (1993) for a survey.

[5] This assumption abstracts from the durability aspect of gold, and makes the gold stock have all the characteristics of a stock of a non-durable resource such as oil.

when the government's stock is exhausted, and in a continuous time model, a jump in the price is clearly not compatible with intertemporal arbitrage. But no-one would want to hold gold for any positive interval of time before the date $T$ (the date at which the fixed price regime ends), because agents with such holding before $T$ would incur the opportunity cost of forgone interest income. It follows that, in equilibrium, just an infinitessimal instant before $T$ there must be a subset of agents who rush to buy gold as an asset. The total stock demand by private agents at that instant must be equal to the amount of stock which would be used up gradually as the price rises (at the proportional rate equal to the interest rate) from $\bar{P}$ to $P^{choke}$. Salant and Henderson called this a "speculative attack". Notice that such an attack is rationally expected, and the "speculators" gain nothing. In fact these speculators would be more correctly described as arbitrageurs. Simply put, if there were no subset of agents who would carry out such "speculative activity", then there would be no equilibrium.

Krugman (1979) used the Salant-Henderson idea to model a balance of payments crisis. A modified version of Krugman's model was developed by Flood and Garber (1984) in which they derived an analytical solution for the time of collapse. In their modified model, also in continuous time, it is assumed that the government maintains a fixed exchange rate $S$ as long as the central bank's stock of foreign reserves remains positive, and that as soon as this stock is exhausted[6], the government will let the exchange rate float. The government creates domestic credit (e.g by printing money) at the rate $\mu > 0$ per period, and will continue to do so even after the stock of reserves is exhausted[7]. During the fixed exchange rate phase, the domestic price $P$ is constant because it is assumed that the foreign price is constant and that purchasing power parity (PPP) holds. The domestic interest rate $r$ is also constant, being equal to the constant foreign interest rate $r^*$ minus the expected rate of depreciation, which is zero, under the fixed exchange regime. Real money demand, $M^d/P = f(Y, r)$ is a constant because $Y$ is a constant by assumption, and so is $r$. Equilibrium in the money market means that the money supply (the sum of the domestic credit $D$ and the central bank's holding of foreign reserves $R$) must be equal to the constant $M^d$. This in turn implies that, since $D$ is growing, the maintenance of the

[6]Of course, one can replace this assumption by an alternative one, postulating that the fixed echange rate regime will end as soon as the reserve stock falls to a given level $R_L$.

[7]This is a simplifying assumption. Alternative assumptions about the behavior of the government after the exhaustion date can be formulated. See for example Obstfeld (1984).

fixed exchange rate $\bar{S}$ involves a loss of foreign reserves at the rate equal to $\mu$. It follows that the fixed exchange rate regime cannot last for ever. (Note the parallel between this model and that of Salant and Henderson.)

When the fixed exchange rate phase ends, the domestic interest rate must jump up (because it must equal the sum of foreign interest rate and the rate of currency depreciation, which is strictly positive in the floating rate phase because domestic credit $D$ still grows at the rate $\mu$). Since the time path of the exchange rate must be continuous in any continuous time model with perfect foresight, and since the path of price is also continuous (by PPP and the continuity of the time path of exchange rate $S(t)$), it follows that the upward jump in the domestic interest rate must be matched by a corresponding downward jump in the economy's money supply. Since the domestic credit does not jump, this money supply jump consists of a downward jump in the stock of foreign reserves. This jump is brought about by arbitrageurs who rush to buy foreign reserves. Just like in the Salant-Henderson model, arbitrageurs gain nothing. The exchange rate path is continuous, and it has a kink at the time the fixed exchange regime collapses.

The above model cannot explain a real world phenomenon: a weak currency's forward exchange rate often exceed the fixed rate for a long time, before the actual collapse. (This is known as the "peso problem".) To explain this phenomenon, Flood and Garber (1984, section 3) present a discrete time model with uncertainty: domestic credit follows a random walk with drift :$D_{t+1} = D_t + \mu_t$ where $E(\mu_t) = \mu > 0$. (We will call this process "FG credit creation process"). This model gives rise to a predictable collapse of the fixed rate regime, though the exact time of collapse is a random variable. Obstfeld (1986, pp. 75-76) reinforces this result by showing that, under the Flood-Garber assumption on domestic credit creation, i.e., the "FG credit creation process", as soon as the shadow exchange rate exceeds the fixed rate, a run is the only possible equilibrium outcome. On the other hand, Obstfeld (1986, pp. 73-74) also shows that if the domestic credit is assumed to have a *constant mean* with a serially correlated disturbance with finite variance, i.e.,

$$D_t = \bar{D} + v_t$$

where $v_t = \rho v_{t-1} + \varepsilon_t$ and $0 \leq \rho < 1$ and $E\varepsilon_t = 0$, and if shocks are small, (we will call this process "CM credit creation process") then an exchange rate collapse almost never happens (i.e. a collapse is a probability-zero event)[8].

[8]As Obstfeld pointed out (p. 74), under the stated assumption, if an attack ever

It would be more interesting to construct a model where there are many equilibria each of which can be brought about by self-fulfilling expectations[9]. One such model is described in Obstfeld (1986, pp.76-78), where it is assumed that agents believe that the government will switch from the "CM credit creation process" to the "FG credit creation process" if a collapse occurs. Then it can be shown that if everyone believes that there will be a run, they will find it advantageous to participate in the run, thus making their expectations self-fulfilling[10]. Runs are made possible by endowing individuals with rational subjective probability of runs[11].

In the preceeding models, the government is assumed to follow an ad hoc rule of credit creation, or an ad hoc response to a run. Realizing that governments perhaps optimize some objective function, Obstfeld (1994) develop two models in which the government's policy is chosen by an optimization process. In both models, the government cannot precommit its future actions. The first model is a simple two-period model. The key factors in that model are the maturity structure of the government's domestic bonds, and the currency composition of the public debt. The second model is an infinite horizon model, in which the wage rate for each period is set in the preceding period, and the government would devalue to offset a substantial negative output shock. (The model is simplified by abstracting from considerations of the stock of reverves, and from other dynamic considerations; in fact, the government's optimization problem becomes a static one.) Private agents believe that a devaluation will be chosen by the government if the adverse output shock exceeds a certain threshold level $\bar{u}$. This level is endogenously determined, by rational expectations and government's optimizing behavior. In both models, multiple self-fulfilling equilibria emerge.

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occurs (an event off the equilibrium path), the exchange rate would appreciate rather than depreciate, and therefore no individual would want to participate in such a run.

[9]This type of models is similar to the bank runs model analyzed by Douglas Diamond and Philip Dybvig (1983). However, unlike the D&D bank runs model, in Obstfeld (1986, pp.77-78), the run can occur only if the government responds to the run by switching to the FG credit creation process.

[10]In this model, the exchange rate will depreciate after a run, because it is assumed that the government will switch from the CM credit creation process to the FG credit creation process as soon as a run occurs.

[11]See also Blanchard (1979), Azariadis (1981), Cass and Shell (1983).

# 3 Moral Hazard

The models surveyed in the preceeding section throw lights on aspects of speculative behavior and self-fulfilling expectations. However, as pointed out by Krugman (1998), the fundamentals that drive those models do not seem to have been present in the Asian economies. He wrote: "On the eve of crisis all of the governments were more or less in fiscal balance; nor were they engage in irresponsible credit creation or runaway monetary expansion. Their inflation rates, in particular, were quite low." He attributed the crisis to unregulated financial intermediation[12].

Financial intermediaries whose liabilities are guaranteed are likely to invest in projects that are too risky, and even prefer projects with low expected return but with a long right-hand tail to those projects that have higher expected return but with a shorter right-hand tail. Here, the expected returns differ from the expected private returns (by private, we mean the return to the FIs), because negative returns will be converted to zero private returns (to the FIs) by government guarantee. This guarantee creates a serious moral hazard problem, which, as Krugman points out, has its parallel in the US: the famous Savings and Loans crisis.

Milgrom and Roberts (1992, p.167) defined moral hazard as "the form of postcontractual opportunism that arises because actions that have efficiency consequences are not freely observable and so the person taking them may choose to pursue his or her private interests at others' expenses." They explained the US "savings and loan crisis" (pp170-176) in the late 1980s in terms of moral hazard. The savings and loan associations (S&Ls) borrowed money from the public and lent it to business enterprises or individuals. The deposits were insured by the Federal Savings and Loan Insurance Corporations (FSLIC). The S&Ls paid (to the FSLIC) insurance premia that were independent of the riskiness of their loans. There were no incentives for S&L owners to avoid excessively risky projects. Over a period of reckless investments, hundreds of S&Ls went bankrupt, and FSLIC's reserves proved

[12] "In Thailand a crucial role was played by the so-called "finance companies"- non-bank intermediaries that borrowed short-term money, often in dollars, then lent that money to speculative investors, largely but not only in real estate. In South Korea more conventional banks were involved, but they too borrowed extensively at short term and lent to finance what in retrospect were very speculative investments by highly leveraged corporations." (Krugman, 1998).

inadequate to pay the depositors. It were the US taxpayers who finally shouldered the burden. The depositors themselves had no incentives to monitor the investments of S&Ls, because their deposits were insured. Politicians were another important group of players in this game. Many politicians received campaign donations from some S&Ls. They had an incentive to protect the S&Ls industry against regulators. According to Milgrom and Roberts (1992, p.177), "these politicians raised the amount of insurance provided by the FSLIC, thereby making it easier to attract large deposits. They relaxed the regulations on the S&Ls and did not provide for an offsetting increase in monitoring. Furthermore, when the S&Ls were first headed for financial trouble, politicians blocked the regulators from intervening."

# 4 A Model of Moral Hazard with Strategic Behaviour

Krugman's analysis of the Asian crisis lays the blame on the moral hazard problem in a financial regime where the FIs are insufficiently regulated. From society's point of view, the FIs overborrowed and took excessive risks in their lending, though it was in their private interest to do so. Overinvestment pushed up asset prices, especially those assets that are in fixed (or inelastic) supply, such as land. Even though investors knew that the returns to investment projects were random, and that "bad states" could occur, as long as they believed that liabilities of the FIs were guaranteed, their calculations would be based on the assumption that they lived in a world where "bad states" did not exist, i.e., in the world of Pangloss, the much ridiculed philosopher in Voltaire's superb satiric work, Candide[13].

However, if agents are rational, they should know that there is a positive probability that the financial regime of government's guarantee will one day come to an end. The bubble may burst after a series of occurence of some bad states. At some stage, after a number of bailouts, the government could find it too costly to continue the guaranteed liability regime. Rational agents know that the probability that a financial regime change occurs at any given time is endogenous. However, if individual agents are "small", each will perceive that his action has "no weight", i.e. no impact on the endogenous

[13]If an investment project has a random return which may be high or low, the high return is called by Krugman the Pangloss value.

probability. A collapse can happen in this "weightless agents" model: if all agents suddenly find themselves in a gloomy mood and believe that everyone else is trying to sell their assets, or withdrawing their deposits, thus creating a liquidity problem for the FIs, and a possible change in financial regime, they will do the same thing to minimize losses. The government will then find it too costly to maintain the existing financial regime.Their prophecy thus becomes self-fulfilled. On the other hand, if investors are in an optimistic mood, and investment projects are not hit by a series of unlucky events, then no collapse will occur. This is basically Krugman's model of multiple equilibria with a possibility of collapse. It adds to the traditional models of multiple equilibria by introducing the moral hazard element.

Our model, which we are going to outline below, is built on the Krugman's model, but we add a new feature: the FIs are modelled as agents that have weight. Unlike the "weightless agents" model, the "weighty agents" model involves strategic behaviour. Implicit in the Milgrom-Roberts analysis of the S&Ls crisis is the recognization that some S&Ls had some weight, for they contributed to campaign funds. However, Milgrom and Roberts did not analyze how such weighty agents would interact with each other. In our model, we propose to do this.

There are two possible approaches to the idea of regime change. The first approach is to model the probability of a change in financial regime as being exogenous, and independent of the actions of the FIs. For example, there could be a change of government, or a change in the economic thinking of the existing government, or a change forced upon the government by some powerful international agencies. The second approach consists of assuming that the government has a "stock of goodwill" (i.e., willingness to provide with bailouts) toward the FIs, and this stock can increase or decrease over time, depending on a number of factors. Lobbying by the FIs can increase the stock, and the accumulated number of bailouts may diminish the stock.

Let $S(t)$ denote the stock of goodwill at time $t$. We assume that its rate of change is given by

$$\dot{S}(t) = \phi(S(t), L(t)) - F(b_1(t), b_2(t), ..., b_n(t))$$

where $L(t)$ denote the total lobbying effort by the FIs and $b_i(t)$ is the amount of bailout money given to firm $i$. We take it that $F$ is an increasing function, and that $\partial\phi/\partial L \geq 0$. The initial stock is $S(0) = S_0 > 0$.

We postulate that the guaranteed liability regime will come to an end at the date when the stock of goodwill is exhausted, i.e. when $S = 0$, unless it has ended before that date. Let $T$ denote the endogenous exhaustion time, i.e. $S(T) = 0$ and $S(t) > 0$ if $t < T$. In addition to this endogenous trigger which forces a regime change, we also allow for the possibility of a regime change that occurs exogenously. To do this, we assume that even before the "exhaustion time" $T$, there is also a positive probability that a regime change will occur over any time interval $[t, t+h)$. This is modelled as follows. For expositional simplicity, suppose, for the moment, that the half real line $[0, \infty)$ that represents time is partitioned into mutually disjoint intervals of equal length $h$: $[0, t_1)$, $[t_1, t_2)$, $[t_2, t_3)$ , etc. where $t_j - t_{j-1} = h$. Let $R(t)$ be a random variable that can take on either of two values, 0 or 1. Initially, $R(0) = 1$, indicating that the government is in favour of liability guarantees. $R(t)$ can change its value only at discrete points $t_1$, $t_2$, $t_3$, etc. Let $t_0 = 0$. The probability that $R(t_i) = 0$ given that $R(t_{i-1}) = 1$ is denoted by $1 - \pi(h)$, where $0 \leq \pi(h) \leq 1$. Furthermore, assume that if $R(t_i) = 0$ then $R(t_{i+1}) = 0$ with probability 1. Given $h$, consider any date $\tau > 0$. Then there exists a unique integer $n$ such that $nh \leq \tau < (n+1)h$. The probability, denoted by $P(\tau)$, that $R(t) = 1$ for all $t \leq \tau$ is $\pi(h)^{nh}$. Now let us take the limiting case where $h$ tends to zero, keeping $\tau$ unchanged (and allowing $n$ to change so that $nh \leq \tau < (n+1)h$). Then

$$P(\tau) = \lim_{h \to 0} [\pi(h)]^{\tau}$$

We assume that $\lim_{h \to 0} \pi(h) = e^{-\mu}$ where $\mu$ is a positive constant. Thus $P(t) = e^{-\mu t}$ is the probability, as perceived at time 0, that $R$ has not been switched to 0 at time $t$, and $\Lambda(t) = 1 - P(t)$ is the probability, as perceived at time 0, that $R = 0$ at or before time $t$. The time derivative of $\Lambda(t)$ is the probability density that an exogenously driven regime change occurs at time $t$ (given that $S$ has not been exhausted at $t$):

$$\dot{\Lambda}(t) = \mu e^{-\mu t}$$

We will refer to the parameter $\mu$ as the "arrival rate" of this Poisson process[14].

Taking into account both the endogenously driven probability of a regime change caused by the exhaustion of $S$, and the exogenous arrival rate $\mu$ of

[14] Thus $\dot{\Lambda}(t)/(1 - \Lambda(t)) = \mu$. This is equivalent to the statement that

$$\lim_{\Delta \to 0} \frac{1}{\Delta} \Pr\{R(t + \Delta) = 0 \mid R(t) = 1\} = \mu.$$

an exogenously driven regime change, we define the variable

$$Z(t) = R(t)S(t)$$

so that $Z(t) > 0$ indicates that at $t$ the regime of liability guarantee is still in operation, and $Z(t) = 0$ indicates that the regime has been changed. $Z(t) = 0$ can be due to $R(t) = 0$ while $S(t)$ is still positive, or it can be due to $S(t) = 0$ while $R(t)$ is still equal to 1.

We now turn to the optimization problem of the representative FI. Assume that at each date $t$, the FI has an opportunity to invest in a risky project, and the rate of return *per dollar* invested in this project is $\beta$, a random variable. We assume for simplicity that the time lag between the investment date and the outcome date is $k \geq 0$ and that there are two possible outcomes: good or bad, with probability $p$ and $(1-p)$ respectively. In the good outcome case, $\beta$ takes the value $\beta_H > e^{rk}$ where $r$ is the risk-free interest rate, and in the bad outcome case, $\beta = \beta_L$ where $0 \leq \beta_L < 1 < e^{rk}$. We assume that $p\beta_H + (1-p)\beta_L < e^{rk}$. This assumption means that the project is not worth undertaking if there is no implicit subsidy, and if the decision maker is risk averse or risk neutral. If the outcome is good, the FI pays the depositor $e^{rk}$ dollars for each dollar deposited, and thus makes a profit of $\beta_H - e^{rk}$. If the outcome is bad, the government will give a bailout amount $1 - \beta_L$ for each dollar invested so that the FI can pay back to the depositor one dollar. If there were no liability guarantee, the expected rate of return to the FI is: $E\beta - e^{rk} = p\beta_H + (1-p)\beta_L - e^{rk}$. With government guarantee, the expected return is $p\beta_H + (1-p) - e^{rk}$ which we assume to be positive..

Let $X_i(t)$ be the amount of dollars invested by firm $i$. The expected amount of money spent to bailout firm $i$ at time $t+k$ is $b_i(t+k) = (1-p)(1-\beta_L)X_i(t)$.

We assume that each FI cares only about the expected value of the integral of the flow of its discounted profits. The rate of discount is $\delta_i$, a positive constant. If there is no possibility of a future change in the financial regime, the FI's expected profit for its investments made at date $t$ is $(p\beta_H + (1-p) - e^{rk})X_i(t)$. However, the FI knows that the financial regime will change when the variable $Z(t)$ becomes zero. This can come about either because the stock of goodwill $S(t)$ falls to zero, or the random variable $R(t)$ jumps to zero. The FI spends an amount $L_i(t)$ on lobbying activities to increase the stock of goodwill. Let $V_i(S, R, t)$ denote the value function of the $ith$ FI, when the stock of goodwill is $S$ and the value of $R(t)$ is $R$. (Recall that $R$ can take on only the value 1 or 0.)

Using the framework expounded in Rishel (1975), Flemming and Soner (1993), Dockner, Jorgensen, Long and Sorger (1998, Chapter 8), we can write fundamental recursive relation for the value function for the FI as follows:

$$V_i(S(t), 1, t) = \max \int_t^{t+dt} \exp(-(\delta_i + \mu)(s-t))\Pi(t)ds$$

$$+\exp(-\delta_i dt)[V(S(t+dt), 1, t+dt)e^{-\mu dt} + V(S(t+dt), 0, t+dt)(1-e^{-\mu dt})] \quad (1)$$

where $\Pi(t) = (p\beta_H + (1-p) - e^{-rk})X_i(s-k) - L_i(s))$, and where $e^{-\mu dt}$ is interpreted as the probability that $R(t)$ remains unchanged (at $R = 1$) over the infinitessimal time interval $dt$. In what follows, to simplify the problem, we assume that the time lag $k$ is insignificant, and set $k = 0$. Then equation (1) gives rise to the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \delta V_i(S(t), 1, t) = \max\{&p(\beta_H - 1)X_i(s) - L_i(s) + \\ &\lambda_i[\phi(S(t), L(t)) - F(b_i(t), b_{-i}(t))] + \\ &\mu_i[V_i(S(t), 1, t) - V_i(S(t), 0, t)]\} + \frac{\partial}{\partial t}[V_i(S(t), 1, t)] \end{aligned} \quad (2)$$

where

$$\lambda_i = \frac{\partial V_i(S(t), 1, t)}{\partial S}.$$

## 4.1 Markov perfect Nash equilibrium

We now analyze a simple version of the game between the FIs. For simplicity, we abstract from lobbying, and assume a specific functional form for $\phi$ and $F$. We let $\phi(S) = aS$, $a \geq 0$, and

$$F(b) = \sum_{i=1}^{n} b_i(t)^{\gamma_i}$$

where $0 \leq \gamma_i \leq 1$. Then

$$\dot{S}(t) = aS(t) - \sum_{i=1}^{n} b_i(t)^{\gamma_i} \quad (3)$$

where $\gamma_i > 1$ and $a \geq 0$. Recall that $b_i$ is the expected amount spent on bailing out firm $i$:

$$b_i(t) = (1-p)(1-\beta_L)X_i(t) \tag{4}$$

Furthermore, we assume that if $S = 0$ (i.e., when the liability guarantee regime is no longer operative), then the value of the firm is zero. Then equation (2) gives the following Hamiltonian function

$$H_i = p(\beta_H - 1)X_i + \lambda_i[aS - \sum_{i=1}^{n} b_i{}^{\gamma_i}] \tag{5}$$

where

$$\dot{\lambda} = (\mu + \delta_i)\lambda - \frac{\partial H}{\partial S}.$$

In what follows, we assume that $\sum_i (1/\gamma_i) < 1$ and that $a/\gamma_i < \delta_i + \mu$. In this case we obtain the following proposition which characterizes the equilibrium of the game:

**Proposition 1**: There exists a Markov perfect Nash equilibrium strategy profile where firm $i$'s investment $X_i$ is an increasing and concave function of the stock of goodwill. Furthermore, $X_i$ is increasing in the probability $p$ of the good state, and increasing in $\beta_L$. The Markov perfect Nash equilibrium strategy profile is given by

$$X_i = \frac{(\varepsilon_i S)^{\alpha_i}}{(1-p)(1-\beta_L)} \tag{6}$$

where $0 < \alpha_i = 1/\gamma_i < 1$, and where the $\varepsilon_i$ $(i = 1, 2, ..., n)$ are the solution of the matrix equation:

$$\begin{bmatrix} 1\text{-}\alpha_1 & \text{-}\alpha_1 & ... & \text{-}\alpha_1 \\ \text{-}\alpha_2 & 1\text{-}\alpha_2 & ... & \text{-}\alpha_2 \\ ... & ... & ... & ... \\ \text{-}\alpha_n & \text{-}\alpha_n & ... & 1\text{-}\alpha_n \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ ... \\ \varepsilon_n \end{bmatrix} = \begin{bmatrix} \delta_1 + \mu - a\alpha_1 \\ \delta_2 + \mu - a\alpha_2 \\ ... \\ \delta_n + \mu - a\alpha_n \end{bmatrix} \tag{7}$$

**Proof**: See the Appendix.

**Remark:** The equilibrium strategies are independent of $\beta_H$. However, $\beta_H$ does affect the value of the firm, which is proportional to $(\beta_H - 1)$. This can be inferred from the proof of Proposition 1.

**Proposition 2**: If all FIs are identical, then the symmetric equilibrium strategy is

$$X_i = \frac{(\varepsilon S)^{\alpha}}{(1-p)(1-\beta_L)} \tag{8}$$

where $\varepsilon$ is an increasing function of the number of FIs:

$$\varepsilon = \frac{(\delta + \mu) - a\alpha}{1 - n\alpha} > 0, \tag{9}$$

and along the equilibrium play, the stock of goodwill will evolve as follows

$$\frac{\dot{S}}{S} = \frac{a - n(\delta + \mu)}{1 - n\alpha} \tag{10}$$

implying that (i) goodwill is depleted gradually if $n(\delta+\mu)$ exceeds $a$, and (ii) the rate of depletion of goodwill is greater, the greater the number of FIs.

**Proof**: Omitted.

It is of some interest to compare the MPNE with the Open-loop Nash equilibrium in which each FI is able to precommit itself to a given time path of investment. We obtain the following result:

**Proposition 3**: Compared with the symmetric Open-loop Nash Equilibrium, the symmetric MPNE results in a faster rate of depletion of the stock of goodwill.

**Proof**: Omitted.

## 4.2 Stability of the Markov perfect Nash equilibrium.

In this subsection we study the stability of the Markov perfect Nash equilibium strategy profile. Our concept of stability is as follows. Assume that initially, all FIs are following their equilibrium strategies, as described in Proposition 1. Suppose that one player, say player 1, decides to deviate from its equilibrium strategy over some time interval, say $[0, \tau]$. We wish to determine the response of other players to this deviation. For concreteness, suppose that FI 1 decides to invest more in the risky project, thus increasing the amount of bailout that it expects to obtain in the event of a bad outcome. Would other FIs respond by investing less, as the theory of oneshot Cournot duopoly would seem to suggest? For concreteness, suppose that player $i$ contemplates a deviation from its equilibrium strategy, and considers playing the following strategy:

$$X_i^d = \frac{((\varepsilon_i + v(t))S)^\alpha}{(1-p)(1-\beta_L)}$$

where $v(t)$ is positive over the interval $[0, \tau]$, and $v(t) = 0$ elsewhere.

It turns out that in our model, a deviation by FI 1 will trigger a deviation **in the same direction** by all other FIs. Furthermore, the magnitude of their responses can be **greater** than the deviation by FI 1. In addition, the magitude of their deviations increases over time over the interval [0, $\tau$].This result is the exact opposite of the kind of response obtained in a static Cournot model. The following proposition summarizes this remarkable result:

**Proposition 4**: The MPNE is not stable with respect to deviation from the equilibrium strategy profile. A deviation by one FI will trigger greater deviations by other FIs, leading to a possible collapse of the existing financial regime.

**Proof**: available upon request.

**Remark**: Our result indicates that if one FI believes that there is an increase in the probability of a regime change, (perhaps because of some observed bad realisations), such belief will tend to be self-fulfilling, as it creates responses which will result in a faster rate of depletion of goodwill.

# 5 Concluding remarks

We have showed that when agents have weights, then a deviation of a single agent from its equilibrium strategy can lead to responses that destabilizes the system, leading to an endogenous change of regime. Unlike the weightless agents models in the litterature on financial crises, where self-fulfilling expectations require a substantial unanimity, our model indicates that a system can collapse with the deviation of a single agent, because responses by other agents tend to exacerbate the overinvestment problem. This result is rather striking, because it stands in sharp constrast with the standard one-shot Cournot model, where if one firm produces more, its opponents will reduce their outputs.

**APPENDIX**

**Proof of Proposition 1:**

The proof relies on a transformation of variables. Define $c_i = b_i^{\gamma_i}$. Then the transition equation becomes

$$\dot{S}(t) = aS(t) - \sum_{i=1}^{n} c_i(t) \tag{11}$$

and

$$X_i = \frac{b_i}{(1-p)(1-\beta_L)}$$

Then player $i$'s control problem becomes

$$\max \left[\frac{p(\beta_H - 1)}{(1-p)(1-\beta_L)}\right] \int_0^\infty e^{-(\delta_i+\mu)t} c_i(t)^{\alpha_i} dt$$

subject to (11). The necessary conditions for player $i$'s problem are (11) and

$$\alpha_i c_i^{\alpha_i - 1} = \lambda_i$$

$$\dot{\lambda}_i = (\delta_i + \mu) - a\lambda_i + \lambda_i \sum_{j \neq i} \frac{\partial c_j}{\partial S}$$

Substituting $c_j = \varepsilon_j S$ into the necessary conditions, we obtain the conditions

$$\alpha_i (\varepsilon_i S)^{\alpha_i - 1} = \lambda_i \tag{12}$$

$$\frac{\dot{\lambda}_i}{\lambda_i} = \delta_i + \mu - a + \sum_{j \neq i} \varepsilon_j \tag{13}$$

$$\frac{\dot{S}}{S} = a - \sum_{i=1}^{n} \varepsilon_i \tag{14}$$

Equation (12) yields:

$$-(1-\alpha_i)\frac{\dot{S}}{S} = \frac{\dot{\lambda}_i}{\lambda_i} \tag{15}$$

Substituting (13) and (14) into (15), we get

$$\delta_i + \mu - a\alpha_i = (1-\alpha_i)\varepsilon_i - \alpha_i \sum_{j \neq i} \varepsilon_j$$

Since this must hold for all $i$, we obtain the matrix equation in Proposition 1. This matrix equation has a unique positive solution $(\varepsilon_1, ..., \varepsilon_n)$. See Takayama (1985, Theorem 4.C.4). Finally, one can verify that the transversality conditions are satisfied.

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