Trade liberalization and endogenous growth

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date :8 April 2001. Filename: WTO2.tex
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Abstract:

1 Introduction

Efforts by the WTO to liberalize trade have met with warm reception in some sections of the community, and outright hostilities from some quarters. Among the questions raised are (i) the validity of the "trickle-down" hypothesis, (ii) the possible destruction of the environment, (iii) the possible negative impact of globalization on cultural diversity, and (iv) convergence or divergence of wealths of nations. In this paper, we will focus on the last issue. Does liberalization in trade flows and international financial flows lead to convergence of income and wealth, or does it lead to divergence, and exacerbate inequalities among nations? Another way of putting this question is: does liberalization reinforce negative feedback forces or positive feedback forces?

Negative feedback is stabilizing: it prevents a system from exploding. Automatic built-in stabilizers such as progressive income tax rates and unemployment insurance payouts aremacroeconomic examples of negative feedback forces [1]. Positive feedback processes have an unstable, runaway quality. Some models of speculative bubbles display positive feedback properties, see De Long et al (1990). In chemistry, the typical positive feedback process is an explosion. Arms races that lead to wars are examples of positive feedback in history. Mathew's Gospel has an example of positive feedback: *"Unto everyone that hath shall be given, and he shall have abundance; but from him that hath not shall be taken away even that which he hath".* Positive feedback can lead to extreme divergence.

Pessimists argue that trade liberalization will eventually make poor countries poorer and the wealthy countries wealthier. For example, deforestation in the poor countries (to supply the wealthy consumers of advanced industrialized economies with forest products) will eventually damage the environmental stock of the poor countries. Another example is that freer trade may force poor economies to specialize in traditional farming activities, resulting in a lower rate of human capital accumulation[2]. (The assumption here is that the manufacturing sector generates beneficial spillovers to the skilled workers).

[1]In engineering, one of the often cited examples of negative feedback is the Watt steam governor. It is an automatic valve regulating the flow of steam into the piston. The faster the engine runs, the more the valve shuts down the steam. This ensures that the engine can run at a constant rate despite considerable fluctuations in the firebox.

[2]See Young (1991), Stokey (1991), Long, Riezman, and Soubeyran (2001).

On the other hand, freer movements of capital mean that workers in poor countries will be able to work with more capital than under autarky. A higher capital labor ratio results in a higher wage rate, which permits more saving and capital accumulation. Potentially, this could lead to equalization of ownnership of wealth.

This paper begins with a selective survey of the literature on trade and growth, and presents two models that highlight the relationship between trade liberalization and growth. The models are not meant to be comprehensive. They serve to point out some negative or positive feedback forces that may be present in a globalized economy.

The first model focuses on a single source of negative feedback that leads to convergence of income. It is based on Long and Shimomura (2001). They postulate that an individual's utility may be a function of several variables, one of which is relative wealth. An individual's concern about his relative position in society may have an influence on his saving behavior. In an international context, with increased opportunities for international travels and interactions, "society" is no longer a local community. The whole world has become a global village. Thus a Vietnamese may not just compare himself with his Vietnamese neighbors: he may want to catch up with his wealthy Singaporian business partner. A simple way to model the process of catching up is to postulutate a *reduced-form utility function*[3] which has relative wealth as an argument. We will show that the recognition of relative wealth in the reduced-form utility function can explain a number of phenomena, such as differences in growth rates, catching-up, etc.

Our second model is based on Long, Riezman, and Soubeyran (2001). Its main concern is the effect of trade liberalization on the relative wage of skilled workers, and hence on the incentive to accumulate skills. We show that trade can make less developed countries specialize in traditional activities, and reduce their welfare in the long run. In other words, the static gains from trade may be offset by a long-term loss that results from failing to specialize in activities that would generate beneficial spillovers.

A brief, selective survey of the literature on trade and growth is presented in section 2. Section 3 provides a motivation for our model on catching-up, and presents an overview of the universal phenomenon of status seeking. Section 4 considers a model of relative wealth and catching up. It contains also the benchmark case with identical agents who seek to maximize the value

[3]For an interesting model leading to reduced form preferences, see Cole et al. (1992)

of their discounted stream of utility. It is shown that their concern about their relative wealths leads to more capital accumulation, as compared to the standard Cass-Ramsey model. We next consider the case with two classes of agents: the poor and the rich. We demonstrate that the poor will finally catch up with the rich if the marginal utility of relative wealth is very high when relative wealth is low. In an Appendix, we show how, in the the framework of the so-called AK endogenous growth model, economies with greater degrees of status-consciousness achieve higher permanent growth rates, but lower welfare.

The remaining sections are devoted to the exposition of a model by Long, Riezman and Soubeyran, on *firm-specific* human capital accumulation.This model differs from existing models in that human capital is assumed to be firm-specific. This means that there is a bilateral monopoly problem between the firm and the worker. Wages are no longer equalized with marginal products. In this context, free trade can be harmful, even though there are no knowledge spillovers.

2 Trade and Growth: A Selective Survey

This section draws attention to some salient features of the literature on trade and growth. It is based on the survey by Long and Wong (1997). (The references cited in this section are as in Long and Wong).

2.1 Learning-by-doing, trade and growth

Much of growth can be attributed to learning-by-doing. The simplest dynamic learning-by-doing model in an international trade context is Lucas (1988). He postulates a two-sector model with Ricardian technology. The marginal product of labor in sector i grows at a rate proportional to that sector's current output. Thus human capital is industry specific.(It is assumed that human capital does not belong to any worker, or any firm; rather it "belongs" to an industry.)Under free trade without government intervention, countries will tend to specialize, therefore each country will accumulate only the type of human capital that is specific to the good it produces. Countries *do not converge* in growth rates. This model implies that protection (at least for some initial phase of development) may be better than free trade.

Young (1991) allows for spillovers across different industries. For example, advances in the pharmaceutical sector may benefits the bio-tech industry. Stokey (1991) distinguishes individual human capital from the country's stock of knowledge. A LDC that embraces free trade may experience a fall in the accumulation of human capital, because free trade reduces the rewards to the skilled labor of LDCs. On the other hand, authors such as Van and Wan (1996) argue that free trade is a channel through which an economy can learn from more advanced economies.

2.2 Spillovers through trade

Rivera-Batiz and Romer (1991a, QJE) show that if there are free flows of ideas, two countries that engage in the exchange of ideas will experience an initial doubling of the stock of knowledge. Even if there is no expansion of employment in the research sector, the growth rate of both countries will double. They will more than double when there is expansion of employment in the research sector.

Walde (1996) shows that with perfect knowledge spillovers across national boundaries, countries will converge to a common growth rate. Feenstra (1996) on the other hand argues that spillovers may depend on the volume of imported foreign inputs, then countries may not converge in growth rates.

2.3 R&D and Impacts of Tariff and Subsidies

Rivera-Batiz and Romer (1991b, EER) show that if two countries produce non-overlapping intermediate goods, a tariff on these goods has a non-monotone effect on their common growth rate. Basically, the tariff has two opposing effects: a distortion effects on the use of foreign inputs, and a R&D resource allocation effect. Grossman and Helpman (1990b) consider two countries with different comparative advantage. Assume country 1 has comparative advantage in R&D. A subsidy by country 2 on its own R&R can hurt country 1's R&D, and the effects can be harmful to both countries (assuming there is spillover.)

3 Catching-up, Relative Wealth and Status Seeking: an Overview

Many economists have pointed out that it is not wealth *per se* that is wanted; rather wealth (in the sense of relative wealth) is valued because it gives access to non-market goods such as status and influence. The relationship between relative wealth and non-market goods was aptly expressed in Adam Smith's 'The Theory of Moral Sentiments':

"To what purpose is all the toil and bustle of the world?...It is our vanity that urges us on...It is not wealth that men desire, but the consideration and good opinion that wait upon riches" .[4]

Status seeking is common in human and animal species. Two major features of social life in many species of animals are territoriality and dominance hierarchies. Hens compete for high positions in the "peck order" (see Dawkins, 1976, p. 88 and 122)."If a batch of hens who have never met before are introduced to each other, there is usually a great deal of fighting. After a time the fighting dies down...It is because each individual 'learns her place' relative to each other individual. This is incidentally good for the group as a whole. As an indicator of this it has been noticed that in established groups of hens, where fierce fighting is rare, egg production is higher than in in groups of hens whose membership is continually being changed, and in which fights are consequently more frequent" (p.88).

Contests among members of a group take time, and, in the long run, a hierarchy is established. "Crickets have a general memory of what happened in past fights. A cricket which has recently won a large number of fights become more hawkish. A cricket which has recently had a losing streak becomes more dovish. This was neatly shown by R. D. Alexander. He used a model cricket to beat up real crickets. After this treatment the real crickets became more likely to lose fights against other real crickets. Each cricket can be thought of as constantly updating his own estimate of his fighting ability, relative to that of an average individual in his population. If animal such as crickets... are kept together in a closed group for a time, a kind of dominance hierarchy is likely to develop"(Dawkins, 1976, p. 88-89).

Why do individuals in an animal society want high social rank? Wynne-

[4]Quoted by Cole et al. (1992, p. 1092). They also quote: "The boy with the cold hard cash is always Mister Right because we are living in the material world and I am a material girl." [Madonna, 'Material Girl'].

Edwards (1962) sees high social rank as a ticket of entitlement to reproduce. "Instead of fighting directly over females themselves, individuals fight over social status, and then accept that if they do not end up high on the social scale they are not entitled to breed. They restrain themselves where females are directly concerned, though they may try every now and then to win higher status, and therefore could be said to compete *indirectly* over females."(Dawkins 1976. p. 123)[5]

In human societies, an agent's status is "a ranking device that determines how well he or she fares with respect to the allocation of non-market goods" (Cole et al., p.1093). Examples of non-market goods are membership of the board of trustees of a prestigious university, and the types of friends or partners for your children. In the model developed by Cole et al., a couple, by deciding how much to bequeath to their son, can influence the quality of his mate: "Parents will be willing to reduce their consumption if it sufficiently increases the quality of their son's mate" (p. 1099).

Status seeking may result in a Rat Race, with negative welfare effects: if everyone tries to run faster, it is possible that while more effort is expended, the relative ranking may remain unchanged. This principle applies not only for races among individuals of a given species, but also for races between different species. The idea of zero change in success rate has been given the name of "the Red Queen Effect" by the American biologist Leigh van Valen (1973). In Lewis Carroll's *Through the Looking-Glass* (1872), the Red Queen seized Alice by hand and dragged her, faster and faster, on a frenzied run, but no matter how fast they ran, they always stayed in the same place. The puzzled Alice commented that " Well in *our* country you'd get to somewhere else- if you ran very fast for a long time as we've been doing". To this the Queen replied: "A slow sort of country! Now, *here*, you see, it takes all the running *you* can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that."

The possible adverse welfare effects of competition have been noted by non-economists as well as economists. The following paragraph from Richard Dawkins's "The Blind Watchmaker" is illuminating:

[5]Malte Andersson, from Sweden, conducted an interesting experiment on the long-tailed widow birds in Kenya. (In the breeding season, the tail of a male long-tailed widow bird can be 18 inches long.) Andersson caught 36 male widow birds, shortened the tails of some and use the feathers thus obtained to lengthen the tails of others using superglue. He found that males with artificially lengthened tails attracted nearly four times as many females as males with artificially shortened tails. (Andersson, 1982).

"Why, for instances, are trees in forests so tall? The short answer is that all the other trees are tall, so no one tree can afford not to be. It would be overshadowed if it did....But if only they were all shorter, if only there could be some sort of trade-union agreement to lower the recognized height of the canopy in forests, all the trees would benefit. They would be competing with each other in the canopy for exactly the same sun light, but they would all 'pay' much smaller growing costs to get into the canopy."(p. 184).

4 A model of an integrated world economy with different relative wealths

4.1 Assumptions and Notation

We assume that all individuals in the world economy have the same reduced-form utility function. Labour must work only in their own countries, but capital is perfectly mobile across countries. It follows that marginal products of capital are the same in all countries. We postulate that technology is identical in the world. Then equalization of marginal products of capital implies equalization of wage rates as well.

Labor does not enter the utility function. Each individual inelastically supplies one unit of labor per unit of time. Let c_i denote individual i's consumption, and k_i his wealth (not including human wealth, which is defined as the present value of the stream of future wage income.) Let k denote the world's per capita wealth.The reduced-form utility function of individual i is assumed take the separable form

$$U(c_i, \frac{k_i}{k}) = u(c_i) + \theta v \left(\frac{k_i}{k} \right)$$

where $u(.)$ and $v(.)$ are strictly concave and increasing functions. The parameter $\theta \geq 0$ is the weight given to the concern about relative wealth. If $\theta = 0$, then the model reduces to the standard text-book version of the Cass-Ramsey model, where wealth does not appear in the utility function.

There is a continuum of individuals, represented by the interval $[0, 1]$. Individuals are price-takers: they take the paths of wage rate $W(t)$ and rental rate $R(t)$ as given, independent of their actions.

The agregate production function, in per capita form, is $y = f(k)$. In this section, we assume that $f(k)$ has the usual neoclassical properties, and satisfies the Inada conditions:

$$\lim_{k\to 0} f'(k) = \infty \text{ and } \lim_{k\to\infty} f'(k) = 0$$

Capital depreciates at the rate $\delta \geq 0$.

Let $c(t)$ denote per capita consumption. Then the stock $k(t)$ evolves according to the differential equation

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t)$$

In a competitive equilibrium, the rental rate is given by

$$R(t) = f'(k(t))$$

and the wage rate is

$$W(t) = f(k(t)) - k(t)f'(k(t))$$

The rate of interest is equal to the rental rate minus the rate of depreciation

$$r(t) = R(t) - \delta$$

Individuals have perfect knowledge of the time paths of future factor prices and per capita capital stock. Individual i seeks to maximize the integral of the discounted utility flow:

$$\max_{c_i(t)} \int_0^\infty \left[u(c_i(t)) + \theta v\left(\frac{k_i(t)}{k(t)}\right)\right] e^{-\rho t} dt \tag{1}$$

where $\rho > 0$ is the utility-discount rate. The maximization is subject to the constraints

$$\dot{k}_i(t) = R(t)k_i(t) + W(t) - c_i(t) - \delta k_i(t) \tag{2}$$

$$k_i(0) = k_{i0} \tag{3}$$

$$\lim_{t\to\infty} k(t) \exp\left[-\int_0^t r(s)ds\right] = 0 \tag{4}$$

4.2 The benchmark scenario: identical agents

It is useful to consider first the benchmark case where individuals have identical initial stocks of capital: $k_i(t) = k_j(t)$ for all i, j. For this benchmark case, we will first consider the solution that a social planner would arrive at, on behalf on the individuals. Next, we will examine the laissez-faire outcome.

4.2.1 The social planner's problem

Since all individuals are identical, the social planner would set $k_i = k$, thus $v(z_i) = v(1)$, where $z_i \equiv k_i/k$. The problem is simply

$$\max_{c(t)} \int_0^\infty [u(c(t)) + \theta v(1)] e^{-\rho t} dt$$

subject to

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t) \quad (5)$$

$$k(0) = k_0 \quad (6)$$

$$k(t) \geq 0$$

The solution of this problem is well known. The optimal consumption path must satisfy the Euler equation

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c(t))} [f'(k(t)) - \delta - \rho] \quad (7)$$

where $\sigma(c)$ is the elasticity of marginal utility

$$\sigma(c) = -\frac{cu''(c)}{u'(c)} > 0$$

Furthermore, let k_{ss} be the capital stock level that satisfies the modified golden rule:

$$f'(k_{ss}) = \delta + \rho \quad (8)$$

Then, as has been shown in the literature, the optimal path $k(t)$ converges to k_{ss}

$$\lim_{t\to\infty} k(t) = k_{ss} \quad (9)$$

The corresponding steady-state consumption is denoted by c_{ss} where

$$c_{ss} = f(k_{ss}) - \delta k_{ss}$$

It is well-known that c_{ss} (called the "modified golden rule" consumption) is smaller than the maximum sustainable consumption $\widehat{c}$, which is defined by

$$\widehat{c} = \max_k [f(k) - \delta k] \quad (10)$$

The intuition behind the result that $c_{ss} < \widehat{c}$ is that, given that utility is discounted, it is not optimal to try to reach the maximum sustainable consumption level.

The two differential equations (5), (7) together with the two boundary conditions (6) and (9) determine a unique pair of optimal paths $(k(t), c(t))$ which can be represented by a trajectory in the (k, c) plane that converges to the point (k_{ss}, c_{ss}).

The converging trajectory defines a function $c^* = \phi(k)$ which is the optimal control rule in feedback form. If the social planner instructs all individuals to follow this rule, i.e., $c_i = \phi(k_i)$ and if they all obey, the socially optimal solution can be achieved. However, in general, individuals will have an incentive to deviate from this recommended rule, because each will seek to achieve a higher social status by increasing his wealth above the recommended path. This results in a Rat Race, making all individual worse off. The laissez-faire outcome is considered in the next sub-section.

4.2.2 Individual optimization under laissez-faire

Each individual i takes the time path of society's per capita capital stock as given.(The time paths of factor prices are also taken as given.) Each contemplates the possibility of steering the ratio k_i/k away from unity. The Hamiltonian for the optimization problem of individual i is

$$H = u(c_i) + \theta v\left[k_i/k\right] + \psi\left[r(t)k_i(t) + W(t) - c_i(t)\right]$$

The necessary conditions are

$$\frac{\partial H}{\partial c_i} = u'(c_i(t)) - \psi(t) = 0 \tag{11}$$

$$\dot{\psi}(t) = \rho\psi_i(t) - \frac{\partial H}{\partial k_i(t)} = [\rho - r(t)] - \frac{\theta}{k(t)}\frac{dv}{dz_i(t)} \tag{12}$$

where $z_i = k_i/k$, and

$$\dot{k}_i(t) = \frac{\partial H}{\partial \psi(t)} = r(t)k_i(t) + W(t) - c_i(t) \tag{13}$$

The boundary conditions are (3) and (4).

From (11) we get

$$u''(c_i)\dot{c}_i = \dot{\psi}$$

hence

$$\frac{c_i u''(c_i)}{u'(c_i)}\frac{\dot{c}_i}{c_i} = \frac{\dot{\psi}}{\psi} = [\rho - r(t)] - \frac{\theta}{u'(c_i)k}\frac{dv}{dz_i} \tag{14}$$

Since $r(t) = f'(k) - \delta$, condition (14) yields

$$\frac{\dot{c}_i}{c_i} = \frac{1}{\sigma(c_i)}\left[f'(k) - \delta - \rho + (\theta/k)\frac{v'(z_i)}{u'(c_i)}\right] \tag{15}$$

It follows that the steady state capital stock under *perfectly competitive* behavior, denoted by k_p, satisfied the condition

$$f'(k_p) = \delta + \rho - (\theta/k_p)\frac{v'(1)}{u'(c_p)} < \delta + \rho \tag{16}$$

where c_p satisfies

$$c_p = f(k_p) - \delta k_p$$

Proposition 1: If individuals are wealth-conscious (i.e., $\theta > 0$) then, under laissez-faire, the steady state capital stock, k_p, is greater than the stock level k_{ss} that the social planner would wish to achieve.

Corollary 1: If individuals are wealth-conscious (i.e., $\theta > 0$) and θ is not too great, then (i) the steady state consumption under laissez-faire exceeds the modified golden-rule consumption c_{ss} that the social planner would wish to achieve, and (ii) their steady-tate saving rate (defined as I/GNP$= \delta k_p/f(k_p)$) exceeds the steady-state saving rate $\delta k_{ss}/f(k_{ss})$ under the social planner.

Proof: If θ is positive but not too large, then $k_p < \widehat{k} \equiv \arg\max\,[f(k) - \delta k]$. Now, $f(k) - \delta k$ is an increasing function of k for all $k < \widehat{k}$. It follows that $c_p > c_{ss}$.This proves (i). To prove (ii), note that $f(k)/k$ is a decreasing function of k. ■

Proposition 2: If σ is a constant, and $\theta > 0$, then at any stock level $k < k_p$ the rate of consumption *growth,* $\dot{c}/c$, under laissez-faire is greater than the rate of consumption growth under the social planner.

Proof: Compare (15) with (7). ■

Remark: Proposition 2 implies that initially (at time $t = 0$), individuals under the laissez-faire scenario have a lower consumption level than they would under the social planner. This higher saving rate (in the initial phase) is the outcome of the individual's desire to accumulate wealth k_i so as not to fall behind the forecasted path of society's average wealth $k(t)$. Each individual thinks that all others are trying to run faster than him, and this results in a "Rat Race"[6], which ultimately makes everone worse off (i.e. the value of the integral of discounted utility flow under laissez-faire is lower than the one achieved under the social planner.)

4.3 Catching-up when households have unequal initial wealths

We now return to the case in which individuals have unequal initial stocks. The measure of the set of all individuals is normalized to unity. There are two groups of individuals: those who are initially poor, and those who are initially wealthy. Their measures are α_1 and α_2 respectively, where $\alpha_1 + \alpha_2 = 1$. The initial capital stock of a poor individual is $k_1(0)$ and that of a wealthy one is $k_2(0) > k_1(0)$. The question that interests us is whether the poor will catch up with the wealthy in the long run.

An early answer to this question was given by Stiglitz (1969), who assumed that individuals *do not* maximize utility over time. Stiglitz postulated that all individuals save a constant fraction s of their income. He demonstrated that in the long run, all individuals will end up with the same amount of capital. Kemp and Shimomura (1992) considered the case where each individual maximizes the discounted value of the stream of his utility of consumption. They showed that inequality persists in the long run.

The model considered in this section differs from the Kemp-Shimomura model in that the utility function has two arguments: consumption, and relative wealth. We will show that if the elasticity of marginal utility of relative wealth is sufficiently high, individuals will end up with equal wealths.

The utility function of individual h (where $h = 1$ or $h = 2$) is

$$U_h(c_h, \frac{k_h}{k}) = u(c_h) + \theta v\left(z_h\right), \qquad z_h \equiv \frac{k_h}{k}$$

where $k = \alpha_1 k_1 + \alpha_2 k_2$, and $\theta_h \geq 0$.

[6]See Akerlof (1976) for an insightful discussion of the Rat Race.

We assume that $v(z_h)$ is increasing and strictly concave in z_h. Individuals earn the same wage rate, independently of their capital ownnership. They take the time path of the overall capital labor ratio, $k(t)$, as given.

The strict convavity of $v(.)$ means that $v'(z_h)$ is a decreasing function. This implies that a poor person gets more pleasure from a marginal increase in his relative wealth than a rich person. This provides a strong incentive for the poor to accumulate.

To simplify notation, we assume $\delta = 0$. We also write v_i' for $v'(z_i)$ and u_i' for $u'(c_i)$. Let

$$\beta = \frac{1}{\sigma} = -\frac{u'}{cu''} > 0$$

The Euler equation for the rich is

$$\frac{1}{\beta}\frac{\dot{c}_1}{c_1} = f'(k) - \rho + \left[\frac{\theta v_1'}{k u_1'}\right] \tag{17}$$

and that for the poor is

$$\frac{1}{\beta}\frac{\dot{c}_2}{c_2} = f'(k) - \rho + \left[\frac{\theta v_2'}{k u_2'}\right] \tag{18}$$

The rate of change of the capital stocks are

$$\dot{k}_1 = rk_1(t) - c_1 + W = f'(k)k_1 - c_1(t) + [f(k) - kf'(k)] \tag{19}$$

and

$$\dot{k}_2 = f'(k)k_2 - c_2 + [f(k) - kf'(k)] \tag{20}$$

4.3.1 Steady states

Consider now the steady state of the system (17)-(20). Let the superscript $*$ denote steady-state values. Then, setting the left-hand sides of (19)-(20) to zero, we have

$$c_1^* - f'(k^*)k_1^* = c_2^* - f'(k^*)k_2^* \text{ (and both are equal to } f(k^*) - f'(k^*)k^* > 0) \tag{21}$$

Divide both sides by k_1^*

$$\frac{c_1^*}{k_1^*} - f'(k^*) = \frac{c_2^*}{k_2^*}\frac{k_2^*}{k_1^*} - f'(k^*)\frac{k_2^*}{k_1^*} \tag{22}$$

Next, setting the left-hand sides of (17)-(18) to zero, we have

$$\frac{\theta v_1'(z_1^*)}{u_1'(c_1^*)} = \frac{\theta v_2'(z_2^*)}{u_2'(c_2^*)} \tag{23}$$

It is easy to see that, for all α_i in $(0,1)$ and $\alpha_j = 1 - \alpha_i$, if all individuals have the same functions $u(.)$ and $v(.)$, there is always a *"symmetric" steady state* with $k_1^* = k_2^* = k^*$ and $c_1^* = c_2^* = f(k^*)$, with the property that

$$f'(k^*) - \rho + \frac{\theta v'(1)}{k^* u'(f(k^*))} = 0 \tag{24}$$

provided that the function

$$\phi(k) \equiv k u'(f(k)) \left[f'(k) - \rho\right]$$

has the property that $\phi(0) > \theta v'(1)$ and $\phi(\infty) < \theta v'(1)$. Note, however, that (24) may give several values k^*. (A sufficient condition for **uniqueness**[7] of k^* is $\beta > k/f(k)$.) Furthermore, in general, for any given steady state aggregate capital stock k^*, one cannot exclude *"asymmetric" steady state* with $k_i^* \neq k_j^*$.

Remark 1: Note that if $\theta = 0$ then, even though the steady-state aggregate capital stock is uniquely determined by $k^* = k_{ss}$, where

$$f'(k_{ss}) = \rho,$$

there are infinitely many steady-state wealth distributions (k_i^*, k_j^*), and which steady state distribution will be reached depends on the initial stocks k_{i0} and k_{j0}. (See Kemp and Shimomura, 1992).

We now seek sufficient conditions that rule out asymmetric steady states, when $\theta > 0$. Note that any steady state (c_1, c_2, k_1, k_2) is a solution to the following system of equations

$$c_1 = f'(k)k_1 + [f(k) - kf'(k)] \tag{25}$$

$$c_2 = f'(k)k_2 + [f(k) - kf'(k)] \tag{26}$$

$$0 = f'(k) - \rho + \frac{\theta v'(k_1/k)}{k u'(c_1)} \tag{27}$$

[7]In general, one cannot be sure that there is only one steady state. It is possible to have several steady states, or even a continuum of steady states in our model of relative wealth. Mathematical biologists have discovered under certain conditions, there are infinitely many equilibria in the races for tail length etc. See Russell Lande (1980,1981).

$$0 = f'(k) - \rho + \frac{\theta v'(k_{21}/k)}{ku'(c_2)} \tag{28}$$

where $k = \alpha_1 k_1 + \alpha_2 k_2$. Suppose there is an asymmetric steady state, $(c_1^*, c_2^*, k_1^*, k_2^*)$ where $k_1^* \neq k_2^*$ and $c_1 \neq c_2$. Then, for a given value $k^* = \alpha_1 k_1^* + \alpha_2 k_2^*$, consider, in the (k_h, c_h) space, the straight line

$$c_h = f'(k^*)k_h + [f(k^*) - k^* f'(k^*)] \tag{29}$$

with slope $dc_h/dk_h = f'(k^*)$, and the curve

$$0 = f'(k^*) - \rho + \frac{\theta v'(k_h/k^*)}{k^* u'(c_h)} \tag{30}$$

If there is an asymmetric steady state, then these two graphs must cut each other twice (at least); one of these points is (k_1^*, c_1^*) and the other is (k_2^*, c_2^*). Now the slope of (29) is $f'(k^*)$, and the slope of (30) is

$$\frac{dc_h}{dk_h} = \frac{u'v''}{k^* v' u''} = \beta(c_h)\eta(z_h)\frac{c_h}{k_h} \tag{31}$$

where $\beta(c_h)$ is the inverse of the elasticity of marginal utility of consumption,

$$\beta(c_h) \equiv -\frac{u'(c_h)}{c_h u''(c_h)} > 0$$

and $\eta(z_h)$ is the coefficient of relative risk aversion with respect to relative wealth,

$$\eta(z_h) = -\frac{z_h v''(z_h)}{v'(z_h)} \geq 0$$

Now if the curve (30) cuts the line (29) twice, then at one of these points, say point A, the curve (30) cuts the line (29) from above. (See Figure 1). At point A, the slope of the curve (30) is smaller than the slope of the ray OA that goes through the origine O. It follows that at A,

$$\frac{k_h}{c_h}\frac{dc_h}{dk_h} < 1 \tag{32}$$

In view of (57), condition (32) cannot be met if

$$\beta(c_h)\eta(z_h) \geq 1 \tag{33}$$

PLEASE PLACE FIGURE 1 HERE

We can now state our proposition on steady-state wealth distribution:

Proposition CU1: If, for all non-negative c_h,z_h and k,

$$\beta(c_h)\eta(z_h) \geq 1 \tag{34}$$

then, given any *steady-state aggregate capital stock* k^*, all individuals have identical steady-state wealth and consumption levels (that is, asymmetric steady states do not exist.)

Remark: a sufficient condition for (34) to hold is the function $v(.)$ is "very" concave.

Example: if

$$v(z_h) = \frac{z_h^{1-\sigma}}{1-\sigma} \text{ and } u(c_h) = \frac{c_h^{1-\sigma}}{1-\sigma} \text{ for } h = i, j \tag{35}$$

then $\beta(c_h)\eta(z_h) = 1$, implying the non-existence of asymmetric steady states, The steady-state aggregate capital stock, denoted by k^*, is determined by the equation

$$f'(k^*) - \rho = -\frac{\theta v'(1)}{k^* u'(f(k^*))} = -\frac{\theta v'(1)(f(k^*))^\sigma}{k^*}$$

4.3.2 Catching up: stability analysis

We now examine the stability properties of symmetric steady states **(without assuming that $\alpha_i = \alpha_j$)**. We must examine the local stability of the system (17)-(20). Rewrite the system as follows

$$\dot{k}_1 = [f'(k)]\, k_1 - c_1 + [f(k) - kf'(k)] \tag{36}$$

$$\dot{k}_2 = [f'(k)]\, k_2 - c_2 + [f(k) - kf'(k)] \tag{37}$$

$$\dot{c}_1 = \beta c_1 \left[f'(k) - \rho + \frac{\theta v_1'}{k u_1'} \right] \tag{38}$$

$$\dot{c}_2 = \beta c_2 \left[f'(k) - \rho + \frac{\theta v_2'}{k u_2'} \right] \tag{39}$$

We linearize the system and then *evaluate all derivatives at the steady state*. We have the following matrix (Recall that at the steady state, $k_1^* = k_2^* = k^*$ and $c_1^* = c_2^* = c^*$.)

$$J \equiv \begin{bmatrix} a_{11} & a_{12} & -1 & 0 \\ a_{21} & a_{22} & 0 & -1 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & 0 & a_{44} \end{bmatrix}$$

where

$$a_{11} = f' - (k^* - k_1^*)\alpha_1 f'' \qquad (= f' \text{ at } k_1^* = k_2^* = k^*).$$

$$a_{12} = -\alpha_2(k^* - k_1^*)f'' \qquad (= 0 \text{ at } k_1^* = k_2^* = k^*).$$

$$a_{21} = -\alpha_1(k^* - k_2^*)f'' \qquad (= 0 \text{ at } k_1^* = k_2^* = k^*).$$

$$a_{22} = f' - (k^* - k_2^*)\alpha_2 f'' \qquad (= f' \text{ at } k_1^* = k_2^* = k^*)$$

$$a_{31} = \beta\alpha_1 c_1^* \left[f'' - \frac{(1-\eta_1)\theta v_1'}{k^2 u_1'} \right] + \frac{\beta c_1^* \theta v_1''}{k^2 u_1'}, \text{ with } \eta_i \equiv -\frac{z_i^* v_i''}{v_i'} = -\frac{(1)v''(1)}{v'(1)}$$

$$a_{32} = \beta\alpha_2 c_1^* \left[f'' - \frac{(1-\eta_1)\theta v_1'}{k^2 u_1'} \right]$$

$$a_{33} = \rho - f' = -\frac{\theta v_1'}{k u_1'} < 0$$

$$a_{41} = \beta\alpha_1 c_2^* \left[f'' - \frac{(1-\eta_2)\theta v_2'}{k^2 u_2} \right]$$

$$a_{42} = \beta\alpha_2 c_2^* \left[f'' - \frac{(1-\eta_2)\theta v_2'}{k^2 u_2} \right] + \frac{\beta c_2^* \theta v_2''}{k^2 u_2'}$$

$$a_{44} = \rho - f' = -\frac{\theta v_2'}{k u_2'} < 0$$

The characteristic equation is obtained by calculating the determinant of the matrix $xI - J$, and equating it to zero, where x is a scalar and I is the 4x4 identity matrix. Since $k_i^* = k_j^*$, we have $a_{12} = a_{21} = 0$. Substracting the third row of $xI - J$ by the first row times $[x - a_{33}]$, and substracting the fourth row by the second row times $[x - a_{44}]$, we obtain

$$\det[xI - J] = \det \begin{bmatrix} x - f' & 0 & 1 & 0 \\ 0 & x - f' & 0 & 1 \\ -a_{31} - Y & -a_{32} & 0 & 0 \\ -a_{41} & -a_{42} - Y & 0 & 0 \end{bmatrix}$$

where

$$Y = (x - f')(x + f' - \rho) = (x - f')\left(x - \frac{\theta v'}{ku'}\right) \tag{40}$$

Therefore

$$\det[xI - J] = (a_{31} + Y)(a_{42} + Y) - a_{41}a_{32} = Y^2 + (a_{31} + a_{42})Y + a_{31}a_{42} - a_{41}a_{32}$$

Let

$$A \equiv f'' - \frac{(1-\eta)\theta v'(z^*)}{k^2 u'(c^*)}$$

$$B \equiv \frac{\theta v''(z^*)}{k^2 u'(c^*)}$$

Then

$$a_{41}a_{31} = \alpha_1\alpha_2 (\beta c^*)^2 A^2 > 0$$

$$a_{31}a_{42} = (\beta c^*)^2 (\alpha_1 A + B)(\alpha_2 A + B) = a_{41}a_{31} + (\beta c^*)^2 \{B^2 + (\alpha_1 + \alpha_2)AB\}$$

Thus

$$a_{31}a_{42} - a_{41}a_{32} = (\beta c^*)^2 B(B + A) \equiv d$$

Note that

$$d \equiv a_{31}a_{42} - a_{41}a_{32} > 0 \text{ if } G \equiv f'' - \frac{(1-\eta)\theta v'(z^*)}{k^2 u'(c^*)} + \frac{\theta v''(z^*)}{k^2 u'(c^*)} < 0 \tag{41}$$

Lemma 1: d is positive at the steady state.

Proof: Using the fact that $z^* = 1$ at a symmetric steady state, write G as

$$G = f'' - \frac{(1-\eta)\theta v'}{k^2 u'} - \frac{\eta\theta v'}{k^2 u'} = f'' - \frac{\theta v'}{k^2 u'} < 0.$$

Next,

$$a_{31} + a_{42} = \beta c^* \left[\alpha_j A + B\right] + \beta c^* \left[\alpha_i A + B\right] = \beta c^*(A + 2B) \equiv b$$

Thus, the characteristic equation is

$$\det\left[xI - J\right] = d + bY + Y^2 = 0 \tag{42}$$

which is a quadratic in Y. The two roots are

$$Y_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2}$$

where

$$\Delta = b^2 - 4d = (\beta c^*)^2 \Big[A^2 + 4B^2 + 4AB - 4B(B + A)\Big] = (\beta c^*)^2 A^2$$

Thus

$$Y_1 = -B\beta c^* > 0 \text{ if } \theta v'' < 0$$

and

$$Y_2 = -\left[B + A\right]\beta c^* = -\beta c^* \left[f'' - \frac{(1-\eta)\theta v'(z^*)}{k^2 u'(c^*)} + \frac{\theta v''(z^*)}{k^2 u'(c^*)}\right]$$

Substituting Y_1 into (40), we get

$$(x - f')\left(x - \frac{\theta v'}{ku'}\right) - Y_1 = x^2 - x\left(f' + \frac{\theta v'}{ku'}\right) + \frac{\theta v' f'}{ku'} + \frac{\beta c^* \theta v''(z^*)}{k^2 u'(c^*)} = 0$$

hence

$$x^2 - \left(f' + \frac{\theta v'}{ku'}\right)x + \frac{\theta v'}{k^2 u'}\left[k^* f' + \beta c^* \frac{v''(z^*)}{v'(z^*)}\right] = 0 \tag{43}$$

Now since $z^* = 1$

$$-\frac{v''(z^*)}{v'(z^*)} \equiv \frac{\eta}{z^*} = \eta$$

Assume (34) holds. Then $\eta\beta \geq 1$, then $k^* f' - \beta\eta c^* \leq k^* f' - c^* = -\left[f(k^*) - k^* f'(k^*)\right] < 0$ (from (36)), and equation (43) has two real roots of opposite sign.

Similarly, substituting Y_2 into (40), we get

$$(x - f')\left(x - \frac{\theta v'}{ku'}\right) - Y_1 =$$

$$x^2 - x\left(f' + \frac{\theta v'}{ku'}\right) + \frac{\theta v' f'}{ku'} + \frac{\beta c^* \theta v''}{k^2 u'} + \beta c^* \left[f'' - \frac{(1-\eta)\theta v'}{k^2 u'}\right]$$

hence

$$x^2 - \left(f' + \frac{\theta v'}{ku'}\right) x + Q = 0 \tag{44}$$

where

$$Q \equiv \frac{\theta v'}{k^2 u'}\left[k^* f' + \beta c^* \frac{v''}{v'}\right] + \beta c^* \left[f'' - \frac{(1-\eta)\theta v'}{k^2 u'}\right]$$

Lemma 2: Q is negative if

$$\beta \geq \varsigma(k) \tag{45}$$

where $\varsigma(k)$ is the share of capital income in national income,

$$\varsigma(k) \equiv \frac{kf'(k)}{f} > 0$$

Proof:

$$Q = \beta c^* f'' + \frac{\theta v'}{k^2 u'}\left[k^* f' + \beta c^* \left(\frac{v''}{v'} - (1-\eta)\right)\right]$$

$$= \beta c^* f'' + \frac{\theta v'}{k^2 u'}\left[k^* f' - \beta c^*\right]$$

where $c^* = f(k^*)$. Thus $Q < 0$ if (45) holds.

Lemma 3: Equation (44) has two real roots of opposite signs if (45) holds.

Proposition CU2: If

$$\beta \geq \max\left[\varsigma(k), \frac{1}{\eta(z)}\right] \tag{46}$$

then there are four real roots, two of which are negative, implying that the steady-state is stable in the saddlepoint sense. This implies that the poor will finally catch up with the rich.

5 A model of human capital accumulation and loss from trade liberalization

In the preceding section, we show that trade liberalization, by equalizing the wage rates in all countries, can help poor countries to catch up with rich countries, provided that individuals care enough about about their relative wealth status in the global economy. In that model, we abstracted from an important consideration: the acquisition of skills. We now turn to this important topic. We wish to point out that trade liberalization may reduce the wage rate of skilled workers in the high-tech sector of developing countries. This may have adverse impact on their rate of human capital accumulation. This point has been made by authors such as Young (1991) and Stokey (1991), and reviewed by Long and Wong (1997), Van and Wan (1996).The effects of trade liberalization on wage structure and employment have been a continuing topic of debate. (See, for example, Freeman (1995), Wood (1994), Krugman (1995), Davis (1998), Falvey (1998), Leamer (1998), Tyers and Yang (1999).) Economists participating in this debate typically use a modified version of the Heckscher-Ohlin model[8], with *fixed* endowments of skilled and unskilled workers. While that framework is a useful starting point, it neglects an important aspect: human capital accumulation. In our view, the decision to acquire skills *in response to* expected changes in trade regime should be modelled.

On the other hand, in the endogenous growth literature, human capital accumulation has received a great deal of attention. See Lucas (1988), Young (1991), Stokey (1991), and, for a survey of the trade and growth literature, see Long and Wong (1996). These authors however focused on long run considerations, and did not consider short-run issues such as the accumulation of industry-specific and firm-specific human capital, in response to trade liberalization. In this paper, we seek to fill that gap.

This paper presents a simple model of firm-specific human capital accumulation in a small open economy. We have two major goals in this paper. We want to find out if trade liberalization will (a) increase or decrease firm-specific human capital accumulation and (b) widen the wage gap between skilled and unskilled workers. This will have implications for the trade pattern, welfare and income distribution in both less developed countries (LDCs)

[8]For an important exception, see Neary (2000) who focused on oligopolistic competition, with *R&D* rivary in the first stage of the game.

and developed countries (DCs). We develop a new framework for the analysis of the impact of trade liberalization on the wage structure. Our model focuses on the decision of workers to accumulate firm-specific skills, knowing that this means their future wages will have to be negotiated, and that the outcome of negotiation will depend on the profitability of firms operating in a new trading environment.

We show that, for a less developed economy (one which imports the high-tech good), the expectation of trade liberalization leads to *less* human capital accumulation for skilled workers in the high-tech industry. In the absence of perfectly competitive labor markets (in our model wages are negotiated between management and workers with firm-specific skills), the effect of free trade on the supply of skills *may well be welfare-worsening*[9]. This argument has received some support from some section of the profession. In fact, the following quotation from Hirschman (1965, p. 5) is quite relevant:

"The opponents of free trade have often pointed out that for a variety of reasons it is imprudent and harmful for a country to become specialized along certain product lines in accordance with the dictates of comparative advantage. Whatever the merits of these critical arguments, they would certainly acquire overwhelming weight if the question arose whether a country should allow itself to become specialized not just along certain commodity lines, but along factor-of-production lines. Very few country would ever consciously wish to specialize in unskilled labor, while foreigners with a comparative advantage in entrepreneurship, management, skilled labor and capital took over these functions, replacing inferior "local talents."

Analysis of our model shows that, for a developed economy (high-tech good exporter), free trade leads to *more* human capital accumulation. In terms of wage gaps, the effect of free trade depends on the pattern of comparative advantage. The wage gap between high and low skill workers increases in the country that exports the high-tech good and decreases in the country which imports the high tech good. In section 4 we also briefly discuss the policy implications of these results as well as the effects of externalities and uncertainty.

[9]This is proved in Proposition 6 and Appendix 2B, under certain plausible assumptions.

6 A Basic Model of Human Capital Accumulation

6.1 Assumptions and Notation

We assume that there are two periods only. As a first step, let us consider a small open economy, consisting of two sectors, denoted by G, and H. (G and H stand for general and specific human capital respectively.) One can think of sector G, which produces Q_G, as the "low-tech" sector consisting of goods such as textiles and clothing. The "high-tech" sector's output, Q_H represents goods such as pharmaceuticals, software, computers, etc. Each individual in this economy possesses one unit of general human capital, and can accumulate firm-specific human capital. Sector G produces the numeraire good, which is exported (or imported) at the price $P_G = 1$. The only factor of production used to produce Q_G is general human capital. Production in sector G is under constant returns to scale: one unit of general human capital produces W_G units of good G. Thus the wage rate in this sector is W_G in both periods.

Sector H produces an output Q_H. Good H is produced using industry-specific physical capital, and human capital. There are N_H firms in sector H, each endowed with one unit of industry-specific physical capital. N_H is exogenously given. For the time being, the price of good H in period t, denoted by P_t , is taken as a parameter. In a subsequent subsection, we shall consider autarky equilibrium and show how P_t is determined endogenously.

We assume that, in sector H, to produce a positive output, a firm must have exactly one unit of industry-specific physical capital, and exactly one worker: a second worker would add nothing to output. If the worker (who works with one unit of industry-specific physical capital) has only one unit of *general* human capital, then the output is 1 unit of good H. If he has accumulated, in addition, h units of *firm-specific* human capital, then the output is $1+\mu h$, where μ is a positive parameter representing the *productivity* of firm-specific human capital in sector H. (Here, h is the worker's decision variable.) Since μ is only relevant in period 2, one may also interpret it as a measure of *technical progress embodied* in firm-specific human capital, and μh is firm-specific human capital measured in *efficiency units*.

Initially,workers in sector H have no firm-specific human capital. In period one each sector H worker decides on h, the amount of firm-specific human capital he wants to acquire. We assume that, without the firm's unit of specific physical capital, the worker cannot acquire firm-specific knowledge.

The cost of acquiring firm-specific human capital depends on the amount h and on the learning ability of the worker. We model this by assuming that there is a parameter θ that represents learning ability, where θ is a positive real number, restricted to lie on the real interval $[\theta_a, \theta_b]$ where $0 < \theta_a < \theta_b$. Workers with higher θ have higher learning ability. The distribution function of θ is $F(\theta)$, where $F(\theta_a) = 0$ and $F(\theta_b) = 1$.

Our reason for introducing differences in learning ability is that we would like to model a *continuous* distribution of wages. On the other hand, it may be convenient, in some contexts, to talk about only *two* types of workers: skilled and unskilled. To accommodate both objectives, we find it useful to introduce a parameter z, a non-negative real number, which we use as an exponent for the parameter θ, and we assume that to obtain h, a worker of type θ must *directly incur* an effort cost which is denoted by $C(h)/\theta^z$, where $C(h)$ is convex and increasing, with $C(0) = 0$, and $z \geq 0$ is a useful parametrization of model type. So, workers with higher learning ability incur lower costs in acquiring human capital. A *special case* is obtained when we set $z = 0$ (or take the limit $z \to 0$). In this case, $\theta^z = \theta^0 = 1$ for all θ, so that effort cost is independent of learning ability, and the model reduces to one with only *two* types of worker: skilled and unskilled. We call the case with $z = 0$, the "benchmark case".

Each worker's θ is common knowledge. We assume for simplicity that for the worker, the cost $C(h)/\theta^z$ can be measured in terms of good G.[10] Let N be the number of individuals in this economy. We assume that $N > N_H$, so that when each firm in sector H employs one worker, there are enough workers left to produce some good G.

At the beginning of period two, a firm in sector H that has hired a worker of type θ in period 1 can rehire this worker, who has acquired $h(\theta) \geq 0$ units of firm-specific human capital, at a wage $W_2(\theta)$ (which is an outcome of a bargaining process between the firm and the worker, to be discussed below), or it can dismiss that worker, and employ a new worker, who, of course, does not have firm-specific human capital. If it takes the latter course of action, its profit is

$$\pi_R = P_2 - W_G. \tag{47}$$

This is the firm's reservation level of profit in its second-period bargaining with its worker. The experienced worker, on the other hand, can work in

[10] Alternatively, we can interpret $C(h)/\theta^z$ as the cost of education, which uses up real resources, identified as good G.

sector G in period two, at the wage W_G (since his firm-specific human capital is useless in other firms in sector H). This is his reservation wage in his bargaining with his existing employer.

6.2 Analysis of Wage Profiles

We now turn to the question of how bargaining determines the wage of the skilled worker of type θ in period two, *given* that the worker has acquired $h(\theta)$ units of firm-specific human capital. To do this, we use the theory of Nash cooperative bargaining, according to which the bargaining outcome in period 2 is a pair $(W_2(\theta), \pi_2(\theta))$ that maximizes the so-called Nash product, $(\pi_2(\theta) - \pi_R)^{\beta}(W_2(\theta) - W_G)^{1-\beta}$ subject to the constraint that

$$\pi_2(\theta) + W_2(\theta) = (1 + \mu h(\theta))P_2 \tag{48}$$

where $h(\theta)$ has been determined in period 1, and is *taken as given*[11] in the bargaining problem. The parameter β represents the relative bargaining power of the firm, where $0 \leq \beta \leq 1$. The constraint (48) may be written as

$$\pi_2(\theta) + W_2(\theta) = \pi_R + W_G + S(\theta)$$

where

$$S(\theta) \equiv \mu h(\theta) P_2 \tag{49}$$

is the surplus to be shared by the firm and the worker.

Solving this maximization problem yields the Nash-bargaining solution

$$W_2(\theta) = W_G + (1 - \beta)S(\theta) = W_G + (1 - \beta)\mu h(\theta) P_2 \tag{50}$$

and

$$\pi_2(\theta) = \pi_R + \beta S(\theta) \tag{51}$$

Equation (50) says that the skilled worker's wage consists of two components: a wage that he would earn elsewhere, plus a share of the surplus that his skills (together with the firm's capital stock) generate. Equation (51) indicates that

[11] An alternative formulation, which would lead to a different result, is that the bargaining would take place in period 1, where the outcome would result in a contract that specifies how much human capital the worker must acquire, as well as wage rates $W_1$and W_2.

firm's profit equals the sum of the profit it would earn if it were to employ a worker without firm-specific skills and its share of the surplus generated by the skilled worker.

We now show how $h(\theta)$ is determined in period one. Assume that there is no uncertainty, and that individuals can borrow and lend at a constant[12] rate of interest r. Then in period one, the worker of type θ in sector H chooses $h(\theta)$ to maximize his lifetime wage income, net of effort cost:

$$M(\theta) \equiv W_1(\theta) - \frac{C(h)}{\theta^z} + \frac{1}{(1+r)}\left(W_G + (1-\beta)\mu h P_2\right) \tag{52}$$

where he takes the first period wage, $W_1(\theta)$ as given. (Note that $\theta^z > 0$.because $\theta > 0$.) Solving this maximization problem yields the first order condition

$$\frac{\theta^z(1-\beta)\mu P_2}{(1+r)} - C'(h(\theta)) = 0 \tag{53}$$

and the second order condition

$$-C''(h(\theta)) < 0.$$

Condition (53) says that a worker acquires firm specific human capital to the point where the discounted marginal gain in wage income in period two is equated to the marginal effort cost that the worker has to pay in period one to acquire the skills.

In order to get a closed form solution for $h(\theta)$ we assume a particular functional form for costs. We parametrize the cost function by

$$\frac{1}{\theta^z}C(h(\theta)) = \frac{Ah(\theta)^{1+v}}{(1+v)\theta^z} \qquad A > 0, v > 1 \tag{54}$$

The parameter A represents the non-idiosyncratic learning cost specific to an individual country. Comparing these costs across countries a large A might be associated with poor schools, inadequate libraries, or lack of access to the internet. Using this parameterization we can solve (53) for the worker's *optimal* level of firm-specific human capital, $h^*(\theta)$. Using (53) and (54)

$$\frac{Ah^*(\theta)^v}{\theta^z} = \frac{(1-\beta)\mu P_2}{1+r} \tag{55}$$

[12]The question of how r is determined will be addressed in a subsequent section.

Solving we get

$$h^*(\theta) = h^*(P_2, \mu, \beta, A, r, \theta) = \left[\frac{\theta^z(1-\beta)\mu P_2}{A(1+r)}\right]^{1/v} \quad (56)$$

It will be useful to define

$$B \equiv \frac{(1-\beta)\mu}{A(1+r)} \quad (57)$$

so that

$$h^*(\theta) = [\theta^z B P_2]^{1/v} \quad (58)$$

From (55) and ((54), the optimized cost is

$$\frac{1}{\theta^z}C(h^*(\theta)) = \frac{[Ah^*(\theta)^v]h^*(\theta)}{(1+v)\theta^z} = \frac{(1-\beta)\mu P_2 h^*(\theta)}{(1+r)(1+v)} \quad (59)$$

Using (56) and considering for the moment the case in which the second period price is exogenous (the small country open economy case) we get a number of very intuitive results. First, in the case $z > 0$, workers with lower learning cost (i.e., higher θ) accumulates more human capital:

$$\frac{\partial h^*}{\partial \theta} = z\theta^{(z-v)/v}\left[BP_2\right]^{1/v} > 0 \text{ if } z > 0. \quad (60)$$

Higher second period prices and more productive technology result in workers accumulating higher levels of human capital:

$$\frac{\partial\left[h^*(\theta)\right]^v}{\partial P_2} = \frac{\theta^z(1-\beta)\mu}{A(1+r)} > 0, \qquad \frac{\partial\left[h^*(\theta)\right]^v}{\partial \mu} = \frac{\theta^z(1-\beta)P_2}{A(1+r)} > 0 \quad (61)$$

On the other hand, more bargaining power on the part of the firm and higher learning cost leads to less investment in human capital.

$$\frac{\partial\left[h^*(\theta)\right]^v}{\partial \beta} = \frac{-\theta^z\mu P_2}{A(1+r)} < 0, \qquad \frac{\partial\left[h^*(\theta)\right]^v}{\partial A} = \frac{-\theta^z(1-\beta)\mu P_2}{A^2(1+r)} < 0 \quad (62)$$

Using (50) and (58) we can solve for $W_2(\theta)$

$$W_2(\theta) = W_G + \mu(1-\beta)\left(\theta^z B\right)^{1/v} P_2^{(1+v)/v} = W_G + (1-\beta)\mu h^*(\theta) P_2 \quad (63)$$

thus $W_2(\theta)$ is increasing in P_2 and non-decreasing in θ.

Substituting (57) into (63) we get

$$W_2(\theta) = W_G + (1-\beta)\left[\theta^z B P_2\right]^{1/v} \mu P_2 \tag{64}$$

Another useful expression for W_2 is obtained by using (64), (57) and (58):

$$W_2(\theta) = W_G + \frac{(1+r)A}{\theta^z} h^*(\theta)^{1+v} \tag{65}$$

Next we determine the wage $W_1(\theta)$ of a sector-H worker of type θ in period one. To do this, we must first pin down the wages of the marginal worker $\widehat{\theta}$ (the one who is indifferent between being employed in the general sector, and being employed in the high-tech sector). We assume that prior to period 1 all workers are mobile. This means that in equilibrium the expected lifetime income (net of effort cost) of the marginal sector-H worker $\widehat{\theta}$ must be equal to the alternative lifetime income that he could obtain in sector G:

$$M(\widehat{\theta}) = W_G[1 + \frac{1}{1+r}] \tag{66}$$

Using (66), (52), for $\theta = \widehat{\theta}$, and (59),

$$W_G[1+\frac{1}{1+r}] = W_1(\widehat{\theta}) - \frac{(1-\beta)\mu P_2 h^*(\widehat{\theta})}{(1+r)(1+v)} + \frac{1}{1+r}\left(W_G + (1-\beta)\mu h^*(\widehat{\theta})P_2\right) \tag{67}$$

This simplifies to

$$W_1(\widehat{\theta}) = W_G + \left[\frac{(1-\beta)\mu P_2 h^*(\widehat{\theta})}{(1+r)(1+v)}\right] - \frac{1}{1+r}(1-\beta)\mu h^*(\widehat{\theta})P_2 \tag{68}$$

Equation (68) says that in period 1, the employer pays the marginal worker $\widehat{\theta}$ his outside wage, plus the cost of firm-specific education (the expression inside the square brackets [...]), minus the discounted value of the surplus[13] that the employee can expect to capture in period 2.

We can write (68) as

$$W_1(\widehat{\theta}) = W_G - \frac{v}{(1+v)(1+r)}(1-\beta)\mu h^*(\widehat{\theta})P_2 < W_G \tag{69}$$

[13]This equation reflects the theory of on-the-job training, developed by Gary Becker (1964).

From (65) and (69), we have the following relationship, for the *marginal* high-tech worker:

$$W_2(\widehat{\theta}) > W_G > W_1(\widehat{\theta}) \tag{70}$$

To explain the second inequality in (70), that is, why the *marginal* high-tech worker gets a lower salary in period 1 than what he would get if he would work in the general sector G, we point to the fact that his employer offers him a lower wage in period 1 because she wants to extract from him, in period 1, the surplus that he expects to get in period 2 ($\beta\mu S(\widehat{\theta})$, net of his directly incurred effort cost), so that the marginal worker is indifferent between employment in sector H and in sector G. The worker is willing to accept a wage lower than W_G because his training would not be possible without access to the firm's unit of capital. His *total* cost of acquiring a positive h consists therefore of the firm's charge for access to capital, as reflected in lower period 1 wage) and his effort costs. Our result is consistent with the literature on "*on-the-job training*", see Gary Becker (1964).[14]

We now turn to the determination of $\widehat{\theta}$ itself. Recall that there are N_H units of capital. Due to our assumption of the Leontief-type technology, there can be at most N_H workers in sector H. We will focus on the case where all units of capital are used. Then $\widehat{\theta}$ must satisfy

$$\int_{\widehat{\theta}}^{\theta_b} f(\theta)d\theta = \frac{N_H}{N} \tag{71}$$

where $f(\theta)$ is the density function, with

$$\int_{\theta_a}^{\theta_b} f(\theta)d\theta = 1.$$

(For example, in the special case of a uniform density, $f(\theta) = 1/\left[\theta_b - \theta_a\right]$, (71) gives

$$\frac{\theta_b - \widehat{\theta}}{\theta_b - \theta_a} = \frac{N_H}{N} \equiv n_H$$

i.e., $\widehat{\theta} = \theta_b - n_H\left(\theta_b - \theta_a\right)$.)

Having determined $\widehat{\theta}$ and $W_1(\widehat{\theta})$, we can now determine the wage structure for workers who have *lower* learning costs than the marginal high-tech worker,

[14]Clearly, in general, there exist intramarginal workers with lower learning costs than the marginal worker (those with $\theta > \widehat{\theta}$) for whom $W_G < W_1(\theta)$.

i.e. all $\theta \geq \widehat{\theta}$. We first solve for the value of $W_1(\theta)$ for all $\theta \geq \widehat{\theta}$, by appealing to the equilibrium condition that all sector -H firms, by competing for workers, must earn the same discounted sum of profit:

$$\pi_1(\theta) + \frac{1}{1+r}\pi_2(\theta) = \pi_1(\widehat{\theta}) + \frac{1}{1+r}\pi_2(\widehat{\theta}) \equiv \Pi(\widehat{\theta}), \qquad \theta \geq \widehat{\theta} \tag{72}$$

where

$$\Pi(\widehat{\theta}) = P_1 - W_1(\widehat{\theta}) + \frac{1}{1+r}\left[\pi_R + \beta\mu h^*(\widehat{\theta})P_2\right] \tag{73}$$

and

$$\pi_2(\theta) = \pi_R + \beta\mu h^*(\theta)P_2$$

It follows that

$$-W_1(\theta) + \frac{1}{1+r}\beta\mu h^*(\theta)P_2 = -W_1(\widehat{\theta}) + \frac{1}{1+r}\beta\mu h^*(\widehat{\theta})P_2$$

i.e.,

$$W_1(\theta) = W_1(\widehat{\theta}) + \frac{1}{1+r}\beta\mu P_2\left[h^*(\theta) - h^*(\widehat{\theta})\right] \geq W_1(\widehat{\theta}) \tag{74}$$

where $h^*(\theta)$ is an increasing function of θ, by (60) for all $\theta \geq \widehat{\theta}$. Workers with lower learning costs earn higher first period wages. They also earn higher wages in the second period as well, as can be seen from (50) and (60).

Using (69) and (74)

$$W_1(\theta) = W_G - \frac{v}{(1+v)(1+r)}(1-\beta)\mu h^*(\widehat{\theta})P_2 + \frac{1}{1+r}\beta\mu P_2\left[h^*(\theta) - h^*(\widehat{\theta})\right] \tag{75}$$

Subtracting (75) from (50), we get a measure of the steepness of the wage profile, for $\theta \geq \widehat{\theta}$

$$\Delta W_t(\theta) \equiv W_2(\theta) - W_1(\theta) =$$

$$(1-\beta)\mu P_2\left[h^*(\theta) - h^*(\widehat{\theta})\frac{v}{(1+v)(1+r)}\right] - \frac{\beta\mu P_2}{1+r}\left[h^*(\theta) - h^*(\widehat{\theta})\right] \tag{76}$$

from which we get:

$$\frac{\partial\left[\Delta W_t(\theta)\right]}{\partial\theta} = (1-\beta)\mu P_2\left[1 - \beta - \frac{\beta}{1+r}\right]\frac{\partial h}{\partial\theta}$$

This shows that the steepness is greater for workers with higher θ if and only if

$$\beta < (1+r)/(2+r). \tag{77}$$

Thus, *workers with higher θ earn higher wages in each period and, if $\beta < (1+r)/(2+r)$, they also have steeper wage profiles.*

Note that if P_2 is exogenous, then, we can easily compute the effect of technological progress (in the sense of an increase in μ) on the wage rates. Since, from (60),

$$\mu h^*(\theta) = \mu^{(1+v)/v}\left[\frac{\theta^z(1-\beta)P_2}{A(1+r)}\right]^{1/v} \tag{78}$$

$$\frac{\partial\left[\mu h^*(\theta)\right]}{\partial\mu} = \frac{1+v}{v}\left[\frac{\theta^z(1-\beta)\mu P_2}{A(1+r)}\right]^{1/v} = h^*(\theta)\left(\frac{1+v}{v}\right) > 0 \tag{79}$$

we obtain from (75)

$$\frac{\partial W_1(\theta)}{\partial\mu} = -\frac{(1-\beta)}{1+r}P_2 h^*(\widehat{\theta}) + \frac{\beta P_2}{1+r}\left(\frac{1+v}{v}\right)\left[h^*(\theta) - h^*(\widehat{\theta})\right]$$

Thus, for the marginal worker

$$\frac{\partial W_1(\widehat{\theta})}{\partial\mu} < 0 \tag{80}$$

(implying that, with a greater productivity parameter, the firm charges the *marginal* worker more for the use of capital in on-the-job learning). On the other hand, from (50) and (79),

$$\frac{\partial W_2(\theta)}{\partial\mu} = (1-\beta)P_2 h^*(\theta)\left(\frac{1+v}{v}\right) > 0, \qquad \theta \geq \widehat{\theta} \tag{81}$$

that is, a higher μ will increase the second period wage. Does a higher μ will *increase the steepness of the wage profile* in sector H ? From (76),

$$\frac{\partial\left[\Delta W_t(\theta)\right]}{\partial\mu} = (1-\beta)P_2\left[h^*(\theta)\frac{1+v}{v} - \frac{h^*(\widehat{\theta})}{1+r}\right] - \frac{\beta P_2}{1+r}\left(\frac{1+v}{v}\right)\left[h^*(\theta) - h^*(\widehat{\theta})\right] \tag{82}$$

Condition (77) is sufficient condition for this expression to be positive.

In the next subsection we determine autarky equilibrium. Here the complication is that the second period price is endogenous.

6.3 Autarkic Equilibrium

To solve for an autarkic equilibrium, we must specify the demand side. The question of how r is determined should also be addressed. This can be done most simply by assuming that individuals maximize life-time utility $U_1 + \delta U_2$ where U_t is quasi-linear, i.e., $U_t = V(X_{Ht}) + X_{Gt}$, and δ is a constant, $0 < \delta < 1$ and $V(.)$ is strictly concave and increasing. Then, in equilibrium, $1/(1+r) = \delta$. We assume that positive amounts of each good are consumed in each period.

The consumer of type θ solves the following intertemporal maximization problem

$$\max_{X(\theta)} X_{G1}(\theta) + V(X_{H1}(\theta)) + \delta X_{G2}(\theta) + \delta V(X_{H2}(\theta)) \tag{83}$$

subject to

$$X_{G1}(\theta) + P_1 X_{H1}(\theta) + \frac{1}{1+r}(X_{G2}(\theta) + P_2 X_{H2}(\theta)) = M(\theta) \tag{84}$$

where $M(\theta)$ is his life-time disposable income (net of learning effort cost) and $X(\theta) = (X_{G1}(\theta), X_{H1}(\theta), X_{G2}(\theta), X_{H2}(\theta))$. We consider interior solutions (i.e., $X_{Ht}(\theta) > 0$ and $X_{Gt}(\theta) > 0$ for $t = 1, 2$.)

For our purposes we are concerned with the demand for period two consumption[15]. Solving the consumer problem above yields the inverse demand function:

$$V'(X_{H2}(\theta)) = P_2$$

from which we obtain the demand function, for all $\theta \in [\theta_a, \theta_b]$,

$$X_{H2}(\theta) = D(P_2) \tag{85}$$

where $D' < 0$. Thus all consumers have the same demand for good H in period 2. (Differences in incomes affect only the levels of consumption of the numeraire good.)

Supply of period two goods by firm θ (for $\theta \geq \widehat{\theta}$) is given by

$$q_{H2}(\theta) = (1 + \mu h^*(\theta)) \tag{86}$$

[15]The problem of first period consumption is separable and has no impact on human capital decisions or on wages.

where $\mu h^*(\theta)$ is given by (78). Since the probability density that a high-tech firm is matched with a worker of type θ, given that only those workers with $\theta \geq \widehat{\theta}$ work in the high-tech sector, is $f(\theta)/[1 - F(\widehat{\theta})]$, and since N_H is the measure of firms, total supply of good H in period 2 is

$$Q_{H2} = \int_{\widehat{\theta}}^{\theta_b} N_H q_{H2}(\theta) \frac{f(\theta)}{1 - F(\widehat{\theta})} d\theta$$

$$= N_H + N_H \mu \left[BP_2\right]^{1/v} R \tag{87}$$

where

$$R \equiv \int_{\widehat{\theta}}^{\theta_b} \frac{\theta^{z/v} f(\theta)}{1 - F(\widehat{\theta})} d\theta \equiv E\left[\theta^{z/v} \mid \widehat{\theta}\right]$$

is the conditional expectation of $\theta^{z/v}$, given $\widehat{\theta}$. (In the special case where $z = 0$ we have $R = 1$.) Notice that even though the number of firms is fixed and each firm employs only one worker, from (87), an increase in P_2 will increase supply, because a higher P_2 encourages human capital accumulation.

Let λ denote the ratio of capitalists to workers. Capitalists have equal shares in all firms of sector H. (There are no profits in sector G.) The total population is $(1 + \lambda)N$. We assume that capitalists have the same utility functions as workers. Then $D(P_{2A})$ is the representative capitalist's demand for good H in period 2. Equating demand to supply we obtain

$$(1 + \lambda) N D(P_{2A}) = N_H + N_H \mu R \left[BP_{2A}\right]^{1/v} \tag{88}$$

This equation determines the equilibrium autarkic price P_2, which we denote by P_{2A}. Next we substitute P_{2A} into (75) to obtain the period one autarkic wage

$$W_{1A}(\theta) = W_G - \frac{v}{(1+v)(1+r)}(1-\beta)\mu h^*(\widehat{\theta}) P_{2A} + \frac{1}{1+r}\beta\mu P_{2A}\left[h^*(\theta) - h^*(\widehat{\theta})\right] \tag{89}$$

where $\mu h^*(\theta)$ is given by (78). Finally, using (63) we have

$$W_{2A}(\theta) = W_G + \mu(1 - \beta)\left(\theta^z B\right)^{1/v} P_{2A}^{(1+v)/v} \tag{90}$$

We are interested in finding out how our endogenous variables $(W_{1A}(\theta), W_{2A}(\theta), P_{2A})$ vary across countries. To do that we determine how these variables change with changes in our parameters (A, μ, β, n_H) where $n_H = N_H/N$.

6.4 Autarky Results

From (88), we get the autarkic equilibrium price P_{2A} of a country as a function of the parameters n_H, μ, A and β. Differentiating (88) totally, we obtain

$$JdP_{2A} = \left(1 + \mu R B^{1/v} P_{2A}^{1/v}\right) dn_H$$

$$+\frac{n_H}{v} \mu R P_{2A}^{1/v} B^{(1-v)/v} dB + n_H R B^{1/v} P_{2A}^{1/v} d\mu \tag{91}$$

where,

$$J \equiv \left((1+\lambda)D' - \frac{n_H}{v} \mu R B^{1/v} P_{2A}^{(1-v)/v}\right) < 0$$

and, using the definition of B given in (57),

$$\frac{dB}{B} = \frac{d\mu}{\mu} - \frac{d(1+r)}{1+r} - \frac{dA}{A} + \frac{d(1-\beta)}{1-\beta} \tag{92}$$

We can therefore state the following comparative statics results:

Proposition B1

(i) An increase in n_H will reduce the second period price P_{2A}

$$\frac{\partial P_{2A}}{\partial n_H} = \frac{1}{J} \left(1 + \mu R B^{1/v} P_{2A}^{1/v}\right) < 0 \tag{93}$$

(ii) An increase in A will increase the second period price P_{2A}

$$\frac{\partial P_{2A}}{\partial A} = -\frac{1}{J} \left(\frac{1}{A}\right) \frac{n_H}{v} \mu R P_{2A}^{1/v} B^{1/v} > 0 \tag{94}$$

(iii) An increase in μ will reduce the second period price P_{2A} :

$$\frac{\partial P_{2A}}{\partial \mu} = \frac{1}{J} \left(\frac{1+v}{v}\right) n_H R P_{2A}^{1/v} B^{1/v} < 0 \tag{95}$$

Remark: Result (i) is obvious because a higher n_H increases supply relative to demand, at any given price. Result (ii) is also plausible, because higher cost of human capital accumulation will result in *a lower rate of accumulation*, thus a fall in output at any given price. This entails a fall in the autarky equilibrium price. Finally, a higher μ will increase human capital accumulation at any given P_2, resulting in a greater supply of good H in period 2 at any given P_2 (i.e., a rightward shift in the supply curve), and hence the equilibrium price must fall in autarky (given that the demand curve is downward-sloping).

The effects of changes in the parameters μ, A, and n_H on second period wage can be computed from (63):

$$\frac{\partial W_{2A}(\theta)}{\partial n_H} = \frac{\partial W_{2A}}{\partial P_{2A}} \frac{\partial P_{2A}}{\partial n_A} > 0 \tag{96}$$

$$\frac{\partial W_{2A}(\theta)}{\partial \mu} = \frac{\partial W_{2A}(\theta)}{\partial \mu} |_{P_{2A} const} + \frac{\partial W_{2A}}{\partial P_{2A}} \frac{\partial P_{2A}}{\partial \mu} \tag{97}$$

which is ambiguous in sign. Similarly, $\frac{\partial W_{2A}(\theta)}{\partial A}$ is also ambiguous in sign.

7 Direction of Trade and Wage Gaps

In this section, we assume that the economy under consideration is under autarky in period 1. We consider two scenarios. Under scenario 1, the economy remains under autarky in period 2, and everyone knows this in period 1. Under scenario 2, the economy will be open to free trade in period 2, and this is also known in period 1. We call the first scenario the autarky scenario, and the second one the free trade scenario. We assume that the country produces both goods in each period, under either scenario. (This is the *incomplete specialization* assumption.)

We consider a two-country world in which countries differ in endowments (n_H), technology (μ), and the cost of education (A).[16] If the two countries differ only in the parameter n_H, then, as shown in the Appendix, the country with a greater n_H will have a lower autarkic price P_{2A}. This country will therefore export the high-tech good under free trade. This is an *endowment-based* explanation of trade.

If the two countries have identical n_H, then, ceteris paribus, the country with a higher μ will have a lower autarkic price P_{2A}, and therefore export the high-tech good under free trade. This is a *technology-based* explanation of trade. Similarly, difference in A provides an *education-cost-based* explanation of trade.

More generally, for any given country, if its second period autarkic price P_{2A} of the high-tech good is smaller [respectively, greater] than the free trade

[16]The cost of education varies across individuals in each country, but differences in A reflect cross-country differences in education cost.

world price P_{2T} of that good, then the opening of trade in period 2 (fully anticipated in period 1) will make that country an exporter[17] [respectively, importer] of the high-tech good in period 2. Let us denote variables of a country by a superscript e (m) if it exports (imports) the high-tech good after the opening of trade. We next determine the effects of free trade on wage gaps.

Let $W^e_{2T}(\theta)$ [respectively, $W^m_{2T}(\theta)$] be the second period wage of a type θ worker in the high-tech sector of a country that exports [respectively, imports] the high-tech good under the free trade scenario. Let $W^e_{2A}(\theta)$ [respectively, $W^m_{2A}(\theta)$] be the second period wage of a type θ worker in the high-tech sector of the same country under the autarky scenario.

We begin by considering *within-country* wage gaps. We call $W^e_{2T}(\theta)$ - W_G the wage gap (between skilled workers of type $\theta \geq \widehat{\theta}$ and unskilled ones) under free trade, of a high-tech exporting economy. We want to compare this gap to the corresponding wage gap under autarky, $W^e_{2A}(\theta) - W_G$. Similarly, $W^m_{2T}(\theta)$ -W_G is called the wage gap under free trade, of a high-tech importing economy. We want to compare this gap to the corresponding wage gap under autarky, $W^m_{2A}(\theta) - W_G$.

Proposition B2: (Effect of trade on wage gaps) For each type θ worker in the high-tech sector,

(i) free trade increases the wage gap in the high-tech exporting country (relative to its wage gap under autarky),

(ii) free trade reduces the wage gap of the high-tech importing country (relative to its wage gap under autarky), and

(iii) the increase (or decrease) is greater for workers who have a greater learning ability, θ.

Proof: For (i), we must show that $W^e_{2T}(\theta)$-W_G exceeds $W^e_{2A}(\theta) - W_G$. For an exporting country, $P^e_{2T} \geq P^e_{2A}$. Therefore, using (63),

$$W^e_{2T}(\theta) \geq W^e_{2A}(\theta) \tag{98}$$

For (ii), note that for an importing country, $P^m_{2A} \geq P^m_{2T}$. Therefore

$$W^m_{2A}(\theta) - W_G \geq W^m_{2T}(\theta) - W_G \tag{99}$$

For (iii), we must show that

$$\frac{\partial}{\partial\theta} W^e_{2T}(\theta) > \frac{\partial}{\partial\theta} W^e_{2A}(\theta) \tag{100}$$

[17]See Appendix 1 for analysis of equilibrium world price P_{2T} in a two-country world.

Using (63),

$$\frac{\partial}{\partial\theta}W^e_{2T}(\theta) = (z/v)\theta^{(z/v)-1}B^{1/v}(1-\beta)\mu\left[P^e_{2T}\right]^{(1+v)/v} > 0$$

Similarly

$$\frac{\partial}{\partial\theta}W^e_{2A}(\theta) = (z/v)\theta^{(z/v)-1}B^{1/v}(1-\beta)\mu\left[P^e_{2A}\right]^{(1+v)/v} > 0$$

Thus (100) is proved. A similar argument applies to the importing country.

Remark: Proposition 2 shows that the effect of international trade on the wage gap between skilled and unskilled workers depends on the pattern of trade. Countries that export the high-tech good will see the wage gap increase, but importing countries will actually find that the wage gap decreases with the opening of trade. The intuition for these results is clear. For example, in the country that exports the high-tech good the opening of trade will increase the price of the high-tech good. This increases profits in that industry and since skilled workers bargain with firms over wages the workers will share in those increased profits through the bargaining process. We can also use these results to say something about *inter-country* wage gaps.

Corollary: The difference between the wage of skilled workers in the exporting economy and that in the importing economy under free trade, $W^e_{2T}(\theta) - W^m_{2T}(\theta)$, exceeds the autarkic difference, $W^e_{2A}(\theta) - W^m_{2A}(\theta)$.

Proof: From (99),

$$-W^m_{2T}(\theta) \geq -W^m_{2A}(\theta) \tag{101}$$

Adding (98) to (101),

$$W^e_{2T}(\theta) - W^m_{2T}(\theta) \geq W^e_{2A}(\theta) - W^m_{2A}(\theta) \tag{102}$$

Remark: Trade will also increase the wage gap between skilled workers across countries. Workers in the high-tech sector in the exporting country will see their wage rise relative to their counterparts in countries that import the high-tech good. This is a direct consequence of the fact that skill premia are increasing in the exporting country, but decreasing in the importing country.

When trade is *endowment-based,* equation (102) has a special interpretation as explained in the next proposition.

Proposition B3: (Wage equalization) If trade is *endowment-based*, second-period wages for type-θ workers are *equalized* (for a given θ) across countries under free trade, given the incomplete specialization assumption. The country that exports the high-tech good under free trade has low autarkic wages of skilled workers.

Proof: From (63), $W^e_{2T}(\theta) = W^m_{2T}(\theta)$ if countries differ only in n_H. With endowment-based trade, the left-hand side of (102) is zero, implying $W^e_{2A}(\theta) \leq W^m_{2A}(\theta)$.

Remark: On the other hand, if trade is *technology-based*, driven by, for example, the difference in μ, $(\mu^e > \mu^m)$, then, as is clear from (63),

$$W^e_{2T} - W^m_{2T} = (1-\beta)\left[\frac{\theta^z(1-\beta)}{A(1+r)}\right]^{1/v}\left[(\mu^e)^{(1+v)/v} - (\mu^m)^{(1+v)/v}\right]P_{2T}^{(1+v)/v} \geq 0 \quad (103)$$

that is, under free trade, the wage of skilled workers in the exporting country is higher than in the importing country. Then the left-hand side of (102) is positive, and the right-hand side may be positive or negative.

We next turn to consideration of how trade affects the decisions of workers about how much human capital to accumulate.

Proposition B4: (Effect of trade on human capital accumulation) The opening of trade increases the accumulation of human capital in the country whose autarkic price $P_{2A}(\theta)$ is lower than the free-trade price $P_{2T}(\theta)$, and decreases the accumulation of human capital in the country whose autarkic price $P_{2A}(\theta)$ is higher than the free-trade price $P_{2T}(\theta)$.

Proof: From (56), and $P^e_{2A} < P_{2T}$

$$h^{*e}_A(\theta) < h^{*e}_T(\theta)$$

Similarly, $h^{*m}_A(\theta) > h^{*m}_T(\theta)$.

Remark: In addition to price effects trade also has an influence on human capital accumulation. Proposition 4 shows that trade enhances human capital accumulation in countries that export the high tech good and reduces human capital accumulation in the importing country. This result is important because it implies that, to some extent, *trade-induced wage gaps* are the result not only of direct price effects on wages but also due to the effect trade has on the *incentive* to accumulate human capital.

We next show that in the country that exports the high-tech good, favorable trade and technology changes tend to increase the wage gap. Define the wage gap for type θ in a high-tech exporting country for a given trade volume and level of technology to be $g(\theta) = W_2(\theta) - W_G$. Let us introduce *two disturbances* for this economy: it experiences a rise in μ (improved technology) and it confronts a flood of excess supply of good G from a collection of developing economies, which causes the relative price of good G to fall (i.e., the price P_2 rises, a favorable terms of trade shock.) Then the change in this economy's wage gap can be decomposed into two effects, namely the technology effect and the trade effect:

$$d\left(W_2(\theta) - W_G\right) = \frac{\partial\left(W_2(\theta) - W_G\right)}{\partial \mu} d\mu + \frac{\partial\left(W_2(\theta) - W_G\right)}{\partial P_2} dP_2$$

where, from (50), (56) and (56), the technology effect is

$$\frac{\partial\left(W_2(\theta) - W_G\right)}{\partial \mu} |_{P_2=const} = (1-\beta)P_2 \left[h^*(\theta) + \mu \frac{\partial h^*(\theta)}{\partial \mu} |_{P_2=const}\right] > 0 \qquad (104)$$

and the trade effect is

$$\frac{\partial\left(W_2(\theta) - W_G\right)}{\partial P_2} |_{\mu=const} = \mu(1-\beta) \left[h^*(\theta) + P_2 \frac{\partial h^*(\theta)}{\partial P_2} |_{\mu=const}\right] > 0 \qquad (105)$$

Taken together equations (104) and (105) imply that favorable technology and trade shocks tend to increase the wage gap in the high-tech exporting country, while unfavorable shocks will reduce the wage gap. From these equations we can state proposition 5.

Proposition B5: In the country that exports the high-tech good, favorable (unfavorable) technology and trade shocks (i.e., increases in both μ and P_2, raise (reduce) the wage gap between skilled and unskilled workers. In the country that imports the high-tech good, a favorable technology shock (i.e., an increase in μ) combined with a favorable terms of trade trade shock (i.e., a fall in P_2, the price of the imported good) may increase the wage gap.

Remark: Notice that the increases in the wage gap will be larger for workers with higher θ.

8 Extensions

8.1 Income Distribution

In this subsection we analyze the effect of trade and capital accumulation on income distribution. We will first consider the case special case where $z = 0$, because it is simpler, and then we will consider the general case $z \geq 0$.

8.1.1 The special case where $z = 0$

In the *special case* where $z = 0$, the parameter θ has no effect on learning cost (because $\theta^z = \theta^0 = 1$), so workers within a given country are ex ante identical, and no matter which industry they choose to work in, their life-time income (net of effort cost), in terms of good G, is $W_G(1 + \frac{1}{1+r})$. By assumption of an interior solution, they consume both goods. Capitalists (owners of the physical capital stock) also consume. To keep the analysis clear we assume that workers are not shareholders. Then, when a country changes its trading status from autarky to exporter of the high-tech good, the domestic price P_2 rises, and workers are worse off. They would therefore prefer autarky to free trade. Capitalist's profit is increasing in P_2. Thus, capitalists in the high-tech sector prefer exporting to autarky, provided the increase in profit can overcompensate their loss of consumer surplus. (This condition is indeed satisfied; see Appendix 2.) For a country that would become an importer of the high-tech good under free trade, capitalists in the high-tech sector would prefer autarky to free trade, while the workers would prefer trade to autarky.

8.1.2 The general case where $z \geq 0$

In the general case, where z is not zero, to determine the effects of trade on welfare, we have to find out how the change in P_2 (from autarky to free trade) affects the real income of workers of type $\theta \geq \widehat{\theta}$. Let $M(\theta)$ denote the life time income net of effort cost. For the marginal worker $\widehat{\theta}$, $M(\widehat{\theta}) = W_G + W_G/(1+r)$ which does not change with trade. For $\theta \geq \widehat{\theta}$, let

$$W(\theta) = W_1(\theta) + \frac{1}{1+r}W_2(\theta)$$

then

$$M(\theta) = W(\theta) - \frac{1}{\theta^z}C(h^*(\theta)) \tag{106}$$

Using (74) and (63), we obtain, for $\theta \geq \widehat{\theta}$,

$$W(\theta) - W(\widehat{\theta}) = \frac{1}{1+r}\mu P_2 \left[h^*(\theta) - h^*(\widehat{\theta})\right] \tag{107}$$

Thus, from (106), (107) and (59)

$$M(\theta) - M(\widehat{\theta}) = \left(\frac{1}{1+r}\right)\mu P_2 \left[h^*(\theta) - h^*(\widehat{\theta})\right]\left[1 - \frac{1-\beta}{1+v}\right] \tag{108}$$

which, given that $z > 0$, is positive.

We need to modify the conclusions above since (108) implies that for workers of type θ sufficiently larger than $\widehat{\theta}$, income increases moving from autarky to free trade. We summarize this in the following proposition.

Proposition 6: (income distribution) For the high-tech good exporter, trade liberalization, by increasing the price P_2, makes workers in sector G and the marginal worker $\widehat{\theta}$ worse off in real terms, and makes some workers in sector H better off if their learning ability θ is sufficiently greater than $\widehat{\theta}$. Owners of high-tech capital are better off. (See Appendix 2A for a detailed proof.) For the the country that imports the high-tech good, with a marginal fall in P_2, if $z = 0$, all workers are better off and owners of capital in the high-tech sector are worse off, and *social welfare* falls. (See Appendix 2B.)

Remark: Proposition 6 implies that in the high-tech good exporting country, capitalists who own high-tech capital and highly skilled workers benefit from trade whereas less skilled workers (and capital owners in the general human capital sector, who earn zero profit) are worse off. For the country that imports the high-tech good, P_2 falls as compared to autarky, so , if $z = 0$, all workers are better off (recall that, with $z = 0$, they all earn $M(\widehat{\theta}) = W_G(1 + 1/r)$, which is unchanged) and capitalists are worse off, and it can be shown that *social welfare falls.* (See Appendix 2B.)

8.2 Externalities

We have assumed that the accumulation of human capital by a worker does not have direct spillover effects on other workers. In the endogenous growth literature, however, many authors argue that there exists significant spillovers. Let us indicate briefly how our model can be modified to take into account such beneficial externalities. An intuitively appealing formulation would be to modify our model by specifying that the parameter μ_i for individual i is an increasing function of the average amount $\bar{h}$ of accumulation of human capital in the industry and of N_H

$$\mu_i = \mu^0 + \phi(\bar{h}N_H), \qquad \phi' > 0 \tag{109}$$

The positive externality displayed in (109) implies that a laissez-faire regime would result in an inefficiently low level of accumulation of human capital. Since one person's investment in his human capital has a positive spillover effect on the human capital of others, the government of a small economy might want to pursue policies that increase the price of the high-tech good. This would mean that the country that imports the high tech good under free trade may have an incentive to prohibit such imports so as to raise the domestic price P_2, thus encouraging more human capital accumulation (as there is under-investment in human capital under laisez-faire). Of course, there would presumably be other policies that would be more efficient ways to deal with such externalities, but political or revenue considerations could lead to the adoption of protection.

8.3 Education Policy

In this subsection we briefly indicate how one might want to analyze education policy in the context of our model. One could think of education policy affecting two variables in our model, the cost of acquiring education, A or the productivity of education μ. Think of education policy as affecting μ. Let μ^0 denote the initial level of the variable μ. What is the marginal social benefit of a policy that directly gives rise to an increase in μ^0? Under free trade, for a small open economy, P_2 is exogenous and hence any increase in μ^0 to a higher value, say $\mu^0 + \varepsilon$, will increase h, i.e. increase human capital accumulation.

Under autarky, things are slightly different: any increase from μ^0 to $\mu^0+\varepsilon$ will cause the autarkic equilibrium price P_{2A} to fall to some level $\widetilde{P}_{2A} < P_{2A}^0$, and this may discourage human capital accumulation. Since the demand curve for good H is negatively sloped, this price fall means that the new equilibrium quantity consumed is greater than the old equilibrium quantity consumed. This in turns means that the new effective supply $(\mu^0 + \varepsilon)h(\mu^0 + \varepsilon, \widetilde{P}_{2A})$ exceeds the old $\mu^0 h(\mu^0, P_{2A})$, but we cannot be certain whether $h(\mu^0 + \varepsilon, \widetilde{P}_{2A})$ exceeds $h(\mu^0, P_{2A})$. Therefore, under autarky, a policy that directly increases μ may indirectly reduce h. If there are spillover effects in the economy, and if these effects depend on h rather than μh, then under autarky, a policy that increases μ could be harmful. To summarize, education policy that makes human capital more productive always improves welfare in a free trading economy. In an autarkic economy such an education policy may actually reduce welfare.

8.4 Uncertainty

We can introduce uncertainty about second period price, but the basic results go through. Let $s_1(\theta)$ be the amount of savings for a worker in period 1. At the beginning of period 2, when the uncertainty about period 2 price has been resolved, the worker knows that his second period wage is $W_G + \mu h(\theta) P_2$ and therefore his second period utility is

$$I_2 \left[P_2, h(\theta), s_1(\theta)\right] = \max_{X_{2H}} V\left(X_{2H}\right) + \left[s_1(\theta)(1+r) + W_G + (1-\beta)\mu h(\theta) P_2 - P_2 X_{2H}\right]$$

$$= V\left[D(P_2)\right] - P_2 D(P_2) + s_1(\theta)(1+r) + W_G + (1-\beta)\mu h(\theta) P_2$$

Notice that, since P_2 is not known in period 1, we cannot use the function $h^*(P_2, \mu, \beta, A, r, \theta)$ given in (56). We have, for any given $h(\theta)$,

$$\frac{\partial I_2}{\partial P_2} = -D(P_2) + (1-\beta)\mu h(\theta)$$

$$\frac{\partial^2 I_2}{\partial P_2^2} = -D'(P_2) > 0$$

$$\frac{\partial I_2}{\partial h(\theta)} = \mu P_2 > 0$$

$$\frac{\partial I_2}{\partial s_1(\theta)} = 1 + r$$

In the first period, before the uncertainty is resolved, the worker chooses $h(\theta)$ and $s_1(\theta)$ to maximize his expected two-period utility

$$EU = V(D_1(P_1)) + \left[W_1(\theta) - \frac{C(h(\theta))}{\theta^z} - s_1(\theta) - P_1 D_1(P_1)\right] + \delta E I_2 \left[P_2, h(\theta), s_1(\theta)\right]$$

where E is the expectation operator (and the random variable is P_2).

It is clear from the above that in this simple formulation, uncertainty does not affect the decision on human capital investment: the worker simply chooses $h(\theta)$ to maximize

$$-\frac{C(h(\theta))}{\theta^z} + \delta(1-\beta) h(\theta) E P_2$$

which is the equivalent of (52).

9 Concluding Remarks

We have shown that if relative wealth appears in the reduced form utility function, then a number of standard results in the literature must be modified. In particular, under suitable curvature conditions, the poor will catch up with the rich in the long run. To fix ideas, we have referred to economic agents as individuals operating in a closed economy, but clearly they can be interpreted as nations in a globalised economy with perfectly mobile capital, so that factor prices are the same in all countries. Then our results say that poor nations will catch up with rich nations, if the elasticity of marginal utility of relative wealth is sufficiently great.

In the context of an AK endogenous growth model (see the Appendix), we showed that higher permanent growth rates will be achieved if individuals (or nations) are conscious about their relative wealth status. Such high growth rates however reduce welfare.

Our second model of trade liberalization serves as a counter-point to our first model, which exhibits convergence of income. Under the assumption that spillovers are important in the high-tech sector, we show that freer trade can be harmful to LDCs if they do not have a suitable man-power policy that encourages skill accumulation.

References

Akerlof, George A. (1976), "The Economics of the Caste and of the Rat Race and Other Woeful Tales," *Quarterly Journal of Economics 90,* 599-617.

Andersson, Malte, (1982), "Female Choice Selects for Extreme Tail Length in Widow Birds," *Nature* 299, pp 818-20.

Becker, Gary, 1964, Human Capital; A Theoretical and Empirical Analysis, with Special Reference to Education, University of Columbia Press.

Borjas, George J. and Valerie A. Ramey, 1995, Foreign Competition, Market Power, and Wage Inequality, *Quarterly Journal of Economics*, November 1995, 1075-1110

Cole, Harold L., George J. Mailath and Andrew Postlewaite, (1992), "Social Norms, Savings Behavior, and Growth," *Journal of Political Economy*, 100 (6), 1092-1125.

Dawkins, Richard, 1976, *The Selfish Gene,* Oxford University Press, Oxford, U.K.

Dawkins, Richard, 1986, *The Blind Watchmaker*, Oxford University Press, Oxford, U.K.

Dockner, Engelbert, Steffan Jorgensen, Ngo Van Long, and Gerhard Sorger (2000), *Differential Games in Economics and Management Science*, Cambridge University Press, Cambridge, U.K.

De Long, Bradford, Andrei Shleifer and Lawrence Summers, (1990), "Positive Feedback Investment Strategies and Destabilizing Rational Speculation", Journal of Finance 45: 379-395.

Davis, D., 1998, Does European Unemployment Prop up American Wage?, *American Economic Review* 88(3), 478-494.

Falvey, Rodney, 1998, Trade Liberalization and Factor Price Convergence, *Journal of International Economics*, forthcoming.

Freeman, R. B., 1995, Are Your Wages Set in Beijing? *Journal of Economics Perspectives,* 9(3), 15-32, Summer 1995.

Fisher, R.A.,(1930), *The Genetical Theory of Natural Selection,* Oxford: Clarendon Press, @nd edition. New York: Dover Publications.

Hirschman, Albert O., 1969, "How to Divest in Latin America, and Why," *Essay in International Finance* No 76, International Finance Section, Princeton University, November 1969.

Kemp, Murray C, 1962, "The Gains from International Trade," *Economic Journal* 72, 303-319

Kemp, Murray C, 1995, *The Gains from Trade and the Gains from Aid*, Routhledge, N.Y.

Kemp, Murray C. and Koji Shimomura, (1992), "A Dymamic Model of the Distribution of Wealth among Households and Nations," *Annals of Operations Research 37*, 245-72.

Konrad, Kai A., (1992), "Wealth-Seeking Reconsidered," *Journal of Economic Behavior and Organization,* 18(2), 215-27.

Krugman, Paul, 1995, Growing World Trade: Causes and Consequences, *Brookings Papers on Economic Activity,* 1, 251-270.

Lande, Russell, (1980), "Sexual dimorphism, Sexual Selection, and Adaptation in Ploygenic Characters," *Evolution 34:* 292-305.

Lande, Russell, (1981), "Model of Speciation by Sexual Selection of Polygenic Traits," *Proceedings of the National Academy of Sciences*, 78: 3721-5.

Long, Ngo Van, Ray Riezman, and Antoine Soubeyran,(2001),"Trade, Wage Gaps, and Specific Human Capital Accumulation", Typescript, McGill University.

Long, Ngo Van, and Koji Shimomura, (1998), "Some Results on the Markov Equilibria of a Class of Homogenous Differential Games," *Journal of Economic Behavior and Organization* 33, 557-66.

Long, Ngo Van, and Koji Shimomura, (2001), "Relative Wealth, Catching-up, and Growth," Typescript, McGill University.

Long, Ngo Van, and Kar-yiu Wong, (1997), "Endogenous Growth and International Trade: A Survey," in B. Jensen and Kar-yiu Wong (eds.), *Dynamics, Economic Growth, and International Trade,* The University of Michigan Press, pp. 11-74.

Leamer, E.E., 1998, In search of Stoper-Samuelson Linkages between International Trade and Lower Wage Rates, in S. Collins (ed.) Imports, Exports, and the American Worker, Washington, D.C.,Brookings Institution, 141-202.

Lucas, Robert E., 1988, On the Mechanics of Economic Development, *Journal of Monetary Economics* 22, 3-42.

Neary, J. Peter, 2000, Competition, Trade and Wages, Typescript, University College Dublin.

Pitchford, John D., (1960), "Growth and the Elasticity of Factor Substitution", *Economic Record 36:* 491-504

Solow, Robert M., (1956), "A Contribution to the Theory of Economic Growth", *Quarterly Journal of Economics, 70:*65-94

Stiglitz, J. E., (1969), "Distribution of Income and Wealth among Individuals," *Econometrica 37*(3), 382-97

Stokey, Nancy, (1991), "Human Capital, Product Quality, and Growth," *Quarterly Journal of Economics* 106: 334-61.

Swan, Trevor W., (1956), "Economic Growth and Capital Accumulation", *Economic Record 32:* 334-61.

Tyers, Rod, and Yongzhen Yang, 1999, European Unemployment , US Wages, and the Asian Emergence, *Working Paper # 367*, Faculty of Economics and Commerce, Australian National University.

Wood, Adrian, 1994, *North-South Trade, Employment, and Inequality: Changing Fortunes in a Skill-driven World,* Clarendon Press, Oxford.

Van, Pham Hong, and Henry Wan, Jr., (1996), "Interpreting East Asian Growth," typescript, Cornell University.

Van Valen, Leigh, (1973), "A New Evolutionary Law," *Evolutionary Theory* 1: 1-30.

Wynne-Edwards, V. C., (1962), *Animal Dispersion in Relation to Social Behaviour,* Oliver and Boyd, Edinburgh.

Young, Alwyn, (1991), "Learning by doing and the Dynamic Effects of International Trade," *Quarterly Journal of Economics* 106: 369-450.

APPENDIX

A Modified Endogenous Growth Model

We now turn to an endogenous growth model of the AK variety: the per-capita production function is either linear in the capital labor ratio, or approaches such a linear function as k tends to infinity[18].

We postulate the production function

$$y = Ak$$

where we assume

$$A \geq \rho$$

that is, the technology is sufficiently productive to overcome the force of discounting. Concerning the utility function, we will consider the following two cases, in the next two subsections:

Case I: The utility function is additively separable:

$$U(c_i, \frac{k_i}{k}) = u(c_i) + \theta v \left[\frac{k_i}{k}\right] = \ln c_i + \theta \ln \left[\frac{k_i}{k}\right], \quad \theta \geq 0 \tag{110}$$

Case 2: The utility function is multiplicatively separable:

$$U(c_i, \frac{k_i}{k}) = \left(\frac{c_i^{\alpha}}{\alpha}\right) \left[1 + \theta \left(\frac{k_i}{k}\right)^{\beta}\right], \quad \theta \geq 0, \quad 1 > \beta > 0, \quad 1 > \alpha > 0 \tag{111}$$

Notice that $\alpha > 0$ ensures that $\partial U / \partial k_i \geq 0$.

We continue to assume that all individuals have identical initial wealths.

Case I: additively separable utility

I.a: The social planner's problem

The social planner seeks to

$$\max_{c(t)} \int_0^{\infty} [\ln(c(t)) + \theta \ln(1)]\, e^{-\rho t} dt$$

[18]For early papers dealing with this second variety of production functions, see Solow (1956) and especially Pitchford (1960) who provided a comprehensive analysis of perpetual growth in per capita consumption without technical progress. Long and Wong (1996) refer to that model as the Solow-Pitchford AK model.

subject to

$$\dot{k}(t) = Ak(t) - c(t), \qquad k(0) = k_0, \text{ and } k(t) \geq 0.$$

We can find an explicit solution for the social planner's problem. We use the dynamic programming approach, and write the Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V(k) = \max_c \left[\ln c + V'(k)(Ak - c)\right] \tag{112}$$

where $V(.)$ is the value function, to be determined as part of the solution of the problem.

The first-order condition is

$$\frac{1}{c} = V'(k) \tag{113}$$

In addition, to ensure sufficiency, we impose the transversality condition[19]

$$\lim_{t\to\infty} V(k(t))e^{-\rho t} = 0 \tag{114}$$

Substituting (113) into (112), we get

$$\rho V(k) = \ln 1 - \ln V'(k) + AkV'(k) - 1 \tag{115}$$

which is a first-order differential equation. We try a solution of the form[20]

$$V(k) = D + B\ln k$$

Then $V'(k) = B/k$ and

$$\rho D + \rho B \ln k = -\ln B + \ln k + AB - 1$$

For this equation to hold as an identity, i.e., for *all* $k > 0$, it must be the case that

$$\rho B \ln k = \ln k$$

and

$$\rho D = -\ln B + AB - 1$$

[19]See Dockner et al. (2000), in particular, Chapter 3.

[20]For the properties of solution of a more general class of problem, see Long and Shimomura (1998).

Thus

$$B = \frac{1}{\rho}$$

and

$$D = \frac{1}{\rho}\left[\ln \rho + (A/\rho) - 1\right]$$

It follows that the optimal solution for the social planner problem consists of the linear consumption strategy

$$c = \rho k = \frac{\rho}{A} y \tag{116}$$

Thus, the optimal average propensity to consume is ρ/A. The rate of growth of the captal stock can be computed from

$$\dot{k} = Ak - c = Ak - \rho k$$

implying that the endogenous growth rate is

$$g = \dot{k}/k = A - \rho \geq 0. \tag{117}$$

It follows that

$$k(t) = k_0 e^{(A-\rho)t}$$

The value of the program is[21]

$$V(k_0) = \frac{1}{\rho}\left[\ln \rho + (A/\rho) - 1\right] + \frac{1}{\rho} \ln k_0 \tag{118}$$

9.0.1 The laissez-faire outcome

Individuals take as given the time path of the economy's per capita wealth k. Individual i seeks to

$$\max_{c_i(t)} \int_0^\infty \left[\ln(c_i(t)) + \theta \ln\left(k_i(t)/k(t)\right)\right] e^{-\rho t} dt \equiv V_i(k_{i0})$$

subject to

$$\dot{k}_i = Ak_i - c_i, \qquad k_i(0) = k_{i0}, \text{ and } k_i(t) \geq 0. \tag{119}$$

[21] It can be verified that the transversality condition (114) is satisfied.

The Hamiltonian is

$$H = \ln(c_i(t)) + \theta \ln\left(k_i(t)/k(t)\right) + \psi(t)\left[Ak_i(t) - c_i(t)\right]$$

The necessary conditions are

$$\frac{\partial H}{\partial c_i} = \frac{1}{c_i} - \psi = 0 \tag{120}$$

$$\dot{\psi} = \rho\psi - \frac{\partial H}{\partial k_i} = (\rho - A)\psi - \frac{\theta}{k_i} \tag{121}$$

Differentiating (120) with respect to t, we get

$$\frac{\dot{c}_i}{c_i} = -\frac{\dot{\psi}}{\psi} \tag{122}$$

On the other hand, using (121),

$$\frac{\dot{\psi}}{\psi} = \rho - A - \frac{\theta}{\psi k_i} = \rho - A - \frac{\theta c_i}{k_i} \tag{123}$$

From (122) and (123), we get

$$\frac{\dot{c}_i}{c_i} = \rho - A - \frac{\theta c_i}{k_i} \tag{124}$$

To find a solution for the pair of differential equations (119) and (124), we guess that c_i is a linear function of k_i :

$$c_i = E_i k_i \tag{125}$$

where E_i is to be determined. If (125) holds, then

$$\frac{\dot{k}_i}{k_i} = \frac{\dot{c}_i}{c_i}$$

Substituting (125) into (119) and (124), we obtain

$$A - E_i = \frac{\dot{k}_i}{k_i} = \frac{\dot{c}_i}{c_i} = \rho - A - \theta E_i$$

This implies that

$$E_i = \frac{\rho}{1 + \theta}$$

It follows that

$$c_i = \left[\frac{\rho}{1+\theta}\right]\frac{y_i}{A} < \frac{\rho}{A}y_i \tag{126}$$

The endogenous growth rate under laissez-faire is

$$\frac{\dot{k}_i}{k_i} = A - \frac{\rho}{1+\theta} > A - \rho \geq 0$$

Proposition 3: Assume that individuals are wealth-conscious ($\theta > 0$). Under laissez-faire, individuals consume a smaller fraction of their income than the fraction that the social planner would choose. This results in a higher growth rate under laissez-faire than under the social optimum. However, individuals are worse off under laissez-faire.

Remark: In the case $A = \rho$, we see that under the planner, there would be no growth, while under laissez-faire, there is exponential growth. Such exponential growth would go unchecked in the absence of resource constraints. This version of endogenous growth based on status-seeking has its counterpart in biology. One of the greatest successors of Darwin, R.A. Fisher, has reached the following conclusion:

"...plumage development in the male, and sexual preference for such developments in the female, must thus advance together, and so long as the process is unchecked by severe counterselection, will advance with ever increasing speed. In the total absence of such checks, it is easy to see that the speed of development will be proportional to the development already attained, which will therefore increase with time exponentially, or in geometric progression."(The Genetical Theory of Natural Selection, 1930, cited by Dawkins, 1986, p 199.)

Remark: To verify that individuals are worse off under laissez-faire, let us calculate the integral of discounted utility under laisse-faire. Since $k_i/k = 1$ in equilibrium,

$$V_i(k_{i0}) = \int_0^\infty [\ln c_i(t) + \theta \ln(1)]\, e^{-\rho t} dt$$

$$= \int_0^\infty \left[\ln\left(\frac{\rho k_{i0}}{1+\theta}\exp\left(A - \rho/(1+\theta)\right)t\right)\right] e^{-\rho t} dt$$

$$= \frac{1}{\rho}\left[\ln \rho - \ln(1+\theta) + \ln k_{i0}\right] + (A - \rho/(1+\theta)) \int_0^\infty t e^{-\rho t} dt$$

$$= \frac{1}{\rho}\left[\ln \rho - \ln(1+\theta) + \ln k_{i0}\right] + \frac{1}{\rho^2}\left[A - \frac{\rho}{1+\theta}\right] \quad (127)$$

The difference between (118) and (127) is

$$V(k_0) - V_i(k_{i0}) = \frac{1}{\rho}\left[\frac{A}{\rho} - 1\right] - \frac{1}{\rho}\left[\frac{A}{\rho} - \frac{1}{1+\theta} - \ln(1+\theta)\right] > 0$$

(To see that the difference is positive, it is sufficient to show that $1 < \frac{1}{1+\theta} + \ln(1+\theta)$ for all $\theta > 0$. This is true because $\frac{1}{1+\theta} + \ln(1+\theta)$ is an increasing function of θ for all $\theta > 0$.)

Remark: If we assume that the utility function is $U(c_i, k_i, k) = \frac{1}{\beta} c_i^\beta + \frac{\theta}{\beta}\left[k_i^\beta - k^\beta\right]$, then the basic results of this section remain unchanged.

Case II: multiplicatively separable utility

II a: The social planner's problem

The social planner sets $k_i = k$ for all i. The objective is to maximize

$$\max_{c(t)} \int_0^\infty \left[\left(\frac{c^\alpha}{\alpha}\right)(1+\theta)\right] e^{-\rho t} dt$$

subject to $\dot{k} = Ak - c$, $k(0) = k_0$, and and $k(t) \geq 0$.The Hamiltonian is

$$H = \left(\frac{c^\alpha}{\alpha}\right)(1+\theta) + \psi\left[Ak - c\right]$$

The necessary conditions are

$$(1+\theta)c^{\alpha-1} = \psi \quad (128)$$

$$\dot{\psi} = \psi(\rho - A) \quad (129)$$

Differentiate (128) with respect to t to get

$$(\alpha - 1)\frac{\dot{c}}{c} = \frac{\dot{\psi}}{\psi} = \rho - A$$

or

$$\frac{\dot{c}}{c} = \frac{A - \rho}{1 - \alpha} \equiv g > 0$$

We also have, along a steady growth path,

$$\frac{\dot{k}}{k} = A - \frac{c}{k} = g$$

Hence, under the assumption that

$$\rho - A\alpha > 0 \tag{130}$$

it is clear that the consumption/capital ratio is a positive constant:

$$\frac{c}{k} = A - g = \frac{A(1-\alpha) - A + \rho}{1-\alpha} = \frac{\rho - A\alpha}{1-\alpha} > 0 \tag{131}$$

The transversality condition is satisfied:

$$\lim_{t\to\infty} e^{-\rho t}\psi(t)k(t) = \lim_{t\to\infty} e^{-\rho t}\psi(0)e^{(\rho-A)t}k(0)e^{gt} = \lim_{t\to\infty} \psi(0)k(0)e^{-(A-g)t} = 0$$

The optimal initial consumption is, from (131),

$$c^*(0) = (A-g)k_0$$

and

$$c^*(t) = c^*(0)e^{gt} = (A-g)k_0e^{gt}$$

and the integral of discounted utility is

$$\int_0^\infty \left(\frac{1+\theta}{\alpha}\right)\left[(A-g)k_0e^{gt}\right]^\alpha e^{-\rho t}dt = \left(\frac{1+\theta}{\alpha}\right)[(A-g)k_0]^\alpha \int_0^\infty e^{-(\rho-g\alpha)t}$$

This integral converges if

$$1 - A\alpha > 0 \tag{132}$$

Since (130) is assumed, (132) is satisfied if $1 > \rho$.

IIb Laissez-faire outcome

It is easy to verfiy that under laissez-faire, the outcome is a faster growth rate, but a lower level of wellbeing for all participants. For reason of space, the details are omitted.

APPENDIX 1 (FOR THE WAGE GAP MODEL)
Determination of free trade equilibrium in a two-country world

Recall that, in the home country, the ratio of capitalists to workers is λ. Let the superscript f indicate variables of the foreign country. We assume that $\lambda^f = \lambda$. The home country's demand for good H in period two is $(1+\lambda)ND_2(P_2)$ and its supply of that good in period two is $Q_{H2}(P_2)$, where, using (87),

$$Q_{H2}(P_2) = N_H + N_H \mu R \left[BP_2\right]^{1/v} \tag{133}$$

Thus, the home country's excess supply function, $ES_2(P_2) \equiv Q_{H2}(P_2) - (1+\lambda)ND_2(P_2)$ is upward sloping, and intersects the price axis at a value denoted by P_{2A}.

The foreign country's demand for good H in period two is $(1+\lambda)N^f D_2(P_2)$ and its supply of that good in period two is $Q^f_{H2} = N^f_H + N^f_H \mu^f R^f \left[B^f P_2\right]^{1/v} \equiv Q^f_{H2}(P_2)$. Since $h^{*\prime}(P_2) > 0$ from (56), the foreign country's excess demand function, $ED^f_2(P_2) = N^f D^f_2(P_2) - Q^f_{H2}(P_2)$ is downward sloping, and intersects the price axis at a value denoted by P^f_{2A}.

The free-trade equilibrium price P_{2T} must satisfy

$$ES_2(P_{2T}) = ED^f_2(P_{2T}) \tag{134}$$

Assume that $P^f_{2A} > P_{2A}$. Then there exists a unique free-trade equilibrium price P_{2T} , such that $P^f_{2A} > P_{2T} > P_{2A}$ and $ES_2(P_{2T}) = ED^f_2(P_{2T}) < 0$, indicating that the home country is the exporter of the high-tech good.

It is easy to see that in our model, $P^f_{2A} > P_{2A}$ if and only if, for all $P_2 \geq 0$,

$$n_H \left[1 + \mu R \left(\frac{(1-\beta)\mu}{A(1+r)}\right)^{1/v} P_2^{1/v}\right] > n^f_H \left[1 + \mu^f R^f \left(\frac{(1-\beta^f)\mu^f}{A^f(1+r^f)}\right)^{1/v} P_2^{1/v}\right] \tag{135}$$

Thus, if $n_H > n^f_H$ (with other parameters being the same for both countries), then the home country will be the exporter of the high-tech good; this is an *endowment-based* explanation of trade. Similarly, $\mu > \mu^f$would give a *technology-based* explanation of home high-tech exports. $A < A^f$ would explain home exports of high-tech good in terms of lower *education cost.* $\beta < \beta^f$would explain home exports of high-tech good in terms of lower *bargaining power* of home firms. $r < r^f$ would explain home exports of high-tech good in terms of *time preference* (a low interest rate encourages accumulation of human capital).

APPENDIX 2(FOR THE WAGE GAP MODEL)
Effects of free trade on income distribution

2 A: The impacts on the country that exports the high-tech good:

We first consider an open economy that *exports* the high-tech good under free trade. For this economy, the move from autarky to free trade may be represented by an increase in P_2 (relative to its autarkic P_{A2}). What are the effects of this increase on the welfare of workers and capital owners?

The indirect utility of a worker of type θ is obtained from the maximization problem described in section 2.3. This yields the demand functions

$$X^*_{H1}(P_1) = D_1(P_1),\ X^*_{H2}(P_2) = D_2(P_2) \tag{136}$$

where $D_t(.)$ is the inverse function of $V_t'(.)$. The demand for the numeraire good G can then be inferred from the budget constraint. Thus

$$X^*_{1G} + \frac{1}{1+r}X^*_{1G} = M(\theta) - P_1D_1(P_1) - \frac{1}{1+r}P_2D_2(P_2) \tag{137}$$

Substituting (136) and (137) into the direct utility function, and recalling that $\delta = 1/(1+r)$, we obtain the indirect life-time utility function

$$\mathcal{U}(P_1, P_2, M(\theta)) = V_1(D_1(P_1)) + \delta V_2(D_2(P_2)) + M(\theta) - P_1D_1(P_1) - \delta P_2D_2(P_2) \tag{138}$$

The effect of an increase in P_2 on the welfare of a worker of type θ is

$$\frac{d\mathcal{U}}{dP_2} = \delta V_2'D_2'(P_2) - \delta\left[P_2D_2' + D_2\right] + \frac{dM(\theta)}{dP_2} = -\delta D_2(P_2) + \frac{dM(\theta)}{dP_2}$$

For workers with $\theta \leq \widehat{\theta}$, their life-time income, net of education cost, is $M(\theta) = W_G(1 + \frac{1}{1+r})$ and therefore $\frac{dM(\theta)}{dP_2} = 0$. They are therefore made worse off by the rise in P_2. For workers with $\theta > \widehat{\theta}$, we use (78) and (108) to get

$$\frac{dM(\theta)}{dP_2} = \left(\frac{1}{1+r}\right) P_2^{1/v}\mu^{(1+v)/v}\left[\frac{1-\beta}{A(1+r)}\right]^{1/v}\left[1 - \frac{1-\beta}{1+v}\right]\left[(\theta)^{z/v} - \left(\widehat{\theta}\right)^{z/v}\right]$$

which is non-negative (positive if $z > 0$). Thus workers with sufficiently high learning ability will be better off under free trade.

For owners of capital in the high-tech sector, the indirect utility function is (138) with $M(\theta)$ replaced by the discounted sum of profits, which we denote by Π (which is the same for all firms in the high-tech sector, as we have argued in section 2.2, see equation (72) in particular.) From (47), (73), (69) and (78),

$$\Pi(\theta) = \Pi(\widehat{\theta}) = P_1 - W_G + \left(\frac{1-\beta}{1+r}\right)\left(\frac{v}{1+v}\right)\mu h^*(\widehat{\theta})P_2$$

$$+\left(\frac{1}{1+r}\right)\left[P_2 - W_G + \beta\mu h^*(\widehat{\theta})P_2\right]$$

i.e.,

$$\Pi(\widehat{\theta}) = P_1 + \left(\frac{1}{1+r}\right)P_2(1+\mu h^*(\widehat{\theta})) - W_G\left(1+\frac{1}{1+r}\right)$$

$$-\left(\frac{1-\beta}{1+r}\right)\left(\frac{1}{1+v}\right)\mu h^*(\widehat{\theta})P_2$$

It follows that

$$\frac{d\Pi(\widehat{\theta})}{dP_2} = \left(\frac{1}{1+r}\right)\left[1+\left(1-\frac{1-\beta}{1+v}\right)\frac{d[\mu h^*(\widehat{\theta})P_2]}{dP_2}\right]$$

where, from (78)

$$\frac{d[\mu h^*(\widehat{\theta})P_2]}{dP_2} = \left(\frac{1+v}{v}\right)\mu h^*(\widehat{\theta}) > 0$$

Thus

$$\frac{d\Pi(\widehat{\theta})}{dP_2} = \left(\frac{1}{1+r}\right)\left[1+\left(\frac{v+\beta}{v}\right)\mu h^*(\widehat{\theta})\right] > 0 \tag{139}$$

The total profit gain to all capital owners in the high-tech sector is $\Omega \equiv N_H\left(\frac{1}{1+r}\right)\left[1+\left(\frac{v+\beta}{v}\right)\mu h^*(\widehat{\theta})\right]$, and, if $z = 0$ so that $h^*(\theta) = h^*(\widehat{\theta})$ for all $\theta \geq \widehat{\theta}$, then Ω exceeds the (discounted) total output of the high-tech sector in period 2, which is $N_H(1+\mu h^*(\widehat{\theta}))/(1+r) = Q_2(P_{2A})/(1+r)$. Thus the gain in profits exceeds the loss of consumers surplus, $N(1+\lambda)D_2(P_{2A})/(1+r) = Q_2(P_{2A})/(1+r)$.

2 B: The impacts on the country that imports the high-tech good:

For a country that imports the high-tech good, the opening of trade amounts to a fall in P_2 relative to its autarkic price P_{2A}. A similar argument shows that, if $z = 0$, the fall in P_2 will lead to a *net loss* of social welfare: the loss of profits outweighs the gains in consumers surplus.