Implications of Many Industries on the Heckscher-Ohlin Model

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Abstract

This paper examines the implications of many industries on the Heckscher-Ohlin (HO) model. Available empirical studies suggest that output prices are interdependent. When output prices are interdependent, the HO Theorem obtained in the 2 × 2 case does not generally hold in the multi-commodity world. It is shown that the mean Stolper-Samuelson elasticities as well as the mean Rybcyznski effects become negligible as the number of industries increases. Due to output indeterminacy, exports of a capital abundant country need not be more capital intensive than imports. Leontief's two empirical studies on U.S. trade patterns were invalid tests of the HO predictions that were derived from the 2 × 2 model. Thus, the so-called Leontief Paradox may be commonly observed. The main results of the 2 × 2 HO model are peculiarities that have little relevance to the real world with many industries.

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"There is not much virtue in simplicity if a result that holds in a model of two countries, two commodities, and two factors does not generalize in any meaningful way to higher dimensions." John Chipman (1988, p. 922).

1. Introduction

The two-factor, two-commodity Heckscher-Ohlin (HO) model contains four elegant propositions that have charmed many trade theorists. For instance, if the United States were a capital-abundant country, the HO theory predicts that it would export capital-intensive goods. Wassily W. Leontief (1953) conducted the first empirical test of the theory, using 1947 U.S. trade data. Contrary to his expectation, however, Leontief discovered that U.S. import-competing industries required 30 percent more capital per worker than exports. This finding has come to be known as the Leontief Paradox.

In all subsequent empirical studies, the number of industries has been much greater than that of factors. For instance, Leontief's (1956) second test included 192 industries. Similarly, in Stern and Maskus (1981) and Trefler (1993), the number of industries was much greater than that of factors. However, as Chipman (1988) noted, regardless of the elegance of the results, a simple 2×2 HO model would lose much of its charm if the results were not robust in the multi-commodity world.

The purpose of this paper is to examine the implications of many commodities on various propositions of the HO model. It is shown that once we depart from the simple 2×2 world, the extended HO model cannot predict the trade pattern using notions of factor abundance and intensities. The impacts of output prices on factor prices and the mean Rybczynski effects are shown to be negligible when there are many industries. The n × 2 model does *not* predict that exports of a capital abundant country will be capital intensive. Leontief's approach was not valid, because he expected the prediction of a 2 × 2 model to be borne out in his two empirical studies that included more than two industries.[1] In the real world, the so-called Leontief Paradox may appear to be a common occurrence.

2. *Interdependence of Output Prices*

At the outset it is important to note that while Leontief had intended to test the 2 × 2 HO model, in his first test, he actually built a 38 × 2 model of U.S. trade in 1947. Two stylized facts have emerged from this and subsequent other empirical studies on trade:

Fact 1. The number of outputs, n, is much greater than that of factors, m, used to produce the outputs.

Fact 2. Typically, a trading country produces k goods, $m < k < n$, and the k/m ratio is closer to n/m than to unity.

These stylized facts suggest that some of the essential results of the 2 × 2 HO theory may not be robust in a higher-dimensional world. The relationship between inputs and outputs is summarized by

$$AY = V, \tag{1}$$

where $A = [a_{ij}]$ is an $m \times n$ matrix, Y is an $n \times 1$ output vector, and V an $m \times 1$ input vector. The trade vector is

$$X = Y - C, \tag{2}$$

where C is an $n \times 1$ consumption vector, and X an $n \times 1$ trade vector. The element x_j is positive (negative) if product j is exported (imported). Given the usual assumption of homothetic preferences, the consumption vector can be written as $C = cI$, where c is an $n \times 1$ vector of the average propensities to consume, and I is consumer income. Thus, the trade vector is

$$X = Y - cI.$$

To predict which product a country will export, it is essential to know the output vector Y. In the 2×2 case ($n = m = 2$), the system of equations in (1) has a unique solution, provided that A is nonsingular, i.e., its inverse exists. Hence, a given factor endowment uniquely determines the output vector, which can then be used, together with the consumption vector cI, to determine the country's trade vector.

Consider the smallest uneven case, a 3×2 model, which is slightly more general than the HO model, but is qualitatively similar to Leontief's first empirical 38×2 model. Predicting the output vector Y amounts to solving for three unknowns with two equations, one for each factor. Obviously, the output vector Y is not unique. Infinitely many different output vectors are consistent with a given factor endowment. In this case, the output vector Y has one degree of freedom. If one output is fixed by government decree or if there is a constraint in the relationship between outputs, the output vector can be uniquely determined. As Leamer (1984, 1987) observed, in general the output vector will have (n - m) degrees of freedom.[2]

Inputs may be classified into many categories just as outputs are differentiated. For example, Trefler (1995) used nine categories of labor inputs. Depending on the type and length of education, workers and wages may be further differentiated. However, it may be argued that when the quality of labor is enhanced by education, the original unskilled labor is transformed into an intermediate input which embodies some human capital. The primary input, unskilled labor, remains the same. Thus, the number of primary inputs is still limited, relative to the ever-increasing variety of outputs produced.

Recall that in his first test of the HO theory, using 1947 U.S. trade data, Leontief's (1953) analysis included 50 sectors, of which 38 industries produced traded goods. Since Leontief assumed only two factors, capital and labor, the *n/m* ratio was approximately 20 in the first test. Using the U.S. trade pattern in 1951, Leontief (1956) conducted a second test, in which he divided the U.S. economy into 192 sectors. Since capital and labor were the only primary factors, the *n/m* ratio was approximately 100 in that study.

Stern and Maskus (1981) constructed another HO model with three inputs (physical capital, human capital, and labor) for the period 1958 to 1976. They classified industries into three categories: the Ricardian goods, the HO goods, and the Product Cycle goods. Intuitively, in the production of Ricardian goods, natural resource components (e.g., weather, mineral deposits) are important. The HO goods are characterized by the use of standardized technology, whereas the Product Cycle goods are produced by constant product innovation. When they focused narrowly on the HO goods, the number of HO industries varied over the years, exceeding 120 industries during most of the period. Thus, in the Stern and Maskus' study, the *n/m* ratio was nearly 40.

In a more recent study, Trefler (1993) converted trade data from the four-digit Standard Industrial Trade Classification (SITC) into 79 sectors and investigated trade flows of ten factors, including capital, cropland, pasture and seven categories of labor. In this case, the *n/m* was close to eight. All these empirical studies of U.S. trade patterns indicate that the *n/m* ratio was much greater than one, exceeding ten in most empirical studies.

We now consider the implications of a large *n/m* ratio on four components of the HO model: the Rybczynski Theorem, the Factor Price Equalization result, the Heckscher-Ohlin Theorem, and the Stolper-Samuelson Theorem.

Long-run Indeterminacy of the Output Vector

The system of equations in (1) has (n - m) degrees of freedom, and for all practical purposes, a country's output vector is unpredictable. If the purpose of a model is to predict whether a sector will export its output, it would be disappointing because of output indeterminacy. Given the assumption of identical technologies, any industry can be induced to export its product. If an industry produces enough to export, then other industries must adjust their outputs accordingly. In fact, (n - m) industries can choose their output levels arbitrarily. Then the outputs of the remaining m industries can be determined uniquely. However, it is not easy to predict how much an industry will actually produce and export because of the large degree of freedom.[3]

One way to resolve this production indeterminacy has been suggested by Leamer (1987). His model does not impose any constraint or relationship among commodity prices. Ethier (1984, p. 143) suggests that commodity prices are not drawn from an urn but are interconnected. However, for the sake of resolving this indeterminacy, first consider Leamer's approach and assume that commodity prices are arbitrarily chosen.

Since all industries are competitive, profit-maximizing efforts of competitive firms collectively maximize national income, *Py*, subject to the resource constraints. Due to constant returns to scale, unit costs are independent of outputs, although the input-output coefficients are still functions of factor prices. The problem then is to choose the output vector y to:

$$\text{maximize } I = Py \text{ subject to: } Ay = V,$$

where P and y are $n \times 1$ vectors of prices and outputs, and V is an $m \times 1$ vector of factor endowments.

The Lagrangian function associated with this problem is:

$$\mathcal{L} = Py + W[V - Ay], \tag{3}$$

where W is an $m \times 1$ vector of Lagrange multipliers, reflecting the shadow prices of the internationally immobile inputs. The solution to the problem yields optimal levels y and shadow prices W. Specifically, Leamer (1987) shows that given an arbitrary price vector P, optimal outputs are positive only for m industries and the outputs of the remaining sectors equal zero. However, most of the empirical studies show that the k/m ratio is closer to n/m than to unity. For instance, in Leontief's first test, 35 industries were net exporters and three were net importers. This implies that output prices are interlinked, as Ethier had suggested, and commodity prices move together, at least among the goods that are actually produced.

3. Futility of the Heckscher-Ohlin Theorem

In its simplest form, the Heckscher-Ohlin Theorem states that in the 2 × 2 case, each country exports the commodity which intensively uses its abundant factor. Here are two notions that beg to be defined in the multi-commodity world: factor abundance and factor intensity. It is not difficult to generalize the abundance concept to a higher dimension. In the two-factor case, a country is abundant in capital if $K/L > K^*/L^*$. Let $I = wL + rK$ and

$I^* = wL^* + rK^*$ denote home and foreign incomes, respectively, and let $\alpha = I/(I + I^*)$ denote the income share of the home country. Then a country may be said to be abundant in capital if

$$\frac{K}{K + K^*} > \alpha, \tag{4}$$

which holds if and only if $K/L > K^*/L^*$. The abundance definition in (4) can be applied to any other factor, regardless of the number of factors. With this definition, it is not possible for a country to be abundant or poor in all factors.

In the multi-commodity world there are at least two reasons why factor intensity definitions—however cleverly designed—cannot be used to predict with certainty which product will be exported. First, in the realistic world where $n > m$, the output vector is indeterminate and hence the trade vector cannot be predicted. Specifically, Leamer has shown that if output prices are independent of one another, only m goods are produced. The contrapositive of Leamer's Theorem is that if more than m goods are actually produced, then output prices are dependent, and hence the output vector will be indeterminate.

Second, even when only m good is produced, it is almost impossible to predict which outputs will be exported using factor intensity definitions—unless the inverse of the input-output matrix is utilized. Moreover, the notion of factor intensity becomes ambiguous, because the choice of numéraire is arbitrary. For instance, if N is a third factor representing natural resources, then $K_1/L_1 > K_2/L_2$ and $K_1/N_1 < K_2/N_2$ can hold simultaneously. Factor intensities can be defined between any pair of industries, and the number of pairwise comparisons increases geometrically as the number of goods increases. Such intensity definitions are of little use if they cannot be used to predict trade patterns. Alternative definitions of factor intensities may be devised and interpreted, but none have been utilized to predict the trade pattern of a country as clearly as in the 2 × 2 case.

Theorists have focused on the even case ($n = m$), where the number of goods is equal to the number of factors. Suppose only m goods are produced. For those goods, the relationship between input and output vectors is written as $Ay = V$. If A is nonsingular, the output vector is written

$$y = A^{-1}V = BV, \tag{5}$$

where $B = A^{-1}$. The trade vector is simply $X = A^{-1}V - C$, and it can be predicted from the country's factor endowment vector V. Obviously, in the 2 × 2 case, the effect of a change in factor endowment on the output vector can be predicted (the Rybczynski Theorem) by linking outputs and factor intensities. However, no such intuitive predictions are possible for $m > 2$.[4]

Consider the output vector in (5). For instance, the output of industry 1 is written as: $y_1 = b_{1L}L + b_{1K}K + b_{1M}M + b_{1M}N + \ldots$. Similarly, $y_2 = b_{2L}L + b_{2K}K + b_{2M}M + b_{2M}N + \ldots$, and so on. Let y^{o} be the hypothetical output vector when the home country has an equal share α of each factor. Now let the labor endowment increase so that the home country is abundant only in labor. Thus, from the initial situation y^{o}, only dL is positive and $dK = dM = dN = \ldots = 0$. In this case

$$\begin{aligned} dy_1 &= b_{1L}dL, \\ dy_2 &= b_{2L}dL, \\ dy_3 &= b_{3L}dL, \\ dy_4 &= b_{4L}dL, \\ &\ldots \end{aligned}$$

The Rybczynski result for a change in a factor endowment requires obtaining m cofactors as well as the determinant of the input-output matrix A. In the 2×2 case, the sign of the determinant means a pairwise ranking of factor intensities of the two sectors. However, in the $m \times m$ case, pairwise rankings of m factor intensities cannot determine the sign of the determinant of the input-output matrix, nor the signs of any cofactors. In short, as the number of commodities increases beyond two, pairwise rankings of factor intensities cannot determine the signs of elements of A^{-1}. This is one reason why the Heckscher-Ohlin Theorem cannot be generalized even to the $m \times m$ world using factor intensity definitions.

How Outputs Are Determined in the Short Run

If n exceeds m, the problem is more complicated. The HO model is based on the assumption that all product and factor markets are perfectly competitive. If all prices are equal to unit costs, the output vector will be indeterminate. How then are the actual outputs determined in practice? Since the long-run output and factor employments in each sector are indeterminate, any trade theory that does not explain how the actual outputs are determined would be of little use.

It is useful to think of the HO model with two time horizons. In the long-run, all primary inputs are variables. In the short run, the actual output of a firm in a given industry is determined by its size or the existing capital stock. If the output price deviates from its unit cost, in the short run firms can vary the quantities of the variable inputs. However, if all output prices are jointly determined, equal to unit production costs, then in the long-run competitive firms in each industry earn zero profits, and no firms have any incentive either to enter or exit the market. This is consistent with the result that optimal size or output of a competitive firm is indeterminate when the production function is linearly homogenous.

Instead of Leamer's problem, we now consider a short-run maximization problem. Since it is straightforward to generalize to the n × m case, it is sufficient to illustrate how the smallest of general uneven models, the 3 × 2 case, works. In the short run, capital input is fixed and the problem is to choose inputs, L_i, to maximize national income, subject to the variable input constraints. Let $F^i(\bullet)$ denote the production function of good i. The Lagrangian function associated with this problem is:[5]

$$\mathcal{L} = p_1F^1(L_1,K_1) + p_2F^2(L_2,K_2) + p_3F^3(L_3,K_3) + w[L - L_1 - L_2 - L_3]. \qquad (6)$$

The first order conditions are:

$$p_iF^i_{Li}(L_i,K_i) - w = 0, \text{ i = 1, 2, 3.}$$

The value of the marginal product of labor, $p_iF^i_{Li}$, can be added horizontally as in Figure 1.[6] The shadow price w is determined by the intersection of the aggregate value of the marginal product of labor and the vertical labor supply curve. Once the shadow price is obtained, it can be treated as the wage by competitive firms or industries. Since capital inputs are fixed in the short run, labor demand functions are written as $L_i(K_i, p_i, w)$. Short-run supply function of good i is written as

$$y_i(K_i, p_i, w) = F^i\left[L_i(K_i, p_i, w), K_i\right].$$

However, industry output is indeterminate and the long-run supply curve is horizontal,[7]

$$p_i = g_i(w,r) = a_{Li}(w,r)w + a_{Ki}(w,r)r.$$

Given an arbitrary capital allocation, $\underline{K} = (K_1, K_2, ..., K_n)$, there exists a unique solution to equation (6). Producers earn zero profits if prices are equal to unit production costs. Thus, in each industry, competitive firms have no incentive either to enter or exit the market. However, this does not mean that the output vector is unique in the long-run. Another capital allocation $\underline{K}'$ will yield a different output vector, which also will be consistent with the given output prices.

4. The Mean Stolper-Samuelson Effects

In the 2 × 2 case, the Stolper-Samuelson Theorem states that an increase in the price of a good increases the return to the factor intensively used in that industry and reduces the return to the other factor. Moreover, since the latter declines, the return to the intensive factor increases more than proportionately, a magnification effect. However, the amplified change in the return to the intensive factor may be a *peculiarity* that occurs in the even case where factor prices are determined uniquely.[8]

Leamer (1987) considered the n × 3 case in which profit maximization results in the production of only three goods. In this case, the Stolper-Samuelson result may be obtained from a relevant 3 × 3 submatrix of A. However, if all product prices are interdependent, the Stolper-Samuelson result is not likely to prevail in the more realistic uneven case. If an increase in one price were to alter factor prices, realignment of most other output prices may be required. If this realignment of output prices is *precluded* artificially, the initial price change may be accompanied by quantity responses in many industries and the survival of only m industries, as indicated by Leamer's result. However, there is no a priori way to predict how output prices will be realigned. If these joint changes in other output prices are known, their effects on the factor prices should also be included.

We now argue that when n is large, a change in the price of one good has negligible effects on factor prices, i.e., factor prices are insensitive to a change in output price. The larger the number of industries, the smaller the impact of a change in the output price on factor prices.

Even when output prices are interdependent, a tariff can be arbitrarily imposed on any imports. Will a change in the tariff on a product affect the returns to the immobile factors? To examine its Stolper-Samuelson effect, first consider how factor prices are determined when n is large and there are two factors, K and L. An alternative formulation of Leamer's problem is to choose L_j and K_j to

$$\text{maximize } \sum_j P_j F^j(L_j, K_j)$$

$$\text{s.t. } \sum_j L_j = L,\ \sum_j K_j = K.$$

The long-run Lagrangian function associated with this problem is:

$$\mathcal{L} = \sum_{j=1}^{n} p_j F^j(L_j, K_j) + w\left[L - \sum L_j\right] + r\left[K - \sum K_j\right], \tag{7}$$

where r is the shadow price of capital. Since the industry outputs are indeterminate, supply curves are horizontal at prices equal to unit costs. If commodity prices are arbitrarily chosen, only two goods would be produced. However, output prices are assumed linked together so that prices are equal to unit costs in all industries. The first order conditions are:

$$P_j \frac{\partial F^j}{\partial L_j} - w = 0, \text{ for } y_j > 0,$$

$$P_j \frac{\partial F^j}{\partial K_j} - r = 0, \text{ for } y_j > 0.$$

How does a change in p_j—if it can be changed alone, for instance, by a tariff—affect the factor markets when n is large? In this case, each industry's contribution to national income is small, and it behaves like a competitive firm or a price taker in factor markets. Since its labor demand accounts for only a small fraction of the aggregate labor demand, an increase in p_j shifts the aggregate labor demand to the right only slightly, resulting in a negligible change in the wage.

Recall that in his second test Leontief examined U.S. trade patterns in 192 industries. In this situation, the labor share of an average industry is about 1/200. Suppose a typical industry's output doubles. At given factor prices, doubling of output results in doubling of input requirements. However, this increase in demand in one industry increases, for instance, the aggregate demand for labor only by 0.5 percent. Thus, doubling of input demands in one sector will have a negligible effect on the aggregate demand for each immobile factor. Accordingly, factor prices cannot change dramatically as in the 2 × 2 case.

Perfect competition implies that output price must be equal to unit cost in any industry that produces some output,

$$P = A'W, \tag{8}$$

where P is an $n \times 1$ vector of output prices and W an $m \times 1$ vector of factor prices. If output prices are independent of one another, only m goods are produced. If the submatrix corresponding to the prices of goods that are actually produced is invertible, equation (8) shows that output prices can be derived from input prices and vice versa, and the Stolper-Samuelson Theorem can be obtained. However, equation (8) does not say whether output prices *cause* input prices or vice versa; it only links input and output prices. Thus, the causal relation between input and output prices must be explained by other means.

When n is much larger than m, each industry becomes a price taker in factor markets, and hence factor prices dictate output prices. Of course, output prices may deviate from their unit costs in the short run, depending on demand and supply conditions. Industries that do not earn zero profits will soon be eliminated. Thus, they cannot long deviate from the unit costs. Output prices are little affected by the demand side; they are solely determined by the supply side. In other words, input prices dictate the levels of output prices. This idea represents a significant departure from the so-called Stolper-Samuelson Theorem, which is based on the notion that output prices affect input prices.

When there are two inputs K and L, the i^{th} row of the system of equations in (8) for product j can be written as

$$P_j = a_{Lj}w + a_{Kj}r, \ j = 1, \ldots, n.$$

Since n is large, each industry behaves as a price taker in the factor markets, and factor prices are determined by the intersection of (domestic) aggregate demands and supplies of the factors. Once these factor prices (r and w) are determined, output prices are completely determined by (8), and in the long-run industries cannot deviate from these equilibrium prices.

Although he was not interested in trade issues *per se*, Alfred Marshall (1890, 1961, p. 620) noted the relationship between output and factor prices in competitive markets:

> "In the first place the undertaker's profits bear the first brunt of any change in the price of those things which are the product of his capital (including his business organization), of his labour and of the labour of his employees; and as a result

fluctuations of his profits generally precede fluctuations of their wages, and are much more extensive…. But experience shows that … they seldom rise as much in proportion as prices; and therefore they do not rise nearly as much in proportion as profits."

Thus, the zero profit condition in (8) suggests that when the *n/m* ratio is large, it is the input prices that determine output prices. Changes in output prices—if they can be altered by a tariff— have little effect on factor prices. Domestic supply conditions of the primary inputs (and aggregate factor demands) determine the factor prices, which in turn dictate the output prices for all surviving industries. Thus, output prices are not free to deviate from the unit costs in the long-run. However, they can be affected by policy variables such as tariffs.

We now consider the average effect on wage of a price change caused by a tariff. Differentiating the Lagrangian function in (7) with respect to L gives

$$\frac{\partial I}{\partial L} = w. \tag{9}$$

Differentiating (9) with respect to p_i, we get the Stolper-Samuelson result,

$$\frac{\partial^2 I}{\partial L \partial p_i} = \frac{\partial w}{\partial p_i} = \frac{\partial y_i}{\partial L}, \tag{10}$$

which shows the short run reciprocity relation between the Stolper-Samuelson result and the Rybczynski Theorem. Let

$$\varepsilon_{wi} \equiv \frac{\partial w}{\partial p_i} \frac{p_i}{w}$$

be the elasticity of wage with respect to p_i and let $\bar{\varepsilon}_w$ be the mean value of these Stolper-Samuelson elasticities on the wage:

$$\bar{\varepsilon}_w \equiv \frac{\frac{\partial w}{\partial p_1}\frac{p_1}{w}+\frac{\partial w}{\partial p_2}\frac{p_2}{w}\ldots+\frac{\partial w}{\partial p_n}\frac{p_n}{w}}{n}=\frac{\frac{\partial y_1}{\partial L}\frac{p_1}{w}+\frac{\partial y_2}{\partial L}\frac{p_2}{w}\ldots+\frac{\partial y_n}{\partial L}\frac{p_n}{w}}{n}.$$

Using (9), the mean value of the Stolper-Samuelson elasticities can be written:

$$\bar{\varepsilon}_w = \frac{1}{n}. \tag{11}$$

Similarly,

$$\bar{\varepsilon}_r \equiv \frac{\frac{\partial r}{\partial p_1}\frac{p_1}{r}+\frac{\partial r}{\partial p_2}\frac{p_2}{r}\ldots+\frac{\partial r}{\partial p_n}\frac{p_n}{r}}{n}=\frac{\frac{\partial y_1}{\partial K}\frac{p_1}{r}+\frac{\partial y_2}{\partial K}\frac{p_2}{r}\ldots+\frac{\partial y_n}{\partial K}\frac{p_n}{r}}{n}.$$

Hence,

$$\bar{\varepsilon}_r = \frac{1}{n}. \tag{12}$$

Intuitively, if all prices increase by 1 percent, the wage rate will increase by 1 percent. When the price of one good alone increases by 1 percent—for instance, due to a tariff—on average, the wage rate increases by $1/n$ percent.[9] Therefore, when the number of commodities is very large, the effects of an increase in a single output price on factor prices become negligible. That is, the magnification effect in the 2 × 2 case is not likely to be observed when the number of goods is large. Thus, the magnification effect of a price change on factor prices is a *peculiarity* of the 2 × 2 model that is unlikely to be observed in the multi-commodity world.

5. The Mean Rybczynski Effects

In the 2 × 2 case, the Rybczynski Theorem states that an increase in factor endowment increases the output of the good which intensively uses that factor and decreases the output of the other industry. Implicit is the assumption that before and after the change, the factor endowment belongs to the same cone of diversification so that factor growth does not affect factor prices. In the realistic case where *n* is much larger than *m*, the output vector is indeterminate and hence, after a change in factor endowment the new output vector is also indeterminate. However, this does not imply that a small change in a factor endowment will cause an erratic response in the output vector.

How does the economy move from one equilibrium to another in response to a change in factor endowment when the output vector itself is indeterminate? Consider, for example, how the output vector will change in response to a change in the labor endowment. Because of constant returns to scale, an increase in factor endowment has no effect in the long-run on factor prices within the cone of diversification. In the case of three industries, differentiating $F^i(L_i, K_i)$ with respect to L yields:

$$\begin{aligned}
\frac{\partial y_1}{\partial L} &= F^1_{K1}\frac{\partial K_1}{\partial L} + F^1_{L1}\frac{\partial L_1}{\partial L},\\
\frac{\partial y_2}{\partial L} &= F^2_{K2}\frac{\partial K_2}{\partial L} + F^2_{L2}\frac{\partial L_2}{\partial L},\\
\frac{\partial y_3}{\partial L} &= F^3_{K3}\frac{\partial K_3}{\partial L} + F^3_{L3}\frac{\partial L_3}{\partial L}.
\end{aligned} \tag{13}$$

It is important to note that an increase in labor endowment does not affect the ratios of inputs used in each industry along its expansion path. An increment in the labor endowment must be used up in at least one industry. Suppose $\partial L_1 / \partial L$ in (13) is positive. As long as factor prices remain constant, the ratio of these factors remains unchanged in each industry as its output expands. In (13), $\partial L_1 / \partial L$ is positive if and only if $\partial K_1 / \partial L$ also is positive, because both factors move together along an expansion path. Thus, an increase in labor endowment always increases the output of at least one sector. Moreover, $\partial K_1 / \partial L > 0$, if and only if $\partial K_2 / \partial L$ or $\partial K_3 / \partial L$ is negative. This implies that since all factors in the long-run move together along each expansion path, industry 2 or 3 must shrink. Thus, *an increase in a factor endowment always causes at least one industry to expand and another to shrink*. However, predicting which industry will expand or shrink amounts to predicting the signs of the determinant and cofactors of the input-output matrix, which cannot be accomplished by pairwise comparisons of the input-output coefficients, except in a low-dimensional case.

Instead of focusing on the physical quantities of output, it is more convenient to examine the effect of factor growth on the industry revenue. Using (10), the effect of an increase in labor endowment on the industry revenue is written as:

$$\frac{\partial R_i}{\partial L} \equiv \frac{\partial p_i y_i}{\partial L} = w\frac{\partial L_i}{\partial L}. \tag{14}$$

The average value of L_i is:

$$\bar{L}_i = \frac{L}{n}.$$

Thus, the mean value of $\partial L_i / \partial L$ is

$$\frac{\partial \bar{L}_i}{\partial L} = \frac{1}{n}.$$

Using (9), the mean revenue effect is written

$$\frac{\partial \bar{R}_i}{\partial L} = \frac{\frac{\partial I}{\partial L}}{n} = \frac{w}{n}. \tag{15}$$

Similarly,

$$\frac{\partial \bar{R}_i}{\partial K} = \frac{\frac{\partial I}{\partial K}}{n} = \frac{r}{n}. \tag{16}$$

Intuitively, if the labor force increases by one worker, national income increases by his wage. If the labor endowment increases by ΔL, national income increases by $w\Delta L$. A typical industry gets only gets a small fraction ($1/n$) of this increased income, i.e., on average, the industry revenue increases by $w\Delta L/n$. Thus, when n is large, it is not likely for a typical industry to display any magnification effect on its revenue.

Since the capital inputs are fixed, an increase in labor supply causes a decline along the aggregate value of the marginal product of labor, and hence reduces the wage or shadow price w, which causes each sector to hire more workers. In the long-run, $\partial y_i / \partial L$ and $\partial L_i / \partial L$ can be negative for some industries. However, in the short run, $\partial y_i / \partial L$ and $\partial L_i / \partial L$ are positive for all industries.

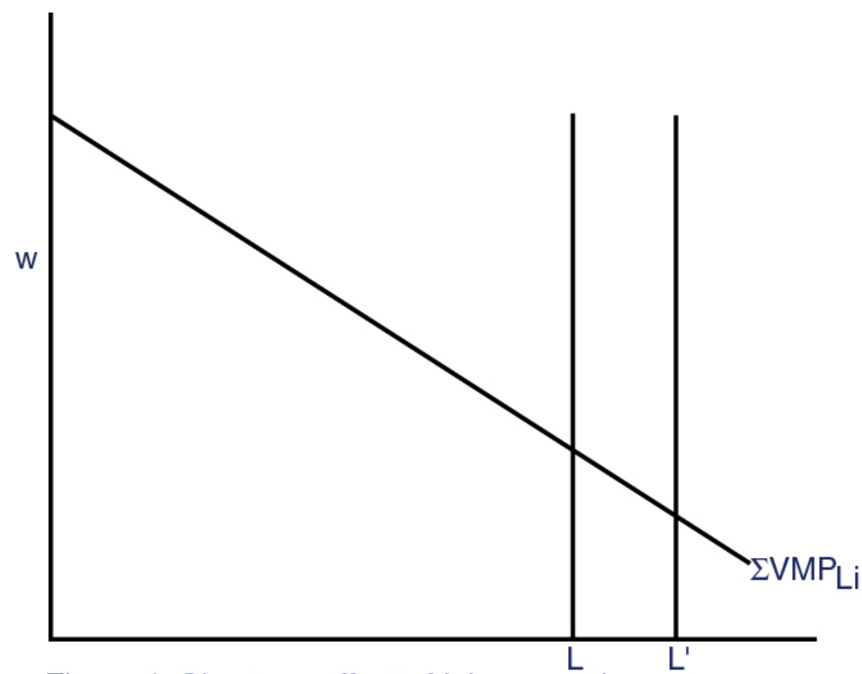

Figure 1. Short run effect of labor growth

6. Factor Price Equalization

The HO theory suggests that under certain conditions free trade of commodities will equalize the returns to immobile factors. Illegal immigration, for instance, is an indication that one country has a higher wage than another. The reasons cited for nonequalization of factor prices include factor intensity reversal and different production technologies, etc.

From (8), if output prices are equal to unit costs, free trade of m goods completely equalizes m factor prices. As the number of commodities increases, holding the number of factors constant, the probability that m goods will be freely traded increases. Accordingly, the probability of factor price equalization increases as n increases.

In the case of 2×2 case, any fluctuation of an output price will cause a ripple in factor prices. Such fluctuations of output prices cannot be treated as errors. However, when n is large as in Leontief's second test, the probability that two goods will be freely traded in long-run equilibrium dramatically increases. From the zero profit conditions of two such markets, factor prices are derived. Long-run equilibrium prices of all other products, consistent with these factor prices can then be derived. All other prices not equal to unit costs that are derived in this manner suggest that these markets are not in long-run equilibrium. Either entry or exit occurs in these markets. Thus, as the number of commodities increases, the international gap between factor prices is more likely to shrink.

7. Leontief Was Not Right

Leontief aggregated industries into 50 sectors, but only 38 industries actually produced commodities that entered the international markets; the remaining sectors were either nontraded goods or accounting identities. He also aggregated factors into two categories, labor and capital. He then estimated the capital and labor requirements to produce $1 million worth of typical exportable and importable goods in 1947. Capital per worker in the export sector was k_x = $14,300, and that in the import sector was k_m = $18,200. Thus, U.S. imports were about 30 percent more capital-intensive than U.S. exports in 1947.[10]

It was pointed out that 1947 was not a typical year to test the HO theory; many industries had not fully recovered from war damages, and postwar reconstruction had not been completed by that time. Leontief (1956) repeated the test for U.S. trade in 1951. In this later study, he disaggregated the U.S. production structure into 192 sectors and found that U.S. import substitutes were still 6 percent more capital-intensive, relative to U.S. exports. Baldwin (1971) found that in 1962 U.S. import substitutes were about 27 percent more capital-intensive than U.S. exports. However, Stern and Maskus (1981) demonstrated that in 1972 the paradox was reversed; the capital-labor ratio in U.S. exports (about $18,700 per worker-year) was higher than in U.S. import substitutes (about $17,300 per worker-year).

In his first test, Leontief used two factors of production, capital and labor. Of the 38 industries, 35 were net exporters, which indicates positive production in those industries. Similarly, Leamer (1987, p. 986) investigated a three-factor (capital, labor, and land) model, reporting that in 1978 at the three-digit International Standard Industrial Classification level, every commodity group was produced by all 38 countries. These empirical results provide concrete evidence that output prices are interdependent. That is, for all of these outputs to be produced, output prices must have moved together to maintain the equality between prices and unit production costs.

Production of more than *m* goods implies that the prices are adjusted to the levels of unit costs. When this occurs, the output vector is indeterminate, and so is the trade vector. Thus, exports of a capital abundant country are not necessarily more capital intensive than their imports.

Was Leontief right when he compared the capital-labor ratios between the import and export sectors? When $n > m$, this extended HO model does *not* predict precisely that exports of a capital-abundant country will be capital-intensive. Recall that in the 3 × 2 case, there is one degree of freedom in the output vector. Thus, for any given output of y_3, the remaining output vector can be uniquely determined. It is then possible to choose a sufficiently large volume of y_3 so that it is exported. Since the remaining two goods cannot both be exported, assume y_1 is exported and y_2 is imported. Then the capital-labor ratio of the export bundle is

$$k_x = \frac{a_{K1}(y_1 - c_1) + a_{K3}(y_3 - c_2)}{a_{L1}(y_1 - c_1) + a_{L3}(y_3 - c_2)}, \qquad (17)$$

and $k_m = a_{K2} / a_{L2}$, where c_i is consumption of good i.

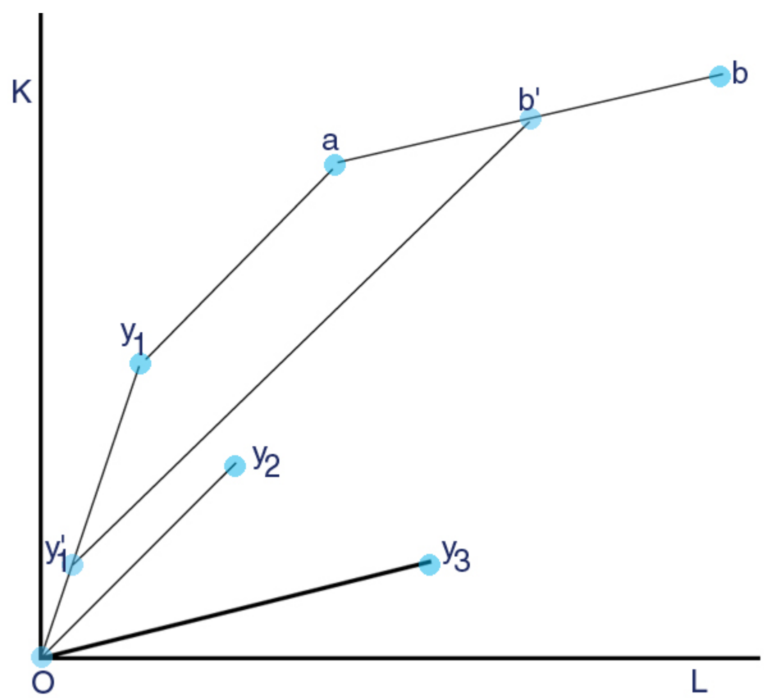

Figure 3. Why a Leontief Paradox may occur

We now show that when $n > m$, it is possible to increase the capital-labor ratio of the export bundle without affecting income or consumption. That is, k_x can be greater than or less than k_m. Since there is one degree of freedom, assume that y_3 is decreased. This causes a movement from b to another point b' in Figure 3. Since industry 3 is the most labor-intensive, a decrease in its production has a Rybcyznski effect, similar to an increase in labor. A new combination of the two products must yield a vector Ob', resulting in a decrease in the production of the most capital intensive good y_1 and an increase in the other good y_2 (not drawn), which is less capital-intensive than y_1. This change in output mix, however, has no effect on income or consumption bundles. Thus, in equation (17) this change in the output mix results in a reduction of the export of the most capital-intensive good y_1 and an increase in the export of the less capital-intensive good y_2, thereby reducing the capital-labor ratio of the export bundle. Output indeterminacy results in indeterminacy of the capital-labor ratios of the export and import bundles, and there is no reason why the export bundle should be more capital-intensive than the import bundle. Thus, a Leontief paradox is likely to be observed frequently in the multi-commodity world.

Trefler (1993) followed Leontief's (1953) hint that American workers may have been more productive than their foreign cohorts and argued that if factor productivity or quality indices were incorporated, Leontief was right to claim that U.S. exports were more labor intensive than U.S. imports in 1947. This analysis shows that the reverse result is equally likely to occur, because even in the absence of factor quality difference, the 38 × 2 model does not predict that exports of a capital abundant country will be more capital intensive than its imports. The Heckscher-Ohlin prediction in the 2 × 2 world simply does not carry over to the n × 2 world. Thus, Trefler tested the validity of a nonexisting theorem. A similar analysis on Leontief's second test may reinforce or reverse Trefler's finding.

Thus, when there are two factors of production and *n* is large, the chance that the HO prediction that the export sector is more capital-intensive than the import sector is likely to be observed is no better than flipping a coin. This conjecture is well supported by the abundant occurrence of the Leontief Paradox in empirical tests of the HO trade theory.

8. The Heckscher-Ohlin-Vanek Theorem

The HOV Theorem explores the factor contents embodied in output trade. Specifically, the HOV theorem states that a capital abundant country exports the services of capital input through commodity trade. Although the trade bundle is indeterminate, the factor contents embodied in the trade bundles are determinate.

Let $V^x = AX$ and $V^c = AC$ denote the vector of factors embodied in the trade bundle and the consumption vector C, respectively. If the j^{th} element of V^x is positive (negative) it shows that product j is exported (imported). Premultiplying (2) by the input-output matrix *A* yields

$$V^x = AY - AC = V - V^c = V - \alpha V^w, \qquad (18)$$

where V^w is the world's factor endowment vector and α is the income share of the home country.[11] Thus, if a country is abundant in factor i ($V > \alpha V_i^w$) then V_i^x is positive. That is, a country exports the services of its abundant factor, despite the indeterminacy of the output and trade vectors. Thus, the HOV theorem survives in the *m* × *n* world. However, this result is predicated on factor price equalization.

When the physical definition of abundance is used, the home country is abundant in capital if $K/L > K^*/L^*$ or $K/K^* > L/L^*$, which holds if and only if

$$\frac{K}{K^*} > \frac{I}{I^*} = \frac{wL + rK}{wL^* + rK^*}. \tag{19}$$

Let $Y^w = Y + Y^*$ denote an $n \times 1$ vector of world outputs, and $C^w = C + C^*$ an $n \times 1$ vector of the world consumption vector. The world as a whole must consume its outputs, and hence $Y^w = C^w$. Let $\alpha \equiv I/(I + I^*)$ be the home country's income share. Then the home country must consume α fraction of the world's output vector. It follows that the factor content of the home country's consumption bundle is

$$V^c = \alpha V^w, \tag{20}$$

where V^w is the world endowment vector of inputs. Then (19) is rewritten as

$$\frac{K}{K^*} > \frac{K_c}{K_c^*}. \tag{21}$$

Thus, a capital abundant country exports capital input through commodity trade.

In the absence of factor price equalization, however, capital abundance ($K/K^* > L/L^*$) does not imply, nor is it implied by

$$\frac{K}{K^*} > \frac{wL + rK}{w^*L^* + rK^*}. \tag{22}$$

Moreover, equation (20) does not hold when factor prices are different between countries. Hence, the HOV Theorem in (21) does not hold when factor prices are not equalized.

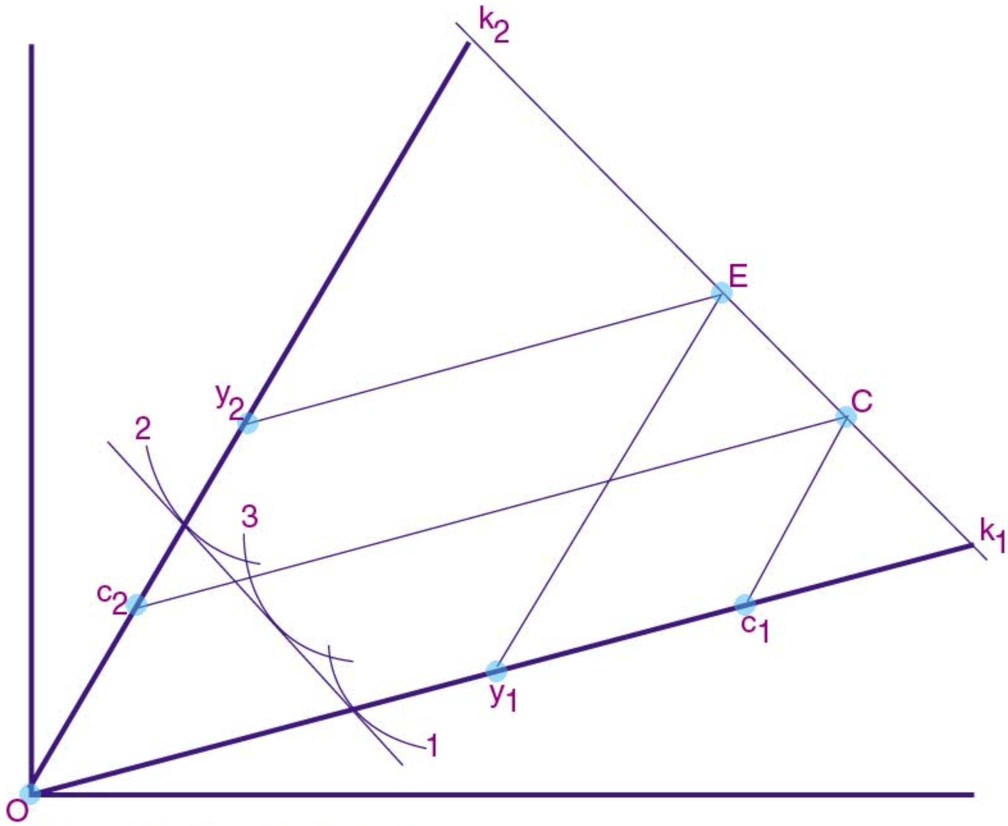

Figure 4. Indirect factor trade

Figure 4 illustrates the HOV Theorem. Point E shows the given factor endowment $(\bar{L}, \bar{K})$. Ok_1 and Ok_2 are the expansion paths of industries 1 and 2 that are generated by a pair of unit value isoquants, labeled 1 and 2, derived from output prices, p_1 and p_2. Points y_1 and y_2 show the factor allocations (L_1, K_1) and (L_2, K_2). If the country exports good 2, the amounts of factors embodied in consumption of good 2 are less than those at point y_2. That is, $(L_2^c, K_2^c) < (L_2, K_2)$. Since good 1 is exported, $(L_1^c, K_1^c) > (L_1, K_1)$. Trade of goods amounts to moving from the endowment point E to another point C on the iso-income line AB, along which national income $wL + rK = I$ remains constant. This diagram illustrates that exports of the capital-intensive good amount to exports of capital and imports of labor services.

Adding more goods does not affect the result. Since all prices are equal to unit costs, the existence of industry 3 simply means that industry 1 or 2 or both industries must reduce production. However, the sum of all these vectors must add up to the endowment point E. Regardless of the composition of the consumption goods, if factor content embodied in the consumption bundle C is to the right of E, the country is indirectly exporting capital and importing labor.

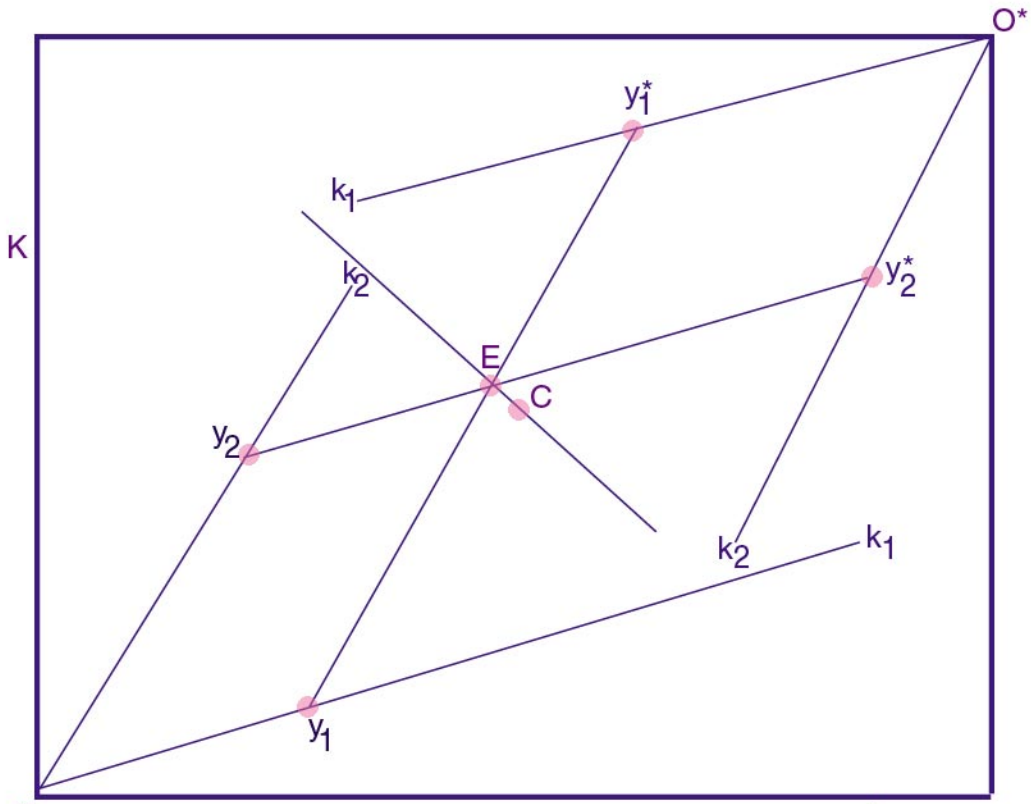

Figure 5. FPE and Factor content of trade

Figure 5 illustrates the factor content of trade when factor price equalization occurs. Point E shows the initial endowment of two factors, K and L. Points y_1 and y_2 show the factor allocations of industries 1 and 2, respectively. If two countries have identical preferences, the ratios of capital-to-labor embodied in consumption are the same, equal to $(K+K^*)/(L+L^*)$.

Consumption point C is the intersection of the straight line (not drawn) connecting the two origins, O and O*, and the iso-income line $I = wL + rK$. Since factor prices are equalized, the sum of any factor exported (or imported) by the two countries is zero. The amount of a factor exported indirectly by the home country is equal to that amount imported by the foreign country.

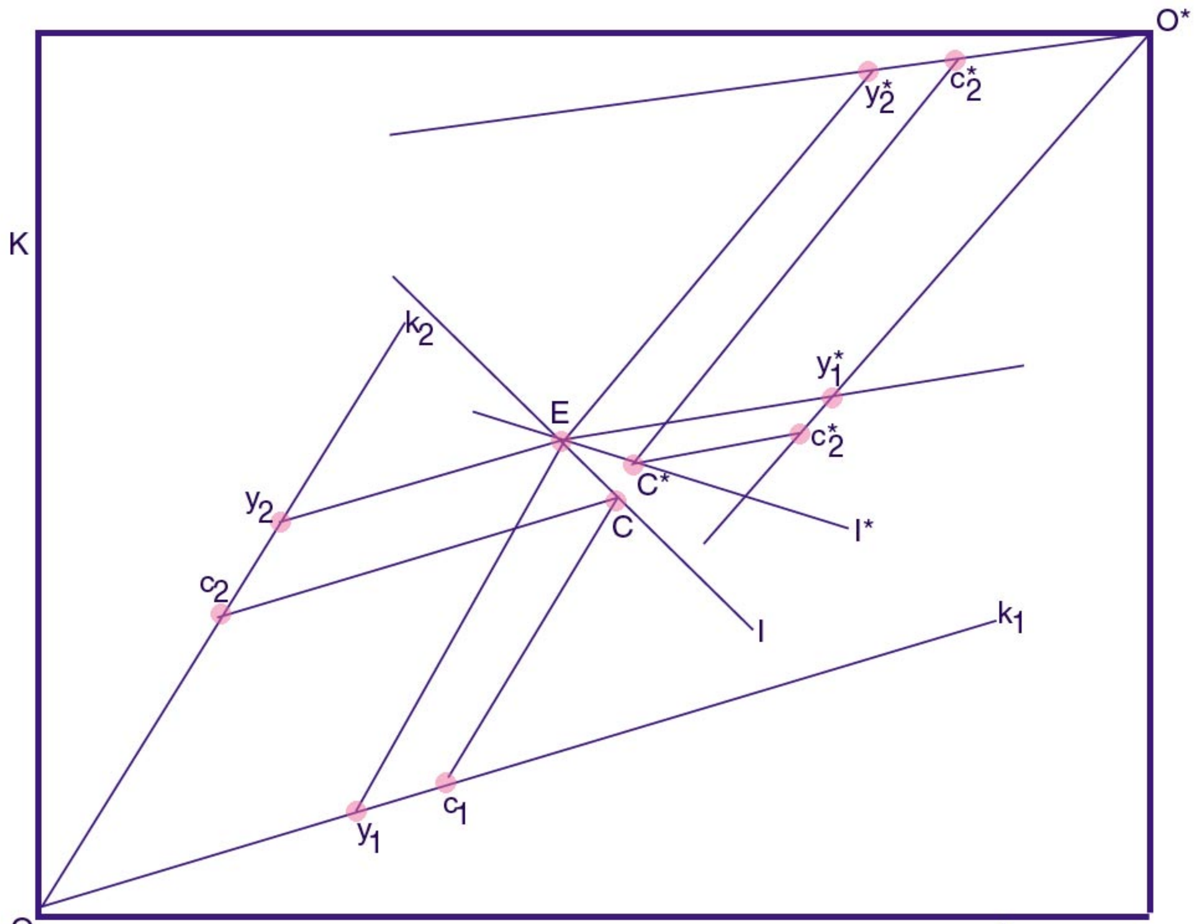

Figure 6. Factor content of trade without FPE.

Figure 6 illustrates that the sum of factors exported by both countries need not be zero when factor prices are not equalized. Points c_1 and c_2 show the amounts of capital and labor inputs embodied in consumption of goods 1 and 2, respectively. Point C shows the factor content of the home country's consumption bundle. Given an iso-income line, the movement from E to C shows the quantities of factor trade. However, in the foreign country factor prices are different and trade in goods causes a movement from E to C*. Thus, the sum of any factor exported by both countries can be either positive or negative.

When a factor intensity reversal occurs, however, it is possible for both countries to claim to have exported the same factor. This can be demonstrated even in the 2×2 case. Suppose the home country exports good 2 which uses capital intensively. Then the foreign country exports good 1, but since factor intensities are reversed, the foreign country also exports capital by exporting good 1. Thus, both countries may appear to be exporting capital and importing labor services via commodity trade.

While the HOV theorem is robust in the multi-commodity world, this result is disappointing because as Chipman (1988, p. 938) noted, it attempts to "replace the problem of explaining trade flows in actual commodities by that of explaining flows of abstract amounts of factors of production 'embodied' in the trade flows." The amounts of factors traded lose much of their significance when a factor intensity reversal occurs.

9. Concluding Remarks

In most empirical studies of the HO model the number of industries is much greater than that of factors. In this case, the output vector is indeterminate and exports of a capital abundant country need not be capital intensive, relative to its imports. Thus, it was an erroneous conjecture to presume that U.S. exports should have been more capital intensive than imports in 1947.

When the number of industries increases, the mean Rybzynski effects and the Stolper-Samuel results become negligible. Factor prices become much more stable than output prices; it is unlikely to observe the magnification effects of the Stolper-Samuelson Theorem in empirical studies. Thus, many of the findings of the HO model are peculiarities that arise from the low dimensionality of the 2 × 2 world. Moreover, wage rates reflect labor productivity in competitive markets. Thus, observed wage disparity among workers in various countries suggests different labor productivity and possibly different technologies.

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ENDNOTES

[1] Casas and Choi (1984, 1985) demonstrate that a Leontief Paradox could occur in the presence of a large trade imbalance. This is because in the presence of a large trade deficit (surplus) a country could import (export) some of the products which it would export (import) under balanced trade.

[2] Jones and Scheinkman (1977) criticized the existing work and investigated the Rybczynski and Stolper-Samuelson propositions in the $n \times m$ world, where the number of factors m exceeds the number of commodities n. The critical assumption in their model is that the number of factors is larger than that of commodities. Although this case is theoretically interesting, in most empirical studies n was much larger than m.

[3] It is interesting to note Deardorff's (1994) result. Using the weak axiom of revealed preference, he shows a negative correlation between trade vector and differences between autarky and free trade prices. That is, if the free trade price is higher than the autarky price, the industry tends to export that product. The fact that the free trade price is higher than the autarky price does not imply that the industry will export the product, because all other prices are also determinants of supply (through the Stolper-Samuelson Theorem) as well as of consumer demand.

[4] Harkness (1978) focused on the even case (n = m) and suggested a hypothesis:

$$(\text{diag } Y)^{-1} E = \Theta\beta + \varepsilon,$$

where diag Y is an (n Η n) matrix with the elements of Y on the main diagonal. However, he provides no theoretical basis for supposing that regression coefficients on factor intensities will duplicate the factor abundance ranking. Leamer and Bowen (1981) even provided a counter-example.

[5] If capital input were also a variable input, another constraint would be included in (6), and since all industries exhibit constant returns to scale, the output vector will be indeterminate if prices were equal to unit production costs.

[6] A solution to the first order conditions yields labor demand functions $L_i(K_1, K_2, K_3, p_1, p_2, p_3)$ and the shadow price $w(K_1, K_2, K_3, p_1, p_2, p_3)$.

[7] In the even case, although prices are equal to unit costs, industry outputs are uniquely determined by the Rybczynski result and industry supply curves are positively sloped. Indeterminacy makes industry supply curves horizontal in the uneven case.

[8] Thompson (1995) reports that eleven magnification effects can occur in the two-good, three-factor model, compared to only one in the 2×2 model. However, this abundance of magnification may be due to the fact that factor prices depend not only on output prices but also on endowments.

[9] Of course, if all prices rise by 1 percent, the wage rate will rise by the same proportion.

[10] See Baldwin (1971) for a number of possible explanations for the Leontief Paradox.

[11] If trade is not balanced, α can be replaced by β, the consumption share of the country.