Speculative Attacks and Financial Crisis: A Dynamic Analysis

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Abstract

This paper analyzes the relationship between speculative attacks and financial crises in a dynamic model in which consumers choose consumption and money demand to maximize their intertemporal welfare. Following Obstfeld (1984), we assume that the government has the option of allowing the exchange rate to float for a certain period of time and then repegging the exchange rate later. As in Calvo (1987), we introduce a cash-in-advance constraint for the consumers. This constraint provides the link between money demand, inflation, and government budget. We derive expressions of the dynamic adjustments of several crucial variables of the model such as the exchange rate, government foreign bonds holding, and domestic consumption.

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1 Introduction

The series of financial crises that occurred in Latin America, Asia, Europe and other places in the past several decades have ignited a large volume of literature on various issues about the causes and impacts of balance-of-payment crises. In particular, people are looking at different government policies that, sooner or later, trigger capital outflow and other financial problems in the economy.[1]

Salant and Henderson (1978), Krugman (1979), and Flood and Garber (1984) examine some government policies that are not consistent with a rigid exchange rate set by the government. They point out that because of the eventual run-out of the foreign reserves held by the central bank, the government will be forced to give up a fixed exchange rate regime. It is further argued that because of speculative attacks on the regime, devaluation of the domestic currency will occur well before the time for the foreign reserve to drop down to a low level along a path due directly to the government policies. They find that inflation, expansionary monetary or fiscal policy will trigger speculative attacks, leading to balance of payment crises (Obstfeld 1988a, 1988b; Edwards 1989; Agenor, Bhandari and Flood 1992). From a policy-making perspective, these results suggest that in a persistent disequilibrium with balance-of-payment deficit, it is almost impossible to regain equilibrium by pegging the exchange rate ultimately unless government decides to change its fiscal policy fundamentally such as "adjusting the government expenditure or their monetary policy (Helpman and Leidman 1987).

The roles of speculative attacks in fiancial cirses have been analyzed in more detail in Obstfeld (1986a, 1984b, 1994), Otker and Pazarbasioglu (1996, 1997), Della and Stockman (1993), and Flood and Marion (1996). In particular, they note that speculative attacks can occur even if government policies are sound and fundamentals of the economy are solid. That is because speculators, knowing that the government will give up the fixed exchange rate regime if the foreign reserves drop to a certain level, have the incentives to buy huge amounts of the central bank's reserves, betting that the domestic currency will be devalued in the near future. The quick change in the value of the domestic currency during a financial crisis represents a large capital gain for those who have just opportunistically converted domestic currency to foreign reserves.[2]

[1]See Saxena and Wong (1999) for a recent survey of some of these issues and models.

[2]If the government can successfully protect the initial exchange rate, the speculators can convert their foreign reserves back to domestic currency after paying minimal transaction fees. The asymmetry between the large capital gain should an attack is successful and the small transaction fees should an attack is not successful is one factor for the frequent

In the analysis in Krugman (1979), Flood and Garber (1984), Calvo (1987), it is assumed that during a financial crisis and a successful speculative attack, the government will choose to let the exchange rate float freely and forever. However, as Obstfeld (1984) argues, there are other options for the local government in the presence of speculative attacks. In particular, the government may choose to devalue the currency immediately, setting a new exchange rate level, or the government may switch to a flexible exchange rate regime for a certain length of time, and then fix the exchange rate at a new level. The option of a temporary flexible exchange rate regime is a sensible and reasonable choice, as allowing the exchange rate to float will lessen the pressure on the foreign reserve holding of the central bank. This approach is fruitful because it can cover the cases of immediate devaluation or permanent floating of exchange rate.

This paper analyzes the relationship between speculative attacks and financial crises in a dynamic model in which consumers choose consumption and money demand to maximize their intertemporal welfare. Following Obstfeld (1984), we assume that the government has the option of allowing the exchange rate to float for a certain period of time and then repegging the exchange rate later. As in Calvo (1987), we introduce a cash-in-advance constraint for the consumers. This constraint provides the link between money demand, inflation, and government budget. Our model has several important features that allows us to obtain more fruitful analysis and results.

1. We assume that the source of an unstable exchange rate system comes from the fiscal side of the government. In other words, it is an incompatible fiscal policy that leads to the eventual collapse of a fixed exchange rate regime.

2. Money has two roles in the present model. First, it is needed for transactions under the cash-in-advance constraint. Second, in the presence of domestic inflation, as in the case of devaluatoin of domestic currency, the consumers are paying an inflation tax. This creates an additional source of revenue to the government.

3. Unlike that in Obstfeld (1984), we assume that the government will revise the fiscal policy as the exchange rate is repegged after a financial crisis. This is needed to maintain a sustainable exchange rate policy.

4. We assume that the government budget is balanced when the exchange rate is allowed to float freely. That is not due to the change in the

speculative attacks in many of these countries.

fiscal expenditure of the government, but due to the new revenues that come from inflation tax and issuance of new money as consumers are demanding for more money as their consumption goes up. This put a constraint on the length of the transitional period: the flexible exchange rate regime. In other words, the length of the transitional period has to be determined endogenously and cannot be arbitrarily chosen by the government.

5. The consumers are maximizing their intertemporal welfare.

The rest of the present paper is organized as follows. Section 2 introduces the model used in this paper. In section 3, we explain how the speculators may react to the policy parameters chosen by the government. We will derive explicitly the adjustment of the exchange rate, the timing of speculative attacks, and other features of the model. Section 4 considers two special cases of devaluation: immediate devaluation and permanent floating of the exchange rate. Section 5 examines the adjustment of the exchange rate in various cases while section 6 analyzes the adjustment of consumption. Concluding remarks are given in section 7.

2 The Model

Consider an economy with one consumption good. Being small in international good market and financial market, it takes the international commodity price and international interest rate as given. Assuming free trade in good and perfect arbitrage, the purchasing power parity (PPP) is assumed to hold, i.e.,

$$p_t = p_t^* + x_t, \tag{1}$$

where p is the domestic price of the consumption good, p^* the foreign price of the consumption good, and x is the exchange rate, expressed as the foreign price of the domestic currency, all in a logarithmic form. We assume that the foreign price is constant over time. Normalizing the foreign price, we let $p_t^* = 0$ for all t, implying that (1) reduces to

$$p_t = x_t. \tag{2}$$

Condition (2) further implies that the domestic inflation rate is the rate of depreciation of domestic currency. Denote the world interest rate by r, which is constant over time.

There are two agents in the economy: an infinitely lived economic agent (consumer) and a domestic government. We describe their actions separately.

2.1 Consumption Maximization

The consumer maximizes his discounted intertemporal utility function, which is defined by

$$U = \int_0^\infty u(c_t)e^{-rt}\mathrm{d}t, \tag{3}$$

where c_t, which the agent chooses, is the consumption level at time t, $u(c_t)$ is the instantaneous utility function, and r is chosen as the discount rate.

Maximization of the agent's intertemporal utility is subject to the cash-in-advance constraint suggested by Lucas (1982), Stockman (1980) and others: For consuming c_t units of the consumption good, an individual has to own a certain amount of domestic currency in advance. So the domestic money balance at time t is assumed to satisfy:

$$m_t = \alpha c_t, \tag{4}$$

where $\alpha > 0$. As suggested by Lucas (1982), the interest rate must be positive when the cash-in-advance is binding. In the presence of domestic inflation, the value of domestic currnecy declines over time. Therefore $\dot{x}_t m_t$ is regarded as an inflation tax.

The agent holds foreign bonds of an amount b_t, which earns him an interest income rb_t. Therefore the wealth constraint of the agent is

$$w_t = m_t + b_t. \tag{5}$$

Wealth accumulates according to the following equation:

$$\dot{w}_t = rb_t + y_t + g_t - \dot{x}_t m_t - c_t, \tag{6}$$

where y_t is the flow of output produced at time t and g_t is a lump-sum transfer received from the government. As explained above, rb_t and $\dot{x}_t m_t$ are the foreign interest income and inflation tax, respectively.

Combining the above condition, the objective of the consumer is

$$\max_{\{c_t, m_t, b_t\}} \int_0^\infty u(c_t)e^{-rt}\mathrm{d}t, \tag{7}$$

subject to (4), (5), and (6).

Using (5), (6) reduces to

$$\dot{w}_t = rw_t + y_t + g_t - (r + \dot{x}_t)m_t - c_t, \tag{8}$$

Multiplying both sides of (8) by e^{-rt} and integrating, using the transversality condition

$$\lim_{t\to\infty} w_t e^{-rt} = 0, \tag{9}$$

the overall budget constraint reduces to

$$\int_0^\infty [(r+\dot{x}_t)m_t + c_t]\, e^{-rt}\mathrm{d}t = w_0 + \int_0^\infty (y_t + g_t)e^{-rt}\mathrm{d}t, \tag{10}$$

where w_0 is the initial holding of wealth. Condition (10) means that the sum of discounted values of expenditures (left-hand side) is equal to the sum of discounted values of wealth (right-hand side).

2.2 Government and Its Fiscal Policy

The domestic government holds foreign real bond b_t^g and issues non-interest bearing money m_t, which satisfies the cash-in-advance constraint given by (4). Its net wealth at time t is

$$w_t^g = b_t^g - m_t. \tag{11}$$

This wealth increases due to the interest income from holding foreign bonds, the revenue of inflation tax, but net of lump-sum transfer g_t,

$$\dot{w}_t^g = rb_t^g + \dot{x}_t m_t - g_t. \tag{12}$$

Substitute (11) into (12) to yield

$$\dot{w}_t^g = rw_t^g + (r+\dot{x}_t)m_t - g_t. \tag{13}$$

Multiplying both sides of (13) by e^{-rt} and impose the transversality condition

$$\lim_{t\to\infty} w_t^g e^{-rt} = 0. \tag{14}$$

We then integrate both sides of (13) from 0 to ∞ to get the overall government constraint

$$\int_0^\infty (\dot{w}_t^g - rw_t^g)\, e^{-rt}\mathrm{d}t = \int_0^\infty [(r+\dot{x}_t)m_t - g_t]\, e^{-rt}\mathrm{d}t, \tag{15}$$

which can be rewritten as

$$w_0^g + \int_0^\infty [(r+\dot{x}_t)m_t]\, e^{-rt}\mathrm{d}t = \int_0^\infty g_t e^{-rt}\mathrm{d}t. \tag{16}$$

This condition implies that intertemporally the government expenditure is equal to its wealth.

The total wealth of the whole economy at time t is the sum of that of the consumer and that of the government, and can be obtained from (5) and (11):

$$W_t \equiv w_t + w_t^g = b_t + b_t^g, \tag{17}$$

which accumulates over time according to

$$\begin{aligned}\dot{W}_t &\equiv \dot{w}_t + \dot{w}_t^g \\ &= rw_t + y_t + g_t - (r + \dot{x}_t)m_t - c_t + rw_t + (r + \dot{x}_t)m_t - g_t \\ &= rw_t + y_t - c_t. \end{aligned} \tag{18}$$

Multiplying both sides of (18) by e^{-rt} and integrating, we have

$$W_0 + \int_0^\infty y_t e^{-rt} \mathrm{d}t = \int_0^\infty c_t e^{-rt} \mathrm{d}t, \tag{19}$$

which states that the initial wealth plus the sum of discounted income are spent on the life-time consumption.

2.3 Maximizing the Consumer's Utility

The government initially sets the exchange rate at x_0. For some reasons to be explained later, the government allows the exchange rate to float freely between the period from T to $T + \tau$, where $\infty > T > 0$ and $\tau \in [0, \infty]$. For $t > T + \tau$, the exchange rate is again fixed at $x_1 > x_0$. The consumer takes the two exchange rates and T and τ as given. That T is positive and finite means that the consumer knows that the domestic currency will devalue in the future, and that the exchange rate will then be fixed. In the special case in which $\tau = 0$, the exchange rate jumps immediately from one level to another. If $\tau > 0$, then the exchange rate will be flexible for some time. If $\tau = \infty$, then the exchange rate is allowed to float freely at $t = T$ and remains flexible forever.

The maximization problem of the consumer can be written as (7) subject to (8). The current value Hamiltonian of the problem is

$$H = u(c_t)e^{-rt} + \lambda_t e^{-rt}[rw_t + y_t + g_t - (r + \dot{x}_t)m_t - c_t], \tag{20}$$

where λ_t is the Lagrange multiplier, or the marginal utility of wealth. With the cash-in-advance constraint (4), the optimal c_t satisfies the first-order condition

$$u'(c_t) = \lambda_t[1 + \alpha(r + \dot{x}_t)], \tag{21}$$

and the following equation

$$\dot{\lambda}_t - r\lambda_t = -\frac{\partial H}{\partial w} = -r\lambda_t. \tag{22}$$

The right-hand side of (21) is the marginal cost of consumption, which includes the cost of inflation tax and the foreign interest incurred by holding the cash in balance as required by the cash-in-advance constraint. The optimality condition (21) requires that the marginal utility of consumption equals the marginal cost of consumption. This condition further implies that at the optimal point $\dot{\lambda}_t = 0$. In other words, to maximize the intertemporal utility, λ_t has to be set to be constant at all times, $\lambda_t = \lambda$ for all t.

Since there are two switches in the exchange rate regimes, first at $t = T$ from fixed to flexible and then at $t = T + \tau$ from flexible to fixed, the first-order condition of the consumer's maximization problem is

$$u'(c_t) = \begin{cases} \lambda(1 + \alpha r) & 0 \leq t < T \\ \lambda[1 + \alpha(r + \dot{x}_t)] & T \leq t < T + \tau. \end{cases} \tag{23}$$

Note that in the first exchange rate regime, $t \in [0, T)$, the exchange rate is fixed so that $\dot{x}_t = 0$. By condition (23), the marginal utility and thus the consumption of good, c_t, are constant. Condition (23) can be written in an alternative form:

$$\lambda = \begin{cases} \dfrac{u'(c_t)}{1 + \alpha r} & 0 \leq t < T \\ \dfrac{u'(c_t)}{1 + \alpha(r + \dot{x}_t)} & T \leq t < T + \tau. \end{cases} \tag{24}$$

Combining (23), (8), and the cash-in-advance constraint, the changes in wealth of the consumer in these two periods are equal to

$$\dot{w}_t = \begin{cases} rw_t + y_t + g_t - (1 + \alpha r)c_t & 0 \leq t < T \\ rw_t + y_t + g_t - [1 + \alpha(r + \dot{x}_t)c_t] & T \leq t < T + \tau. \end{cases} \tag{25}$$

3 The Timing of Speculative Attacks

In this section, we are more explicit about the speculative attacks and when a finacial crisis occurs. As mentioned earlier, the government gives a lump-sum transfer to the consumer equal to g_t, while its wealth changes according to (12).

3.1 The Deterioration of Foreign Bonds Holding

Differentiating both sides of (11) and rearranging terms, we get the change in the foreign bond held by the government:

$$\dot{b}_t^g = rb_t^g + \dot{x}_t m_t + \dot{m}_t - g_t. \tag{26}$$

Let us focus on the first exchange rate regime, i.e., for $t \in [0, T)$. In this regime, the exchange rate is fixed so that $\dot{x}_t = 0$. By (23), consumption by the consumer is constant. By the cash-in-advance constraint, the demand for money is constant, meaning that $\dot{m}_t = 0$. Thus under this regime, (26) reduces to

$$\dot{b}_t^g = r b_t^g - g_t, \tag{27}$$

which means that the change in the foreign bond holding of the government depends on the interest it earns and the transfer it gives to the consumer. For the purpose of this paper, let us assume that the transfer is constant over time initially until sometime later, and is large enough so that

$$g_t = g > r b_0^g. \tag{28}$$

In other words, the transfer is initially greater than the foreign bond interest earning. By (27), the amount of foreign bond held by the government declines. This lowers the government's interest earning, leading to further decline in its foreign bond holding. Substitute equation (28) into (27) to give a differential equation for the government foreign bonds holding:

$$\dot{b}_t^g = r b_t^g - g,$$

which can be solved for an explicit adjustment equation for b_t^g when the exchange rate and the government transfer are constant:

$$b_t^g = -\frac{g}{r} + \left(b_0^g + \frac{g}{r} \right) e^{rt}. \tag{29}$$

From (29), foreign bonds holding is declining. It is obvious that the above fiscal policy cannot continue forever. Either there is a change in its fiscal policy or a change in the exchange rate regime. We assume that its fiscal policy is rigid and cannot be changed in the short run. The government chooses to change the exchange rate regime at a time $t = T$. For the period from $t = T$ or $t = T + \tau$, the exchange rate is allowed to float freely. In this period, depreciation of the exchange rate leads to domestic inflation, contributing an inflation tax to the government. Furthermore, as private consumption is rising, so is money demand. The issuance of money gives the government extra revenue. This helps balance the government budget. In the period beyond $T + \tau$, the exchange rate is fixed again at $x_1 > x_0$. In the first period when the exchange rate is fixed at x_0, the government is losing foreign bonds. However, beyond T, the holding of foreign bonds will be constant.

We assume that the consumer has perfect information about the foreign bond holding of the government and the exchange rate and fiscal policies so that there are no surprises.

How is T determined? We assume that the government will give up its fixed exchange rate regime when its foreign bond holding drops down to a pre-chosen level $\tilde{b}^g$. When the exchange rate is flexible, the government is free from bond depletion pressure, and its holding of foreign bonds is constant at $\tilde{b}^g$. Equation (26) implies that

$$\dot{m}_t + \dot{x}_t m_t = g - r\tilde{b}^g. \tag{30}$$

We also assume that the government maintains the same level of government transfer from $t = 0$ to $t = T + \tau$. When $t > T + \tau$, the transfer level drops down to $g_{1.}$ In order to maintain a substainable foreign exchange rate policy, we assume that the transfer is set so that

$$g_1 = r\tilde{b}^g. \tag{31}$$

The changes in government transfer and the foreign bonds holding are illustrated in Figure 1. More discussion about the foreign bonds holding path will be given below.

3.2 The Timing of Devaluation

To determine the precise timing of devaluation, we make an assumption about the functional form of the consumer's instantaneous utility function, namely, $u(c_t) = \ln c_t$. Therefore

$$u'(c_t) = \frac{1}{c_t}. \tag{32}$$

Using (32) and (24), the change in the consumer's wealth is equal to

$$\dot{w}_t = rw_t + y_t + g_t - \frac{1}{\lambda}, \tag{33}$$

for $t \in [0, T + \tau]$. Condition (33) is applicable in the first two exchange rate regimes: with x fixed at x_0 and x made flexible. Multiply both sides of (33) by e^{-rt}, impose the transversality condition, $\lim_{t\to\infty} w_t e^{-rt} = 0$, and integrate the resulting condition to get

$$\int_0^\infty (\dot{w}_t - rw_t)e^{-rt}\mathrm{d}t = \int_0^\infty (y_t + g_t - \frac{1}{\lambda})e^{-rt}\mathrm{d}t. \tag{34}$$

In the special case in which both y_t and g_t are constant, condition (34) yields

$$y + g + rw_0 = \frac{1}{\lambda}, \tag{35}$$

which is substituted back into (23) to give

$$c_t = \begin{cases} \dfrac{y + g + rw_0}{1 + \alpha r} & 0 \leq t < T \\ \dfrac{y + g + rw_0}{1 + \alpha(r + \dot{x}_t)} & T \leq t < T + \tau. \end{cases} \tag{36}$$

As explained earlier, the government will abandon the fixed exchange rate regime when the foreign bond holding drops down to $\tilde{b}^g$. The consumer is aware of this exchange rate policy. During the period with a flexible exchange rate, $t \in [T, T + \tau)$, condition (30) and the cash-in-advance constraint can be combined to give

$$\alpha \dot{c}_t + \alpha c_t \dot{x}_t = g - r\tilde{b}^g. \tag{37}$$

Differentiate the second part of condition (36). Using this result, we can rewrite condition (37) as a second-order differential equation of x:

$$\frac{\alpha \dot{x}_t}{\lambda[1 + \alpha(r + \dot{x}_t)]} - \frac{\alpha \ddot{x}_t}{\lambda[1 + \alpha(r + \dot{x}_t)]^2} = g - r\tilde{b}^g, \tag{38}$$

which can be rearranged to give

$$\frac{\alpha^2 \ddot{x}_t}{1 + \alpha(r + \dot{x}_t)} - \alpha \dot{x}_t + \lambda(g - r\tilde{b}^g)[1 + \alpha(r + \dot{x}_t)] = 0. \tag{39}$$

The boundary conditions for x are

$$\begin{cases} x_T = x_0 \\ x_{T+\tau} = x_1 \end{cases} \tag{40}$$

Using the boundary conditions, equation (38) can be solved for the exchange rate dynamic path[3]

$$x_t = x_0 + \theta(T - t) + \frac{1}{\beta} \ln \left\{ 1 + \beta \left[1 + \frac{1}{\theta} \dot{x}(T) \right] \left(1 - e^{\theta(T-t)} \right) \right\}, \tag{41}$$

where $\beta = \lambda(g - r\tilde{b}^g) - 1$ and $\theta = (1 + \alpha r)/\alpha > 0$. Both β and θ are constants, and it is shown in the appendix that $\beta < 0$. The value of $\dot{x}(T)$ can be expressed as

$$\dot{x}(T) = \theta \frac{e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1 - \beta(1 - e^{-\theta\tau})}{\beta(1 - e^{-\theta\tau})}, \tag{42}$$

which is a function of τ. Using (41), we have[4]

[3]See the Appendix.

[4]See the appendix.

$$\dot{x}_t = -\theta + \frac{\phi\theta e^{\theta(T-t)}}{1 + \phi\beta(1 - e^{\theta(T-t)})}, \tag{43}$$

where $\phi = 1 + \dot{x}(T)/\theta$. Setting $t = T + \tau$, condition (41) reduces to

$$x_1 = x_0 - \theta\tau + \frac{1}{\beta} \ln \left\{ 1 + \beta \left[1 + \frac{\dot{x}(T)}{\theta} \right] \left(1 - e^{-\theta\tau} \right) \right\}. \tag{44}$$

Condition (44) gives the relationship between the repegged, higher exchange rate and the duration of the flexible exchange rate regime. Recall that x_1 is a government parameter. There is only one endogenous variable in (**??**), which thus represents an implicit function of τ in terms of some parameters. As shown in the appendix, this relationship depends on the value of $\beta + 1$, which in turn depends on the fiscal policy adjustment. Let us consider the following two cases:

Case (a): $\beta + 1 < 0$. This is equivalent to $g < r\tilde{b}^g$. Then τ is an increasing function of x_1, i.e., the higher the new target fixed exchange rate, the longer transitional period for a flexible exchange rate regime is needed. (See the appendix.) Since the government successfully reduces its spending, inflation should become milder and easier to control.

Case (b): $\beta + 1 > 0$. This is equivalent to $g > r\tilde{b}^g$. In this case, the dependence of τ on x_1 is ambiguous.

We now turn to the determination of T as a function of g, $\tilde{b}^g$, and τ. The question is, when will speculative attacks occur?

To answer this question, we have to give more details about speculative attacks. When individuals attack the exchange rate regime, they swap domestic money they hold for foreign bonds of the government. This causes a drop in the government's foreign bond holding. The severity of an attack is often measured by the rate of decrease of the government's foreign bond/reserve holding. As the government is losing foreign bonds, the attackers are losing domestic money. Let m_{T^-} and m_{T^+} be the attackers' cash holdings just before and after time T, respectively, $m_{T^-} \geq m_{T^+}$. They are related to each other by

$$m_{T^-} - m_{T^+} = b^g_{T^-} - \tilde{b}^g, \tag{45}$$

where $b^g_{T^-} \geq \tilde{b}^g$ is the government's foreign bonds holding just before the attacks at $t = T$. Recall condition (27), which is valid in the period $t \in [T, T + \tau)$. Multiply both sides of (27) by e^{-rt} and integrate it from 0 to T^-.

We have

$$\int_0^{T^-} \dot{b}_t^g e^{-rt}\mathrm{d}t = \int_0^{T^-} (r\tilde{b}^g - g)e^{-rt}\mathrm{d}t.$$

Integrating by parts, we obtain,

$$b_{T^-}^g = b_0^g e^{rT^-} + \frac{g(1-e^{rT^-})}{r}, \tag{46}$$

where b_0^g is the initial foreign bonds holding of the government. Because e^{-rt} is a continuous function, $e^{rT^-} = e^{rT}$. Using the cash-in-advance constraint as given by (4), equation (45) and equation (46), we have

$$m_{T^-} - m_{T^+} = \alpha(c_{T^-} - c_{T^+}) = \alpha^2 \frac{\dot{x}(T)}{\lambda(1+\alpha r)[1+\alpha(r+\dot{x}(T))]}. \tag{47}$$

Equations (45) and (47) are combined together to solve for the inflation rate at $t = T$, i.e.,

$$\dot{x}(T) = \frac{\eta\lambda\theta^2}{r - \eta\lambda\theta}, \tag{48}$$

where θ is defined before and η is defined as

$$\eta = (rb_0 - g)e^{rT} - (r\tilde{b}^g - g). \tag{49}$$

Combining (42) and (49), we get an explicit expression of η:

$$\eta = \frac{[e^{\beta(x_1-x_0)+\theta\beta\tau} - 1 - \beta + \beta e^{-\theta\tau}]r}{\lambda\theta[e^{\beta(x_1-x_0)+\theta\beta\tau} - 1]}. \tag{50}$$

Using (50), (48) can be solved for the timing of the speculative attack:

$$T(\tau; g, b_0^g, x_1 - x_0) = \frac{1}{r}\ln\left[\frac{\lambda\theta(g - r\tilde{b}^g) + r\left[\frac{(1-e^{-r\theta\tau})\beta}{e^{\beta(x_1-x_0)+\theta\beta\tau} - 1} - 1\right]}{\lambda\theta(g - r\tilde{b}^g)}\right]. \tag{51}$$

Condition (51) shows that the timing of a financial crisis is a function of government expenditure, the initial and the minimum foreign bonds holdings of the government, the two fixed exchange rates, and the length of the transitional period. As explained earlier, the length of the transitional period is in turn a function of the higher fixed exchange rate, x_1.

As Krugman (1979), Flood and Garber (1984), and Obstfeld (1984) argue, in a perfect foresight model like the present one, given enough of domestic resources owned by speculators, there will not be any expected jump in the exchange rate when the government gives up the fixed exchange rate regime and the exchange rate is allowed to float.[5]

As proved in the appendix, the timing of devaluation has the following properties:

(1) $\partial T/\partial g < 0$, i.e., the larger the government expenditure is, the earlier the crisis will occur.

(2) $\partial T/\partial b_0^g > 0$ and $\partial T/\partial \tilde{b}^g < 0$, i.e., the larger the initial foreign bonds holdings the government has or the lower the reserve limit is set, the later the crisis will occur.

(3) $\partial T/\partial(x_1 - x_0) < 0$, i.e., the greater amount of devaluation the government sets, the earlier the crisis will occur.

4 Two Special Cases of Devaluation

Before we derive the adjustment path of the exchange rate, it is useful to use the present model to analyze two special cases, both of which have received much attention in the literature: the case of a flexible exchange rate permanently after time T and the case of an immediate and discrete devaluation at time T.[6] In the first case, which is equivalent to the present model with τ set to infinity, the government allows the exchange rate to float freely after time T. In the latter case, with τ set to be zero, the government immediately sets the exchange rate at a higher level at time T. We examine these two cases first before we show τ is determined in the present model. In both cases, the consumer is well aware of the policy goal of the government. The timing of the speculative attacks and the financial crisis depends on the length of flexible exchange rate regime.

[5]If individuals expect that there is a discrete jump in the exchange rate at time T, they can sell domestic currency for foreign currency just before T, and buy back domestic currency right after the devaluation. This will give them huge profit rate. However, if such selling and buying is profitable, every individual will try to do it slightly earlier than all others. As a result, every individual will start selling domestic currency as long as it is profitable to do so, thus putting pressure on the exchange rate and preventing any sudden movement of the exchange rate.

[6]This section temporarily treats τ as a parameter and examines how the time of speculative attacks may depend on the length of the transitional period. However, it should be noted that contrary to what is in Obstfeld (1984), the length of the transitional period in the present model is determined endogenously in order to maintain a balanced government budget in this period. Alternatively, we can imagine that the repegged exchange rate (and possibly some other parameters as well) is chosen at an appropriate level so that we get the values of τ considered in this section.

4.1 Permanent Floating of Exchange Rate after the Crisis

In the present case, setting $\tau \to \infty$, equation (51) reduces to

$$\lim_{\tau\to\infty} T(\tau) = \frac{\ln(r\tilde{b}^g - g) - \ln(1+\alpha r)(rb_0^g - g)}{r} > 0. \tag{52}$$

Condition (52) implies that even if the government declares that the exchange rate will be left to float after the crisis, a crisis will still occur. We can further derive more properties of this case. First, we note that there exists τ' so that $T(\tau') = \lim_{\tau\to\infty} T(\tau)$, i.e.,

$$T(\tau') = \frac{\ln(r\tilde{b}^g - g) - \ln(1+\alpha r)(rb_0^g - g)}{r}. \tag{53}$$

Condition (53) can be solved for τ':

$$\tau' = \frac{\alpha[1+\lambda(r\tilde{b}^g - g)](x_1 - x_0)}{\lambda(r\tilde{b}^g - g)(1+r\alpha)}. \tag{54}$$

It is easy to check that $T(\tau)$ is monotonically increasing around τ'. There also exists a unique $\tau^* > \tau'$ so that $T(\tau^*) \geq T(\tau)$ for all $\tau \geq 0$. In other words, $T(\tau^*)$ is the maximum value of T and $\mathrm{d}T/\mathrm{d}\tau = 0$ at $\tau = \tau^*$. Let $T^* = T(\tau^*)$.

We summarize the above results as follows.

(a) Even if the government declares that the exchange rate will be left to float freely and permanently should the foreign bonds holding drop down to $\tilde{b}^g$, speculative attacks will occur at $t = T(\tau')$.

(b) The timing of speculative attacks and a financial crisis is the same as if the government declares that the exchange rate is allowed to float freely for a period of τ'.

(c) For all $\tau \in (\tau', \infty)$, $T(\tau) > T(\tau')$.

(d) There exists τ^* so that $T(\tau^*) \geq T(\tau)$ for all $\tau \geq 0$.

4.2 Instant Devaluation

We now turn to the case in which the government devalues the exchange rate immediately from x_0 to x_1 at $t = T$. The timing of devaluation is again given by (51). This condition shows that $T(\tau)$ does not exist when $\tau = 0$. The reason is that if devaluation occurs at $t = 0$, there is not any time for any speculative attacks. However, without any speculative attacks, the exchange rate is save and thus there is no need to devalue the domestic currency.

We can, however, evaluate $T(\tau)$ when τ is sufficiently small:

$$\lim_{\tau \to 0} T(\tau) = \frac{1}{r} \ln \left[\frac{\lambda\theta(g - r\tilde{b}^g) - r}{\lambda\theta(g - r\tilde{b}^g)} \right]. \tag{55}$$

As shown in the appendix, the expression in condition (55) is greater (or less) than zero if and only if $\tilde{b}^g - b_0^g >$ (or $<$) m_1, where m_1 is the real cash holding of the consumer in the fixed exchange rate regime x_0.[7] Let us analyze these two cases separately.

(A): $m_1 > \tilde{b}^g - b_0^g$. In this case, $T(\tau)$ is negative when τ is sufficiently small. In other words, speculative attacks will occur instantaneously at $t = 0$. This will occur because the speculators see the profit opportunity by buying foreign bonds from the government just before devaluation on the one hand, and on the other hand, the speculators have enough of domestic resources to purchase sufficiently foreign bonds from the government to drop b^g down to the critical level $\tilde{b}^g$, forcing the government to devalue the domestic currency immediately. The dependence of T on τ in this case is shown in Figure 2. Let us define τ_0 where $T(\tau_0) = 0$. For $\tau \leq \tau_0$, devaluation will occur instantaneously at $t = 0$. For $\tau > \tau_0$, devaluation will not occur instantaneously.

(B): $m_1 < \tilde{b}^g - b_0^g$. In this case, $T(\tau)$ is greater than zero from all positive values of τ. This means that at $t = 0$, the speculators do not have enough of domestic money to bring the government's foreign bonds holding down to $\tilde{b}^g$. As time goes on, the government is losing foreign bonds, and less enough domestic money will be needed for a successful speculative attack. The government will be forced to devalue as soon as the speculators have just enough domestic resources to lower b^g down to $\tilde{b}^g$. The relationship between T and τ in this case is shown in Figure 3, in which the vertical intercept represents the time of speculative attacks.

[7] As explained earlier, the consumption of the consumer is constant in the fixed exchange rate regime x_0. By the cash-in-advance constraint, the money demand in this regime is also constant.

5 Exchange Rate Adjustment

Recall equation (41), which represents the dynamic path of the exchange rate. This equation shows that $x(T) = x_0$, meaning that the boundary condition is satisfied at $t = T$. On the other hand,

$$x(T+\tau) = x_1 = x_0 - \theta\tau + \frac{1}{\beta}\ln\left[1 + \beta\left(1 + \frac{\dot{x}(T)}{\theta}\right)(1 - e^{-r\theta\tau})\right]. \tag{56}$$

This expression determines implicitly τ, the length of the flexible exchange rate regime. As shown in the appendix, if x_1 is treated as a function of τ, $\mathrm{d}x_1/\mathrm{d}\tau > 0$ under certain conditions. Apparently if these conditions are satisfied, the inverse is true, i.e., $\mathrm{d}\tau/\mathrm{d}x_1 > 0$. This means that the higher the new exchange rate the government sets when the exchange rate is fixed again, the longer the flexible exchange rate regime is. A policy implication is that the government can choose x_1, together with an appropriate adjustment of its expenditure, to minimize the length of the flexible exchange rate regime.

Differentiate both sides of (41) to yield the inflation rate:

$$\dot{x}_t = -\theta + \frac{\left(1 + \frac{\dot{x}(T)}{\theta}\right)\theta e^{\theta(T-t)}}{1 + \beta\left(1 + \frac{\dot{x}(T)}{\theta}\right)(1 - e^{\theta(T-t)})}. \tag{57}$$

Since $\dot{x}(T)$ is a function of τ, the inflation rate in (57) is a function of t as well as τ. At $t = T + \tau$, the inflation rate is equal to

$$\dot{x}(T+\tau) = -\theta + \frac{\left(1 + \frac{\dot{x}(T)}{\theta}\right)\theta e^{-\theta\tau}}{1 + \beta\left(1 + \frac{\dot{x}(T)}{\theta}\right)(1 - e^{-\theta\tau})}. \tag{58}$$

Substituting (42) into (58), $\dot{x}(T+\tau)$ can be expressed as a function of τ.

In order to find out the features of the adjustment path of the exchange rate, it is important to determine $\dot{x}_t$ at $t = T$ and $t = T + \tau$. Since T is a function of τ, choosing different values of τ will change the values of $\dot{x}_t$ at these two points. The appendix shows that at $T(\tau)$, for any value of τ, $\dot{x}(T) > 0$. Furthermore, at $\tau = \tau^*$, T achieves a maximum, and at this point the inflation rate is also at a maximum. The appendix also shows that the sign of $\dot{x}(T+\tau)$ is the same as that of $\mathrm{d}T/\mathrm{d}\tau$ at the same time t. This result has the following implications:

1. At $\tau = \tau^*$, $\mathrm{d}T/\mathrm{d}t = 0$ and $\dot{x}(T+\tau) = 0$. This means that x_t reaches x_1, its maximum value, at $t = T + \tau$.

2. For $\tau > \tau^*$, $\mathrm{d}T/\mathrm{d}t < 0$ and $\dot{x}(T+\tau) < 0$. This means that x_t drops down to x_1 at $t = T + \tau$.

3. For $\tau < \tau^*$, $\mathrm{d}T/\mathrm{d}t > 0$ and $\dot{x}(T+\tau) > 0$. This means that x_t rises up to x_1 at $t = T + \tau$.

4. More results can be obtained in the special case in which the government chooses to let the exchange rate float permanently after a financial crisis, i.e., τ approaches infinity. We have
$$\lim_{\tau \to \infty} \dot{x}(T(\tau)) = -\theta \frac{1+\beta}{\beta}. \tag{59}$$
Similarly,
$$\lim_{t \to \tau'} \dot{x}_t = \lim_{\tau \to \infty} \dot{x}(T(\tau)) = -\theta \frac{1+\beta}{\beta}. \tag{60}$$
It is easy to check that at $\tau' = \beta(x_0 - x_1)/[\theta(1+\beta)]$ the inflation rate is the same as $\lim_{\tau \to \infty} \dot{x}(T+\tau)$. Thus by either choosing the length of the flexible exchange rate period equal to τ' or abandoning the fixed exchange rate forever, the inflation rate is constant, meaning that the exchange rate changes linearly.

5. If $\tau > \tau'$, $\dot{x}(T) < -\theta(1+\beta)/\beta$ so that $\ddot{x}_t > 0$. In other words, the exchange rate adjustment path is concave.

6. If $\tau < \tau'$, $\dot{x}(T) > -\theta(1+\beta)/\beta$ so that $\ddot{x}_t < 0$. In other words, the exchange rate adjustment path is convex.

The exchange rate adjustment paths in various cases are illustrated in Figure 4.

6 Consumption Adjustment Path

We now derive the adjustment path of consumption, which can be used to shed more light on the dynamic path of $T(\tau)$. Using equation (36) and recalling that λ is a constant, we have
$$c_t = \begin{cases} \dfrac{1}{\lambda(1+\alpha r)} & 0 \le t < T \\ \\ \dfrac{1}{\lambda(1+\alpha(r+\dot{x}_t)} & T \le t < T+\tau \end{cases}.$$

Inflation is a function of τ: for any τ, there is a floating exchange rate path. As argued above, τ is either directly chosen by the government or indirectly affected by the chosen x_1. In the flexible exchange rate regime, consumption is a function of inflation, meaning that c_t is a function of τ.

When the exchange rate is fixed, the inflation and thus the inflation tax are zero. By (23), consumption is constant. Therefore different consumption adjustment paths as caused by different values of τ differ from each other only in the transitional period when the exchange rate is flexible. For precisely, consumption remains constant at c_0 in the first period when x_t is fixed at x_0. In the second period when the exchange rate is flexible, consumption is lower because of spending money on attacking the exchange rate, and because of the inflation tax and consumption may or may not be a constant. In the third period, consumption goes back up to c_0. Therefore our analysis will focus on the adjustment path of c_t while the exchange rate is flexible. Five different cases, depending on the value of τ, can be distinguished. These five cases are analyzed below, and are illustrated in the five panels of Figure 5.

(a) $\tau < \tau'$. Consumption decreases over time when the exchange rate is flexible.

(b) $\tau = \tau'$. Consumption is higher than the case with $\tau < \tau'$, and remains constant over time when the exchange rate is flexible.

(c) $\tau' < \tau < \tau^*$. Consumption increases over time when the exchange rate is flexible, but does not exceed its pre-crisis level until it reaches at the new exchange rate x_1.

(d) $\tau > \tau^*$. Consumption increases in the transitional period and exceeds its pre-crisis level c_0 before its drops back to c_0. That is because the inflation rate becomes negative in a short period before the exchange rate is repegged again.

(e) $\tau \to \infty$. In this case, the exchange rate is left flexible permanently. Because domestic currency depreciates at a constant rate, the inflation tax is constant, too. This causes consumption to drop to a low level and stay there permanently as the exchange rate is floating.

The above analysis shows that there exists a trade-off between consumption and foreign bonds holding. Specifically, the later a speculative attacks starts, the less the consumption level will be affected.

7 Concluding Remarks

In this model, we examine the relationship between speculative attacks, consumption maximization, and government fiscal and exchange rate policies. We obtained some results similar to what has been derived in the literature, but the present framework allows us to analyze the adjustment of the government fiscal policy after a financial crisis, the length of adjustment of the exchange rate before it can be repegged again, and the impacts of devaluation on the consumption of individuals.

One extension of the present model would be to examine how the new, higher exchange rate is chosen. To do that, a suitable objective function of the government has to be introduced. Such a function is absent in most of the papers in the literature, including the present one. Without such a function, the policy parameters assumed are arbitrary.

Another direction of extension is to analyze a seemingly inconsistency between what is usually assumed in the literature and what is usually observed in countries that experience financial crises. In these papers, including the present one, the exchange rate cannot jump. That is because people have perfect foresight, and if a jump in exchange rate is expected, they will act accordingly to capture capital gain. In the real world, it is often the case that after a financial crisis the exchange rate jumps rapidly to a higher level within a very short period of time. How to introduce a model to capture this phenomenon is an interesting topic of research.

Appendix

This appendix proves some of the results mentioned in the paper.

1. To determine the Dynamic Path of Inflation

From equation (39) we have the second-order differential equation:

$$\alpha^2 \frac{\ddot{x}_t}{1+\alpha(r+\dot{x}_t)} - \alpha\dot{x}_t + (g - r\tilde{b}^g)[\lambda(1+\alpha(r+\dot{x}_t)] = 0.$$

It can be rewritten as:

$$\alpha \frac{\mathrm{d}}{\mathrm{d}t} \ln\left[\frac{1}{1+\alpha(r+\dot{x}_t)}\right] + \alpha\dot{x}_t = \lambda(g - r\tilde{b}^g)[1+\alpha(r+\dot{x}_t)]. \tag{61}$$

Integrating both sides of (61) from T to any t, we get

$$\alpha \int_T^t \left[\frac{\mathrm{d}}{\mathrm{d}t} \ln\left(\frac{1}{1+\alpha(r+\dot{x}_t)}\right) + \dot{x}_t\right] \mathrm{d}t = \lambda(g - r\tilde{b}^g) \int_T^t [1+\alpha(r+\dot{x}_t)]\mathrm{d}t. \tag{62}$$

Solving equation (62) to give

$$\ln\left[\frac{1+\alpha(r+\dot{x}(T))}{1+\alpha(r+\dot{x}_t)}\right] = \frac{\lambda(g - r\tilde{b}^g)(1+\alpha r)}{\alpha}(t-T)+[\lambda(g-r\tilde{b}^g)-1](x_t-x(T)).$$

Defining $\beta = \lambda(g - r\tilde{b}^g) - 1$ and $\theta = (1+\alpha r)/\alpha$, and taking exponential on both sides, we have

$$\ln\left[\frac{1+\alpha(r+\dot{x}(T))}{1+\alpha(r+\dot{x}_t)}\right] = \theta(\beta+1)(t-T) + \beta[x_t - x(T)],$$

and furthermore,

$$(\theta + \dot{x}(T)) = [e^{\theta(\beta+1)(t-T)}e^{\beta(x_t - x(T))}](\theta + \dot{x}_t). \tag{63}$$

Let $z_t = e^{\beta(x_t - x(T))}$. Condition (63) can be rewritten in terms of z:

$$\frac{1}{\beta}\dot{z} + \theta z = (\theta + \dot{x}(T))e^{-\theta(\beta+1)(t-T)}. \tag{64}$$

Multiplying both sides of (64) by $e^{\beta\theta t}$ and rearranging terms, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}e^{\theta\beta t}z = \beta(\theta + \dot{x}(T))e^{-\theta(\beta+1)T}e^{-\theta t},$$

which is solved to yield

$$ze^{\theta\beta t} = -\frac{\beta}{\theta}(\theta + \dot{x}(T))e^{(\beta+1)\theta T}e^{-\theta t} + C, \tag{65}$$

where C is a constant. Setting $t = T$ in condition (65) and using the fact that $z(T) = 1$, we have

$$C = \left[1 + \frac{\beta}{\theta}(\theta + \dot{x}(T))\right]e^{\theta\beta T}. \tag{66}$$

Substitute (66) into (65) to yield

$$z = \left[1 + \frac{\beta}{\theta}(\theta + \dot{x}(T))(1 - e^{\theta(T-t)})\right]e^{\theta(T-t)}. \tag{67}$$

The definition of z, which contains x_t, and condition (67) can be combined together to solve for the exchange rate dynamic path given by (41).Taking derivatives of both sides of (41) with respect to t, we get (43). We then set $t = T + \tau$ and use (40) to yield (42).

2. To Determine the Sign of $\mathrm{d}x_1/\mathrm{d}\tau$

Using equation (41) and setting t to be $T + \tau$, we get

$$x_1 = x_0 - \theta\tau + \frac{1}{\beta}\ln\left[1 + \beta b(1 + \beta(1 + \frac{1}{\theta}\dot{x}(T))(1 - e^{-\theta\tau})\right]. \tag{68}$$

Differentiate x_1 with respect to τ to give

$$\begin{aligned}\frac{\mathrm{d}x_1}{\mathrm{d}\tau} &= -\frac{\theta[1 + \sigma\beta - \sigma(\beta+1)e^{-\theta\tau}]}{1 + \beta[1 + (1 + \dot{x}(T)](1 - e^{-\theta\tau})/\theta} \\ &= -\frac{\theta[1 + \sigma\beta - \sigma\lambda e^{-\theta\tau}(g - r\tilde{b}^g)]}{1 + \beta[1 + (1 + \dot{x}(T)](1 - e^{-\theta\tau})/\theta},\end{aligned} \tag{69}$$

where $\sigma = 1 + \dot{x}(T)/\theta$.

We consider two cases. First, if after devaluing the domestic currency, the government reduces the spending so that $g < r\tilde{b}^g$, then $\beta + 1 < 0$. Because $e^{-\theta\tau} < 1$, we have $e^{-\theta\tau}(\beta + 1) > (\beta + 1)$ and

$$\begin{aligned}1 + \sigma\beta - \sigma e^{-\theta\tau}(\beta+1) &< 1 + \sigma\beta - \sigma(\beta+1) \\ &= 1 - \sigma \\ &= -\frac{\dot{x}(T)}{\theta},\end{aligned} \tag{70}$$

which implies $\mathrm{d}x_1/\mathrm{d}\tau > 0$.

Second, if $g > r\tilde{b}^g$, $\beta + 1 > 0$. Let the numerator of the left-hand side term of (69) be

$$f(\tau) = -\theta[1 + \sigma\beta - \sigma(\beta + 1)e^{-\theta\tau}],$$

which depends on τ. We also have

$$f'(\tau) = -\theta^2[\sigma e^{-\beta\tau}(\beta + 1)] < 0.$$

When $\tau \to 0$, $f(\tau)$ approaches $-\theta^2(1-\sigma) = \dot{x}(T) > 0$. When $\tau \to \infty$, $f(\tau)$ approaches $-\theta(1+\sigma\beta)$, which may be positive or negative, depending on the value of β.

3. To Determine the Derivatives of T

Taking exponential of both sides of (49) and defining $\hat{T} = e^{rT}$, we write (49) as

$$\hat{T} \equiv e^{rT} = \frac{r\tilde{b}^g - g}{rb_0^g - g} + \frac{r}{\lambda\theta(rb_0^g - g)}\left(1 - \frac{\beta(1 - e^{-\theta\tau})}{e^{\theta(x_1 - x_0) + \theta\beta\tau} - 1}\right). \tag{71}$$

Differentiate both sides of (71) with respect to g to give

$$\frac{\partial\hat{T}}{\partial g} = \frac{1}{rb_0^g - g}\left[\frac{r\tilde{b}^g - rb_0^g}{rb_0^g - g} + \frac{r}{\theta\lambda(rb_0^g - g)}\left(1 - \frac{\beta(1 - e^{-\theta\tau})}{e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1}\right)\right],$$

which can be rewritten as

$$\frac{\partial\hat{T}}{\partial g} = \frac{\dfrac{\lambda\theta(r\tilde{b}^g - g) + r\left(1 - \dfrac{\beta(1 - e^{-\theta\tau})}{e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1}\right)}{\lambda\theta(b_0^g - g)} - 1}{rb_0^g - g}, \tag{72}$$

where the term in the parenthesis of the numerators is identical to the term in the log function in (51). From (51), we know that in order for $T > 0$, the term in the bracket must be greater than 1. Thus the term in the numerator is positive. Noting that $rb_0^g - g < 0$, the expression in (72) is negative. Note also that $\mathrm{sign}(\partial\hat{T}/\partial g) = \mathrm{sign}(\partial T/\partial g)$. Thus we have $\partial T/\partial g < 0$.

Other partial derivatives of T can be obtained by partially differentiating (71):

$$\begin{aligned}
\frac{\partial\hat{T}}{\partial b_0^g} &= -\frac{r\hat{T}}{rb_0^g - g} > 0 \\
\frac{\partial\hat{T}}{\partial\tilde{b}^g} &= \frac{r}{rb_0^g - g} < 0 \\
\frac{\partial\hat{T}}{\partial(x_1 - x_0)} &= \frac{r\beta^2(1 - e^{-\theta\tau})e^{\beta(x_1 - x_0) + \theta\beta\tau}}{\lambda\theta(rb_0^g - g)(e^{\beta(x_1 - x_0) + \theta\beta\tau} - 1)} < 0.
\end{aligned}$$

4. To Determine $\dot{x}(T)$

Note that because T is a function of τ, so $\dot{x}(T)$ is also a function of τ. Conditions (44) and (57) give the exchange rate and inflation adjustment at $t = T$. Let us rewrite $\dot{x}(T)$ as

$$\dot{x}(T) = -\frac{\theta}{\beta(1-e^{\theta t})}\left[(1+\beta) - \beta e^{-\theta t} - e^{\beta(x_1-x_0)+\theta\beta t}\right].$$

Because for any $\tau > 0$, $-\theta/[\beta(1-e^{-\theta\tau})] > 0$, we define

$$h(\tau) = (1+\beta) - \beta e^{-\theta\tau} - e^{\beta(x_1-x_0)+\theta\beta\tau}.$$

Then $h(0) = -e^{\theta(x_1-x_0)} > 0$, $h(\infty) = 1+\beta > 0$, and $h'(\tau) = \theta\beta(e^{-\theta\tau} - e^{\beta(x_1-x_0)+\theta\beta\tau})$. Define $\hat{\tau}$ so that $h'(\hat{\tau}) = 0$. We then get

$$\hat{\tau} = -\frac{\beta(x_1-x_0)}{\theta(1+\beta)}.$$

Furthermore,

$$h''(\tau) = \beta\theta^2\lambda(g - r\tilde{b}^g)e^{\beta(x_1-x_0)/(1+\beta)} < 0.$$

Therefore $\hat{\tau}$ is an unique value of τ that gives a maximum $h(\tau)$. Also, $h(\hat{\tau}) > 0$. Thus $\dot{x}(T) > 0$ for any $T(\tau)$.

5. To Determine $\dot{x}(T+\tau)$

We first derive the relationship between $\dot{x}(T+\tau)$ and $\dot{x}(T)$:

$$\dot{x}(T+\tau) = -\theta + \frac{[1+\dot{x}(T)/\theta]\theta e^{-\theta\tau}}{\beta\{1+[1+(1+\dot{x}(T)/\theta)]\}(1-e^{-\theta\tau})}. \tag{73}$$

Substitute (42) into (73) and rearranging terms to yield

$$\dot{x}(T+\tau) = -\Theta\left[\frac{\theta}{\beta(1-e^{-\theta\tau})e^{\beta(x_1-x_0)+\theta\beta\tau}}\right], \tag{74}$$

where

$$\Theta = \left[\beta e^{\beta(x_1-x_0)+\theta\beta\tau} - (1+\beta)e^{\beta(x_1-x_0)+\theta\beta\tau-\theta\tau}\right].$$

The first term of the right-hand side of (74) is positive. Differentiate condition (49) with respect to τ and rearrange terms to give

$$\frac{\partial T}{\partial\tau} = \Omega\left[\beta e^{\beta(x_1-x_0)+\theta\beta\tau} - (1+\beta)e^{\beta(x_1-x_0)+(\beta-1)\theta\tau} + e^{-\theta\tau}\right],$$

where

$$\begin{aligned} \Omega &= \frac{r\beta\theta}{\Psi\lambda\theta(rb_0^g - g)(e^{\beta(x_1-x_0)+\theta\beta\tau} - 1)^2}, \\ \Psi &= \frac{\lambda\theta(g - r\tilde{b}^g) + r\left[\dfrac{(1-e^{-r\theta\tau})\beta}{e^{\beta(x_1-x_0)+\theta\beta\tau} - 1} - 1\right]}{\lambda\theta(g - r\tilde{b}^g)} \end{aligned}$$

We note $\Omega > 0$. Thus the sign of $\dot{x}(T+\tau)$ is the same as that of $\mathrm{d}T/\mathrm{d}\tau$.

6. To Determine the Curvature of the Adjustment Path of the Exchange Rate

Twice differentiate x_t with respect to time as given in (57). We have

$$\begin{aligned} \frac{\mathrm{d}^2 x_t}{\mathrm{d}t^2} &= \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\Pi\theta e^{\theta(T-t)}}{1+\Pi\beta(1-e^{\theta(T-t)})}\right] \\ &= -\frac{\Pi\theta^2 e^{\theta(T-t)}(1+\Pi\beta)}{[1+\Pi\beta(1-e^{\theta(T-t)})]^2}, \end{aligned}$$

where $\Pi = 1 + \dot{x}_t/\theta$ is independent of t. Therefore the sign of $\ddot{x}_t$ is opposite to the sign of $1+\Pi\beta$. In other words, if $1+\Pi\beta$ is positive, zero, or negative, then $\ddot{x}_t$ is negative, zero, or positive.

7. To Show that $\beta < 0$.

From the definition, $\beta = \lambda(g - r\tilde{b}^g) - 1$. Using (35), it reduces to

$$\begin{aligned} \beta &= \frac{g - r\tilde{b}^g}{y + g + rw_0} - 1 \\ &= -\frac{r\tilde{b}^g + y + rw_0}{y + g + rw_0} < 0. \end{aligned}$$

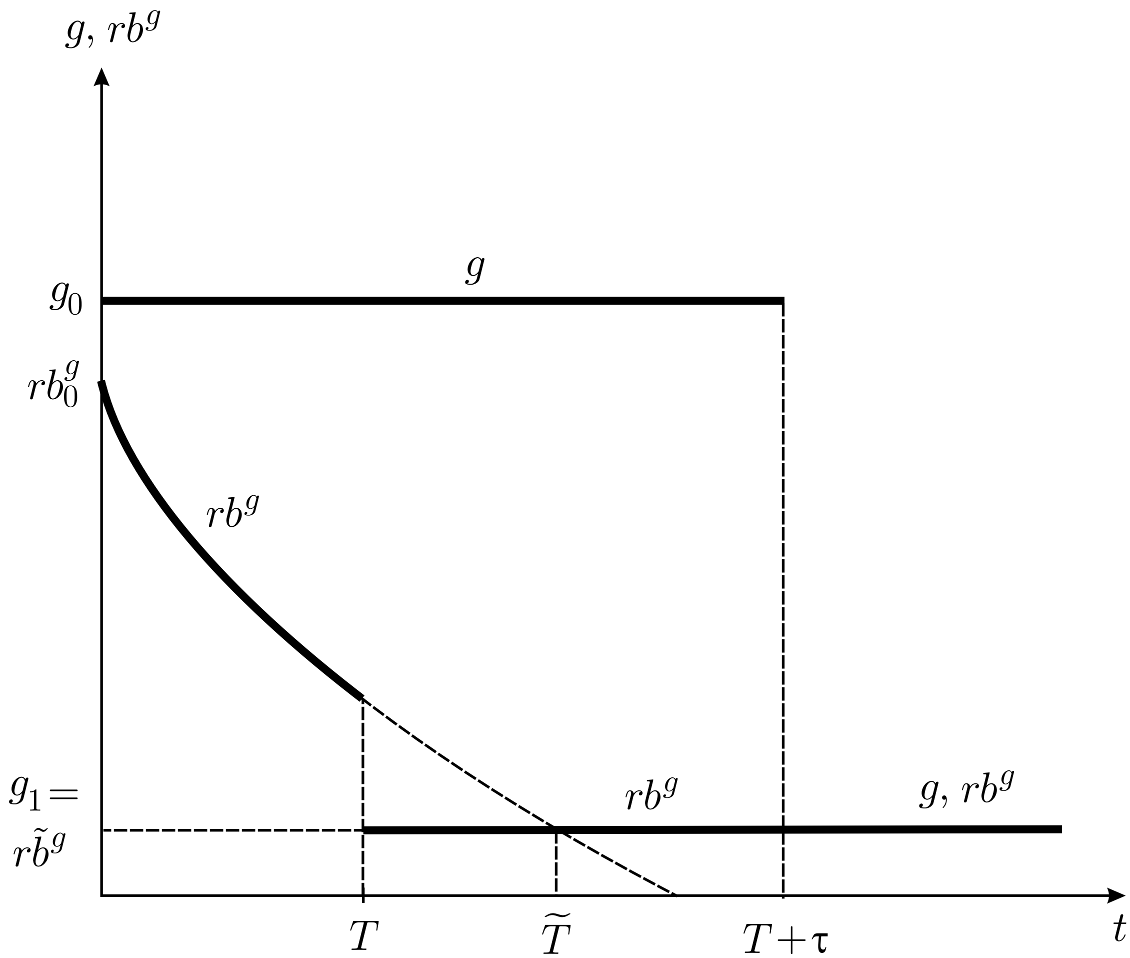


Figure 1

Government Expenditure and

Foreign Bonds Holding

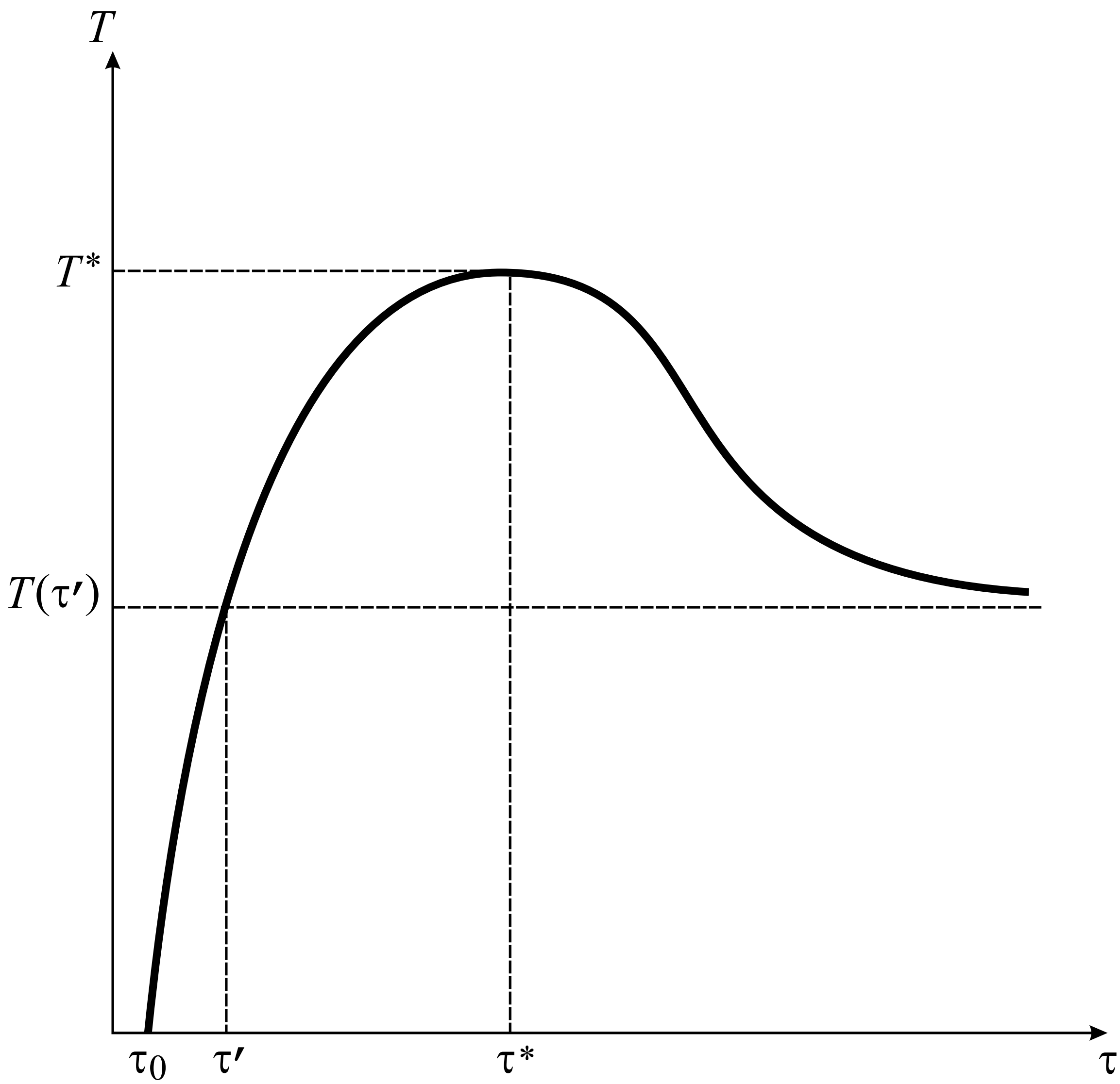


Figure 2

Timing of A Financial Crisis: Case (a)

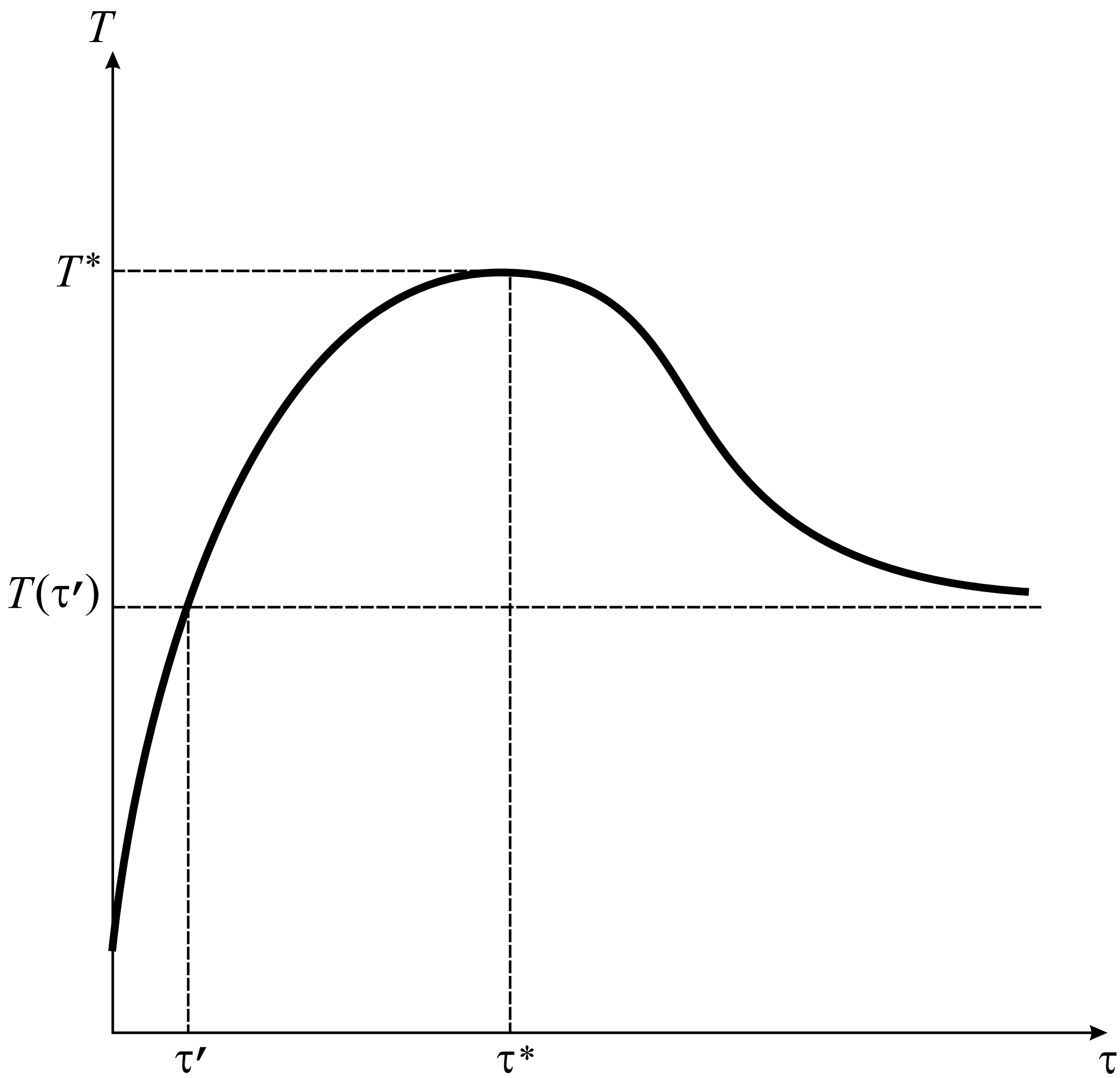
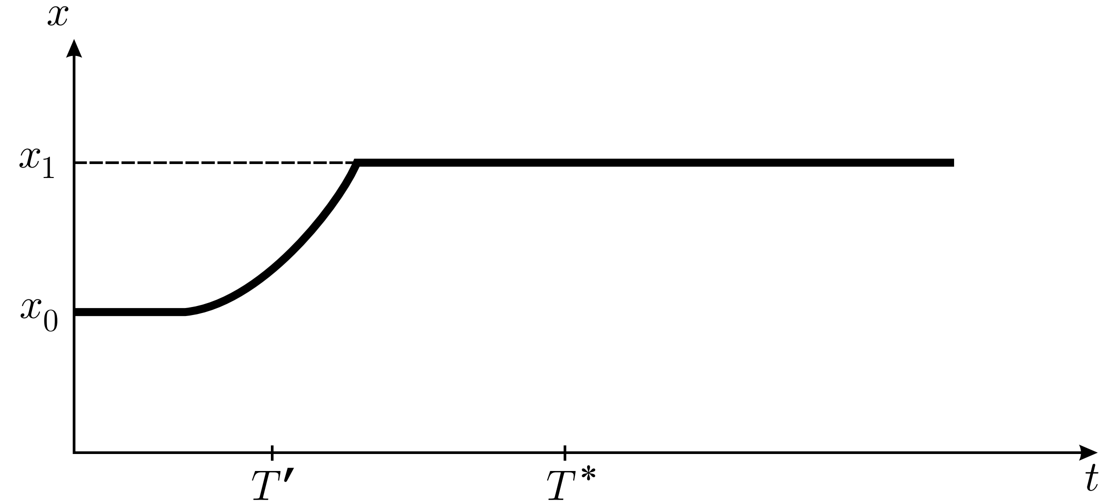


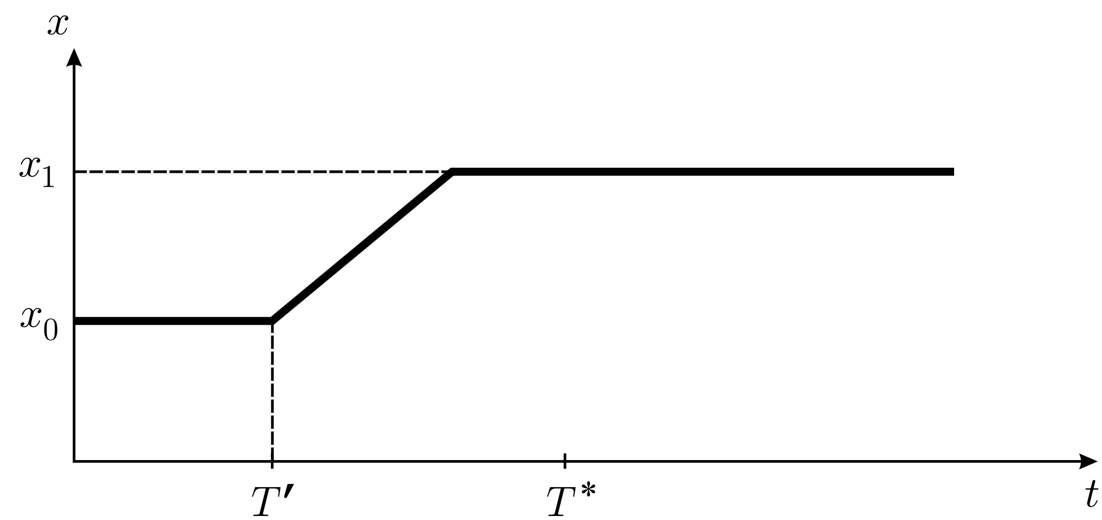
Figure 3

Timing of A Financial Crisis: Case (b)

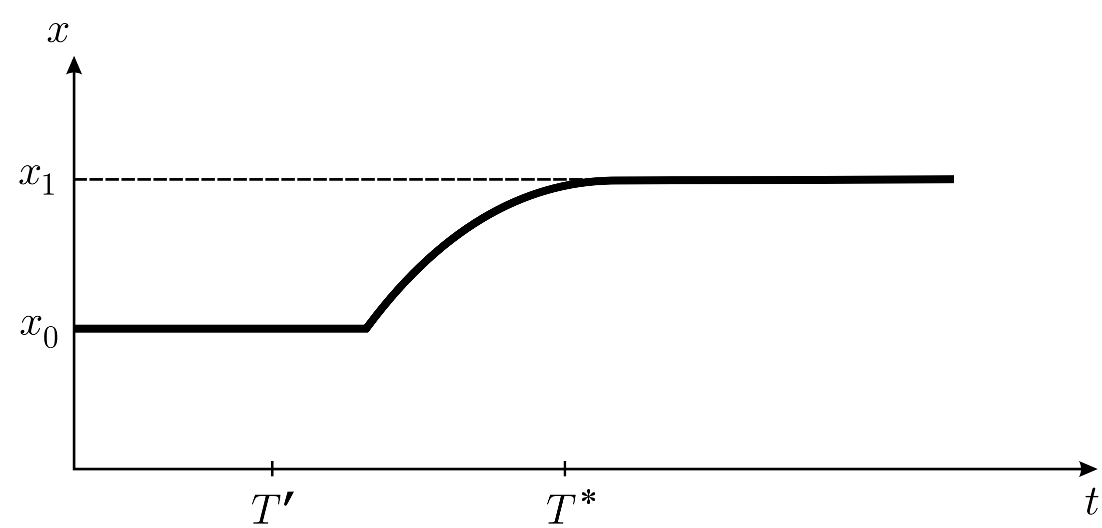
(a): $\tau < \tau'$



(b): $\tau = \tau'$



(c): $\tau' < \tau < \tau^*$



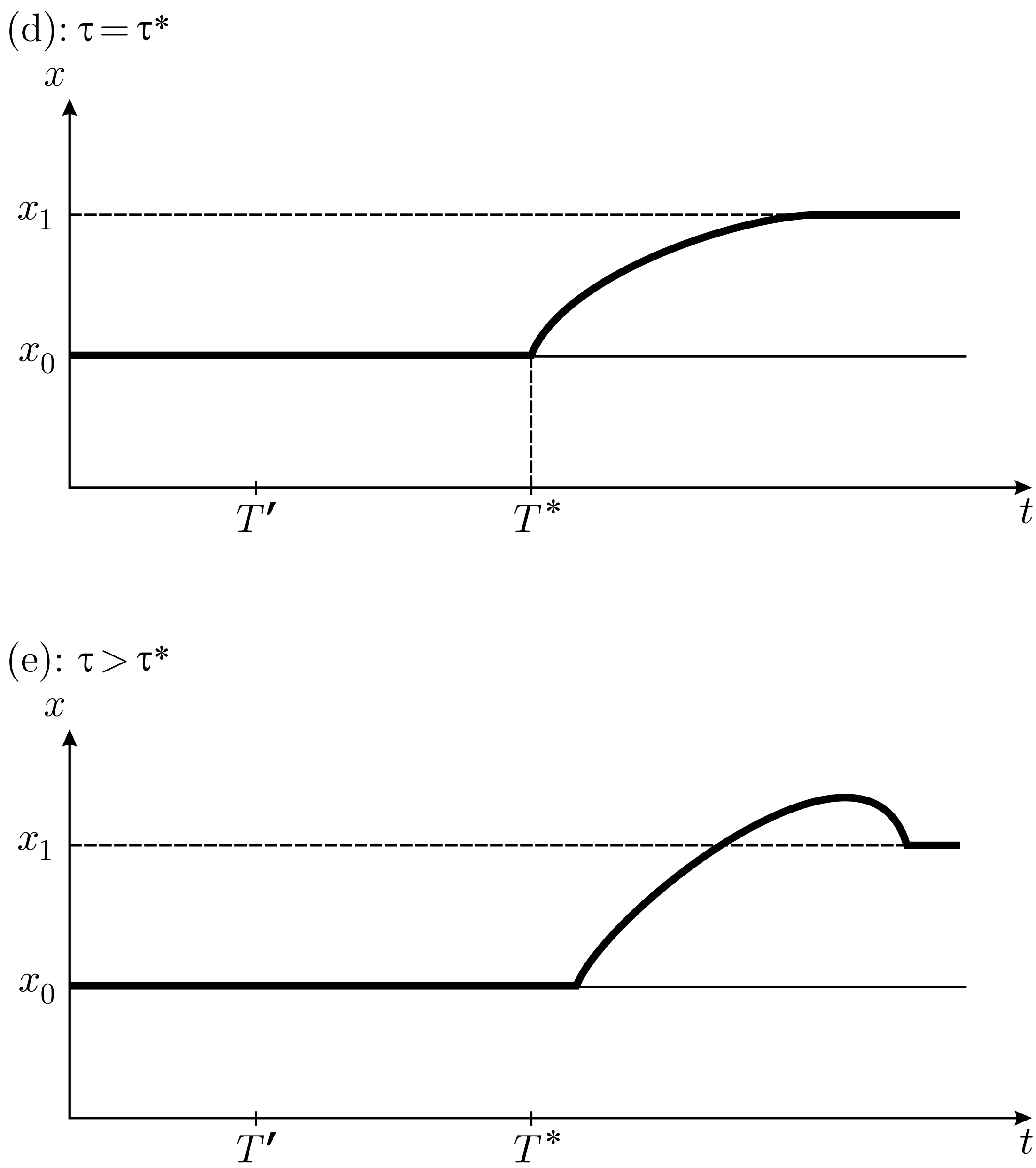


Figure 4

Dynamic Paths of the Exchange Rate

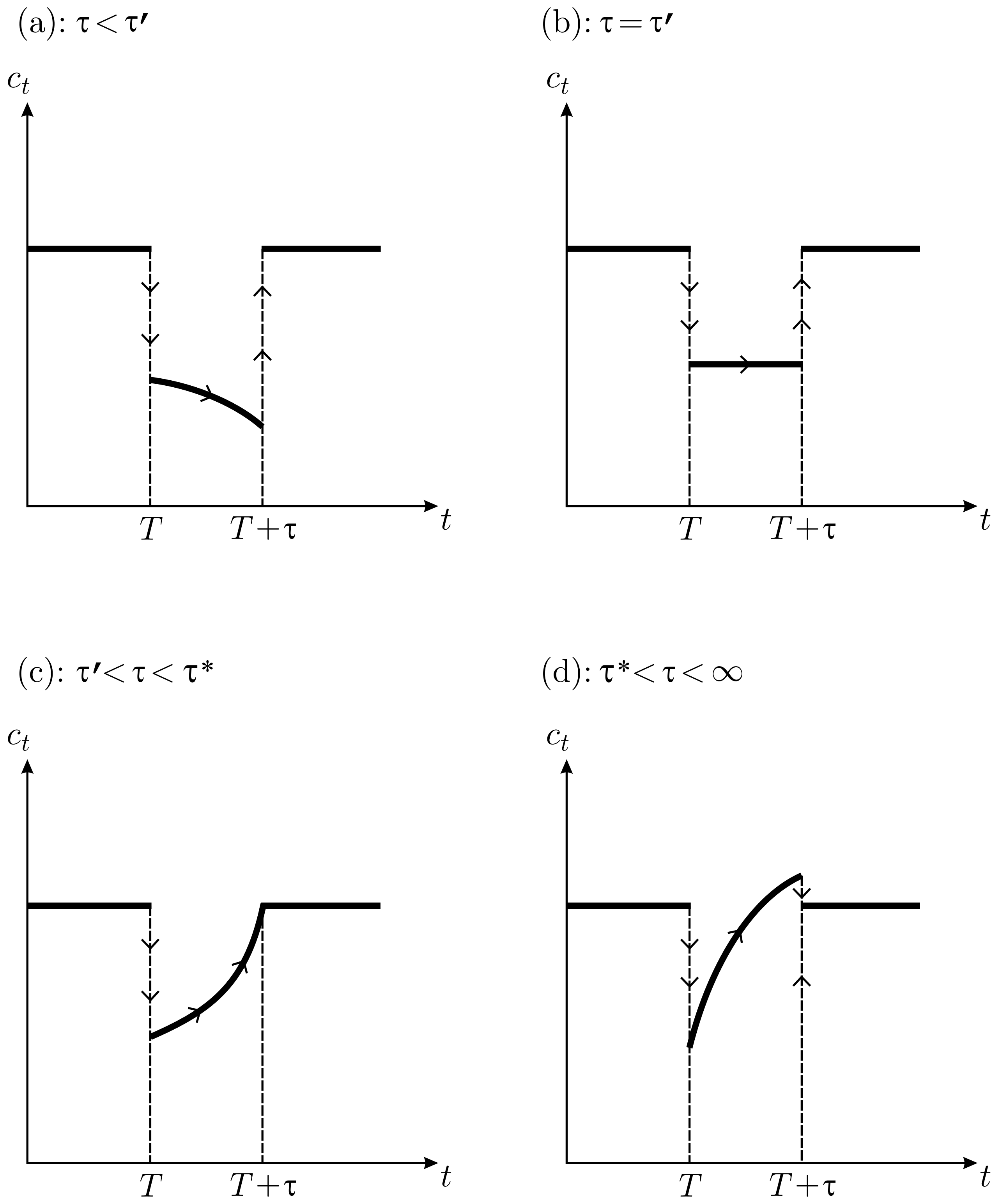


Figure 5

Consumption Adjustment Path

(e): $\tau = \infty$

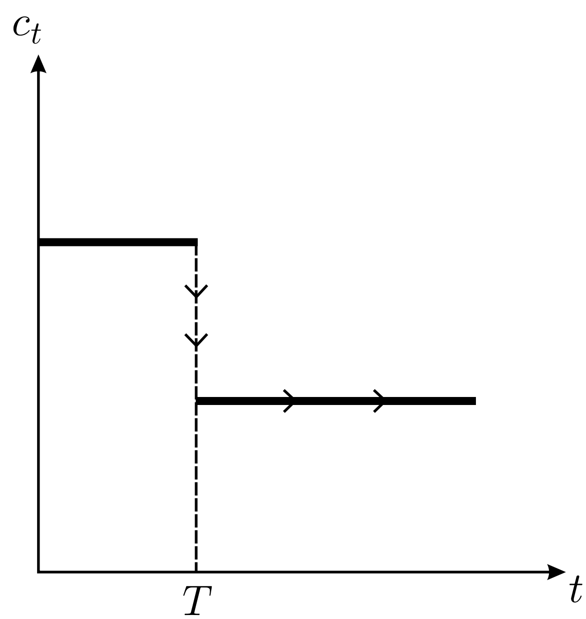


Figure 5

Consumption Adjustment Path

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