

# Bank Deregulation and Welfare

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## Abstract

The currency crisis in Asia has accelerated banking and financial deregulation in many of the affected countries. However, relatively little is known about the macroeconomic consequences of bank deregulation. Previous studies tend to focus on its impact on the banking sector with partial equilibrium analyses. This paper instead develops a general equilibrium model which provides a framework to examine the effects of banking deregulation at the macroeconomic level. In particular, the impact of lowering entry barriers in the banking sector is considered. The effects on inflation, government spending and borrowing are analyzed and are found to depend on the degree that increased deposits help to allay transaction costs associated with purchasing consumption goods. As for welfare, the impact of deregulation may be either positive or negative, with the critical determinant being the elasticities of substitution between deposits and money holdings.

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\*I would like to thank Franklin Allen for helpful comments. I am however solely responsible for any remaining shortcomings.

## LIST OF VARIABLES

$c_t$  : quantity of consumption good consumed.

$g_t$  : quantity of the public good, which is provided by the government.

$\bar{y}$  : Fixed endowment of consumption good per period.

$P_t$  : Price of consumption good.

$r$  : Fixed technological parameter as well as real interest rate.

$\pi_t$  : Rate of inflation, which equals  $(P_t - P_{t-1})/P_{t-1}$ .

$i_t$  : Nominal interest rate. By Fisher's equation  $1 + i_t = (1 + r)(1 + \pi_t)$ .

$r_t^d, i_t^d$  : Real and nominal interest rates on bank deposits or checking accounts.

$M_t, m_t$  : Nominal and real money supply

$D_t, d_t$  : Nominal and real amounts of bank deposits

$h_t$  : Real value of the monetary base, which equals  $m_t + \rho d_t$ , where  $\rho$  is the required reserves ratio.

$s_t$  : Real amount of seigniorage, which equals  $h_t - [h_{t-1}/(1 + \pi_t)]$ .

$\left[ \Phi \left( \frac{m}{c} \right) + \Psi \left( \frac{d}{c} \right) \right] c$  : Real transaction costs incurred in carrying out purchases of the consumption good.  $\Phi$  and  $\Psi$  are functions of the cash-consumption and deposit-consumption ratios respectively.  $\Phi', \Psi' < 0$  while  $\Phi'', \Psi'' > 0$ .

$b_t$  : real amount of government borrowing (i.e. budget deficit) in the form of government bonds. This are purchased by the private banks.

*Note : When both upper and lower case letters appear, they denote the nominal and real values respectively. For example,  $B_t$  represents nominal government borrowing while  $b_t$  denotes the corresponding quantity in real terms.*

# 1 Introduction

The recent currency and financial crisis in Asia has brought worldwide attention to the disastrous effects of closed and protected banking systems. Most analysts agree that one of the main causes of the Asian crisis is the lack of competition in the banking sector, which distorts the market mechanism that allocates credit to the most efficient companies. Bank regulatory bodies throughout the globe are thus increasingly aware of the importance of having greater bank competition in building a resilient banking sector that supports the growth of the financial industry and the overall economy. Thus in the aftermath of the crisis, many countries are taking steps to deregulate their banking sector. In particular, entry barriers in their banking systems are lowered in order to facilitate healthy competition and reduce inefficiencies. This paper aims to examine the impact of this increased bank competition on government policy and on welfare.

While a number of papers have examined the effects of bank deregulation, relatively little work has been done on its impact at the macroeconomic level.<sup>1</sup> This paper develops a theoretical model for analyzing the macroeconomic effects of an important aspect of bank deregulation, which is that of lowering entry barriers into banking. Previous studies on the effect of increasing bank competition tend to be based on partial equilibrium models that emphasized the banking sector. Besanko and Thakor (1992), for instance, show how greater competition in banking improves the welfare of borrowers and savers at the expense of bank stockholders. Chan *et al* (1992) also explore the implications of increased competition (as represented by lowered bank charter values) for the implementability of incentive compatible risk-sensitive deposit insurance pricing. In addition, Baltensperger and Jordan (1997) analyze the effect of greater bank competition on government seigniorage and bank profits, while keeping inflation and other macroeconomic factors exogenously fixed.

While these studies have shed light on important issues that relate to lowering entry barriers to the banking sector, their partial equilibrium models are unsuitable for analyzing the wider effects on the overall economy. By developing a dynamic model in a general equilibrium setting, this paper presents a framework to examine the effects of deregulation on macroeconomic policy variables such as inflation, government spending and budget deficits. In addition, the impact of deregulation on aggregate welfare will be considered.

An important component of the present model is the role of money and deposits as a medium of exchange. There is a number of studies in the macroeconomics literature that considers the use of both cash and checks in conducting purchases (eg Lucas (1984), Lucas and Stokey (1983), Englund and Svensson (1988), Englund (1989)). Most of these models are, however, often used in conjunction with cash-in-advance or deposit-in-advance constraints that require fixed quantities of money and deposits before any transaction can occur. In this paper, however, money and deposits are instead modelled as assets that help to reduce the transaction costs associated with purchasing consumption goods. In this way, transactions may take

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<sup>1</sup>There is a literature that analyzes the effects of free banking on controllability of monetary aggregates (White (1984), Goodhart (1986), Podolski (1986), Selgin (1987, 1994)). The focus of these papers on monetary instability is however very different from that of the present one.

place for any positive level of money or deposit holdings. Besides being more intuitive, this setup allows the amounts of money and deposit holdings to be variable.

An important difference that the present paper has with these studies in the macroeconomic literature is with regard to the industry structure of the banking industry. These other studies are based on a perfectly competitive banking sector which implicitly fixes the number of banks at infinity. Since the number of banking firms is not variable, these models are therefore not suitable for analyzing the effects of varying entry barriers into the industry. To conduct such an analysis, the banking sector has to be instead modelled as being imperfectly competitive. Since the pioneering work of Monti (1972) and Klein (1971), the feature of imperfect competition in banking has received much attention in the banking literature. The present paper draws from this literature to build a general equilibrium model that integrates a banking industry with imperfect competition.

The general equilibrium setting in this paper considers the maximizing behavior of three groups of private agents : the firms, the households and the banks. For the households, they are assumed to hold, among other assets, money and deposits to conduct purchases of the consumption good. To model the usefulness of these two types of monetary assets for meeting transactionary needs, there is assumed to exist a transaction cost function which varies positively with the quantity of transactions but negatively with money and deposits held. Following the iceberg formulation, these transaction costs are measured in terms of the quantity of consumption goods that have to be given up when conducting such transactions.

In this general equilibrium framework, the government's policy decisions have to be also taken into account. The government is assumed to have control over three policy instruments : inflation, government spending and public borrowing. The government's provision of the public good is financed either through seigniorage or public borrowing in the form of government bonds. The impact of deregulation on the optimal levels of these three instruments can then be derived. It is found that the impact of greater bank competition on government spending, inflation and the budget deficit depends on the degree which higher deposits help to reduce transaction costs of purchasing consumption goods.

The rest of this paper proceeds as follows. In section 2, the basic structure of the model used in this paper is presented. The maximization problems of households, firms, banks and the government are considered and the economy-wide equilibrium is derived. In section 3, the impact of varying bank competition is considered. It is shown how the degree of competition in the banking sector affects the optimal levels of macroeconomic variables and policy decisions. Following that, section 4 considers the welfare issues relating to deregulation in the banking sector. Finally, section 5 concludes.

## 2 An Illustrative model

This paper considers a closed economy with three representative agents : a firm, a household and a bank. There are two goods, which include a consumption good and a public good, the latter of which is provided by the government. In this model, transaction costs are associated with purchases of the consumption good, with these costs being modelled as the loss of part of the consumption good during the transaction (i.e. Samuelson's iceberg assumption). Such costs may however be mitigated through using either money or checking deposits; the transaction cost function is specified as  $\{\Phi(m/c) + \Psi(d/c)\}c$ , where  $(m, d, c)$  refers to real money holdings, real deposits and consumption respectively. The usefulness of money and deposits in lowering transaction costs is reflected in having  $\Phi', \Psi' < 0$ . It is also assumed that the second derivatives  $\Phi''$  and  $\Psi''$  are positive. In addition, the additive nature of the transaction cost function in  $m$  and  $d$  implies that money and checking deposits are not interchangeable in usage; one may think this as having an exogenously fixed fraction of all purchases being conducted in cash, while the rest are in checks. Having additional cash may reduce transaction costs for those transactions conducted in cash, but does not affect the rest of the purchases in which checks are used.<sup>2</sup> Financial innovation will obviously increase the ratio of transactions conducted with checks, but such changes are beyond the scope of the present paper, which simply assumes that this fraction is exogenously fixed.<sup>3</sup>

The timing of events in this model is as follows. At the beginning of each period, the goods market opens and the representative household purchases the consumption good from the representative firm. After the market closes, the household consumes. When consumption is complete, the household receives a fixed endowment of the consumption good, and the asset markets then open. The possible assets that the household may hold between periods include money and bank deposits. As was mentioned earlier both money and deposits are useful in reducing transaction costs. The convex nature of these costs implies that both money and deposits are held in equilibrium.

### *Firms*

The production process is exogenous in this economy. The representative firm possesses a production technology which transforms one unit of the consumption good at time  $t$  into  $1+r$  units of the same good at time  $t+1$ . This model abstracts from technological progress and so  $r$  is assumed to be constant throughout the paper. In order to engage in production, however, the representative firm has to borrow from the banking sector. For simplicity, it is assumed that banks offer one-period loans to the firms. With firms borrowing only from banks, the present model thus assumes, as is common in the literature, that households do not have the ability to lend directly to firms.<sup>4</sup> This therefore creates an intermediary role for the banking sector.

It is assumed that firms are perfectly competitive, and hence they earn zero profits. This implies that the

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<sup>2</sup>The consequences of relaxing this assumption that money and deposits are not substitutable in transactions will be examined in section 4.1 of this paper.

<sup>3</sup>Several authors have considered the effect of financial innovations on money demand, such as Dotsey (1984), Westaway and Walton (1991), and Ireland (1995).

<sup>4</sup>Some reasons emphasized in the literature include factors such as high fixed costs and indivisibilities in lending.

real rate of return on loans has to equal the technological parameter  $r$ , which is therefore the real interest rate in this model. As is well-known, in an infinite-horizon model this real rate is closely tied to the rate of time preference, which is here equal to  $1/(1+r)$ .

### *Government*

The government is assumed to possess a technology that costlessly converts the consumption good into the public good, one for one. To provide the public good, the government has to first obtain financing to purchase the consumption good. These revenues are either raised through seigniorage or from issuing government bonds that are held by the banks. Since the real return to bank loans is equal to  $r$ , for government bonds to be sufficiently attractive its real return has to be comparable. Abstracting from differences in risk and other factors between public bonds and private loans, the real return on both types of liabilities has to be equal in order for both to exist in equilibrium. The budget constraint of the government is therefore given by <sup>5</sup>

$$g_t \leq h_t - \frac{h_{t-1}}{1 + \pi_t} + b_t - (1 + r)b_{t-1} \quad (1)$$

where  $g_t$  is the real quantity of public goods provided while  $b_t$  is the real amount of government-issued bonds and  $h_t$  is high-powered money (or monetary base) as defined by the sum of currency and bank reserves, i.e.  $m_t + \rho d_t$  where  $\rho$  is the required reserves ratio and  $d$  is real deposits. The first two terms on the right-hand side of equation (1) therefore constitute (real) seigniorage revenues. In this model, the real interest rate is fixed at  $r$ . Given the Fisherian equality, control over the inflation rate is tantamount to control over the nominal interest rate. Hence, the nominal interest rate is positively correlated with inflation throughout this paper.

### *Households*

All households are identical and have homothetic preferences and so it suffices to consider a representative household. Its preferences are time-separable and depend positively on both the consumption good and the public good:

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) + V(g_t)] \quad (2)$$

where  $\beta$  lies in the unit interval and is equal to  $1/(1+r)$ , where  $r$  is the (fixed) real interest rate.  $c_t$  and  $g_t$  respectively denote the consumption of the consumption good and the public good.  $U(\cdot)$  and  $V(\cdot)$  are time-invariant functions satisfying the usual requirements of positive first-order and negative second-order derivatives. The budget constraint for the representative household is therefore<sup>6</sup>

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<sup>5</sup>In nominal terms, the budget constraint of the government may be written as

$$P_t g_t \leq H_t - H_{t-1} + B_t - (1 + \pi_t)(1 + r)B_{t-1}$$

where uppercase letters represent nominal values.

<sup>6</sup>This budget may be written in nominal terms as

$$P_t c_t (1 + \Phi + \Psi) + M_t + D_t \leq (1 + i_{t-1}^d) D_{t-1} + M_{t-1} + P_t \Pi_t$$

Dividing throughout by  $P_t$  and using Fisher's equation  $1 + i_t = (1 + r)(1 + \pi_{t+1})$  yields equation (3) in the text.

$$c_t + \left\{ \Phi \left( \frac{m_t}{c_t} \right) + \Psi \left( \frac{d_t}{c_t} \right) \right\} c_t + m_t + d_t \leq \bar{y} + (1 + r_{t-1}^d) d_{t-1} + \frac{m_{t-1}}{1 + \pi_t} + \Pi_t \quad (3)$$

where  $m_t$  and  $d_t$  are the real values of money holdings and bank deposits respectively.  $r$  is the (fixed) real interest rate applicable for government bonds, while  $r_t^d$  is the corresponding real rate on bank deposits. In addition,  $\Pi_t$  refers to real profits of banks, which accrue to households since they are the banks' owners, and  $\pi_t$  is the rate of inflation as defined by  $(P_t - P_{t-1})/P_{t-1}$ . The maximization problem of the representative household is as follows. It chooses  $\{c_t, m_t, d_t\}_{t=0}^{\infty}$  to maximize the utility function (2) subject to the budget constraint (3). In doing so, it takes to be constant other variables such as the inflation rate, interest rate on deposits, government spending and bank profits.

Solving this maximization problem of the representative households yields the following conditions (see Appendix A for detailed derivation):

$$-\Phi'(\cdot) = \frac{i_t}{1 + i_t} \quad (4)$$

$$-\Psi'(\cdot) = \beta(r - r_t^d) \quad (5)$$

Since  $\Phi$  is a function of the cash-consumption ratio  $m/c$  the first of these two conditions is an implicit function for money demand. It generates a standard relationship in which real money demand is positively related to consumption and negatively related to the nominal interest rate. Likewise, the second condition implicitly expresses deposit demand as a function of consumption and the interest rate differential between storage and deposit interest rates. Real deposits increases when consumption expands or when there is a rise in the interest differential  $r - r^d$ .

### *Banks*

There is imperfect competition in the banking industry, which has  $n \geq 1$  identical banks. When the number of banks increases, so does the degree of competition. The special case of  $n = \infty$  therefore represents perfect competition. The other extreme is when  $n = 1$ , which is the case of a monopoly. It is assumed throughout this paper that the number of banks is exogenously determined. The lowering of entry barriers in the banking sector is one event that exogenously raises the number of banks in this model. The consequences of such an increase in bank competition will be considered in detail subsequently.

Banks are assumed not to hold any excess reserves and their only costs in creating deposits arises from the reserves that have to be held with the central bank in order to satisfy the reserve requirements. The required reserves ratio,  $\rho$ , therefore determines the amount of bank reserves that are held; these reserves equals  $\rho d_t$ , where  $d_t$  denotes the real amount of deposits at time  $t$ . Consider now the deposits for a particular bank, say bank  $i$ . Given the assumption that all banks are identical, bank  $i$  may also be considered as the representative bank. Bank  $i$  takes as given the nominal interest rate, which through the inflation rate is controlled by the government since the real interest rate is fixed at  $r$ . The total amount of deposits in the banking system is therefore given by

$$d_t = d_{i,t} + \widehat{d}_t \quad (6)$$

where  $d_{i,t}$  and  $\widehat{d}_t$  respectively denote the real quantity of deposits at bank  $i$  and at the other  $n - 1$  banks. Under the Cournot conjecture, bank  $i$  takes  $\widehat{d}_t$  to be fixed when deciding on its own level of deposits. Bank  $i$ 's profits in real terms may be expressed as

$$\Pi_{i,t} = \left[ r(1 - \rho) - r_t^d - \frac{\rho\pi}{1 + \pi} \right] d_{i,t} \quad (7)$$

where  $r$  is the interest rate on loans or government bonds,  $r_t^d$  is that on deposits and  $d_{i,t}$  denotes the real quantity of deposits at bank  $i$ . With perfect competition, the term in the square brackets would become zero, as would bank profits. However, the introduction of imperfect competition allows banks to enjoy positive profits. Using equation (5), the deposit interest rate  $r_t^d$  may be substituted away in equation (7). Assuming that the Cournot conjecture holds, bank  $i$  maximizes its profits while holding constant the deposits at the other  $n - 1$  banks. The first-order condition for bank  $i$ 's maximization problem is therefore given by

$$\frac{d\Pi_i}{dd_i} = -\rho r - \frac{\rho\pi}{1 + \pi} - \frac{\Psi'}{\beta} - \frac{\Psi''d_i}{\beta c} = 0 \quad (8)$$

$$\text{where } (n + 1)\Psi''c + \Psi'''d > 0 \quad (9)$$

and where time subscripts are dropped to simplify notation. The inequality in (9) is assumed to hold in order to satisfy the second-order condition for this maximization problem.<sup>7</sup> Since all banks are identical, both sides of the first order condition (8) may be summed over the  $n$  banks to obtain  $n\beta\rho r + (n\beta\rho\pi)/(1 + \pi) + n\Psi' + \Psi''d_t/c_t = 0$ . This is an implicit function of total deposits  $d_t$  in terms of the number of banks  $n$  and consumption  $c_t$ . Totally differentiating this expression yields:

$$\left( \beta\rho r + \frac{\beta\rho\pi}{1 + \pi} + \Psi' \right) dn + \frac{(n + 1)\Psi''c + \Psi'''d}{c^2} dd - \frac{d[(n + 1)\Psi''c + \Psi'''d]}{c^3} dc + \frac{n\beta\rho}{(1 + \pi)^2} d\pi = 0 \quad (10)$$

The quantity of deposits is therefore positively related to the number of banks and to consumption, while negatively related to inflation.<sup>8</sup> These relationships may be explained as follows. As in any other industry, increased competition results in higher production (i.e. deposits). Deposits also rise with consumption because of the usefulness of deposits in reducing transaction costs associated with consumption (medium-of-exchange function). Lastly, an increase in inflation raises the implicit inflation tax on bank reserves, thus reducing profits and hence the profit-maximizing level of deposits.

### *Consumption and Inflation*

In equilibrium, the real values of consumption, money and deposit holdings are all constant. Using this in the budget constraint (3) implies

<sup>7</sup>The second-order condition is given by  $d^2\Pi_i/dd_i^2 \leq -(2\Psi''c + \Psi'''d_i)/(\beta c^2) < 0$ . Since all banks are identical and hence  $d_i = d/n$ , the second-order condition may be rewritten as  $2n\Psi''c + \Psi'''d > 0$ , which has to hold for all  $n \in \{1, 2, 3, \dots\}$ . Since  $2n \geq n + 1$  for all  $n$  in this range, the inequality in (9) follows.

<sup>8</sup>Note that the coefficient of  $dn$  is negative since  $(n\beta\rho r + \beta\rho\pi/(1 + \pi) + \Psi') = -\Psi''d_i/c < 0$  from (8)



$$c \left[ 1 + \Phi \left( \frac{m}{c} \right) + \Psi \left( \frac{d}{c} \right) \right] = \bar{y} - \frac{\pi(m + \rho d)}{1 + \pi} + (1 - \rho)r d \quad (11)$$

where  $\Pi$  is substituted for using the aggregated version of equation (7), i.e.  $\Pi = [(1 - \rho)r - r^d]d$ . Note that time subscripts are dropped since all real quantities are constant in the steady state. The condition in (11) states that total consumption of the private good, which includes the associated transaction costs for the private good, is equal to the income available to the representative consumer in each period. This income is equal to the sum of the endowment  $\bar{y}$  and the interest on bond holdings (through ownership of banks), and net of the inflation tax paid to the government. Totally differentiating this expression and dividing by  $di$ , it is straightforward to obtain

$$\left[ 1 + \Phi + \Psi - \Phi' \frac{m}{c} - \Psi' \frac{d}{c} \right] \frac{dc}{di} = r(1 - \rho) \frac{dd}{di} - \frac{\pi}{1 + \pi} \frac{dm}{di} - \frac{\beta m}{(1 + \pi)^2} \quad (12)$$

where the last term on the right-hand side is obtained from having  $d\pi/di = \beta$  (due to Fisher's equation  $\beta(1 + i) = 1 + \pi$ , where  $\beta = 1/(1 + r)$ ). From equation (4),  $dm/di$  may be expressed as  $(m/c)(dc/di) - c/[\Phi''(1 + i)^2]$ . Similarly,  $dd/di$  may be obtained from equation (10) as  $(d/c)(dc/di)$ , if the number of banks  $n$  is held fixed (i.e.  $dn = 0$ ). Substituting both of these into equation (12) above yields<sup>9</sup>

$$\frac{\partial c}{\partial \pi} = \frac{\partial c}{\partial i} \cdot \frac{\partial i}{\partial \pi} = -\frac{1}{\beta} \left[ \frac{\pi}{1 + \pi} \frac{c}{\Phi''(1 + i)^2} - \frac{\beta m}{(1 + \pi)^2} \right] \Bigg/ \left[ 1 + \Phi + \Psi + \frac{\pi}{1 + \pi} \frac{m}{c} - r(1 - \rho) \frac{d}{c} \right] < 0 \quad (13)$$

The inequality in (13) indicates that as inflation or the nominal interest rate rises, consumption falls. The intuition is as follows. When the nominal interest rate or inflation rate is raised, money and deposit holdings are reduced because of the higher opportunity cost of holding them (note that there is no direct impact on deposit holdings, which are proportional to consumption as long as the number of banks is unchanged (see equation (10)). Since money serves a medium-of-exchange role, these reduced holdings of monetary assets increase the transaction cost of engaging in consumption. Furthermore disposable income is reduced due to the higher inflation tax, a consequence of the higher inflation rate (or nominal interest rate). With a higher inflation tax and greater transaction costs, the quantity of the consumption good that is actually consumed is therefore decreased. Hence, the consumption of the private good ( $c$ ) falls when the inflation rate or the nominal interest rate rises.

### *Monetary Policy and Seigniorage Revenues*

In order to evaluate the effect of a higher nominal interest rate on real government spending, it is necessary to first ascertain the impact on seigniorage revenues. Let  $s_t$  be the real seigniorage collected in period  $t$ , i.e.  $s_t = h_t - h_{t-1}/(1 + \pi_t)$ . The change in seigniorage  $s$  from an increase in inflation rate, or equivalently from a higher interest rate, is given by<sup>10</sup>

<sup>9</sup>Note that from the Fisherian equality  $1 + i = (1 + r)(1 + \pi) = \beta^{-1}(1 + \pi)$ .

<sup>10</sup>Equation (14) is obtained as follows:

$$\frac{\partial s}{\partial \pi} = \frac{\pi}{1 + \pi} \frac{\partial h}{\partial \pi} + \frac{h}{(1 + \pi)^2}$$

$$\frac{ds_t}{d\pi_t} = \frac{\partial h}{\partial \pi} + \frac{h}{(1+\pi)^2} \quad (14)$$

Clearly the sign of  $ds/d\pi$  is ambiguous. This ambiguity is consistent with a substantial literature which, originating with Cagan (1956), emphasized an inverted U-shaped Laffer curve for seigniorage. The existence of such a curve implies that on its downward-sloping portion, seigniorage will actually fall when inflation (or nominal interest rate) is increased. Since it is widely agreed that inflation is undesirable, a rational government will therefore never be on this downward-sloping section because it can achieve the same seigniorage at a lower inflation rate. The exception is when the economy is trapped with reputation and time inconsistency problems that seem to fit the experience of some high-inflation countries. Since such problems are beyond the scope of this paper, the positive effect is assumed to dominate, i.e. seigniorage increases with the inflation or interest rate. As will be apparent later, the assumption of an interior solution in the government's maximization problem will guarantee that this restriction holds.

### 3 Macroeconomic Impact of Deregulation

In this section we examine the effect of deregulation on the aggregate economy and on the impact it has on the optimal macroeconomic policies. Consider that entry barriers in the banking sector are lowered, thus causing the number of banks to increase. Reducing entry barriers is therefore tantamount to increasing competition in the banking sector.

To derive the optimal policies for the government, the indirect utility function of the representative household is maximized, subject to the government's budget constraint in (1). The government's decision variables are borrowing  $b$  and nominal interest rate  $i$ , with respective first-order conditions being<sup>11</sup>

$$V'(g_{t+1}) = V'(g_t) \quad (15)$$

$$-\lambda_t + V'(g_t) [1 - (1+i)\varepsilon^{hi}] = 0 \quad (16)$$

where  $\varepsilon^{hi}$  is the interest elasticity of the monetary base as defined by  $(i/h)(\partial h/\partial i)$ . The first first-order condition (15) equates the marginal cost of an increase in government borrowing  $b$  to the corresponding marginal benefit. Given that the function  $V'$  is time-invariant, this equation also implies that real government spending  $g$  is constant in the steady state. With respect to the second first-order condition in (16), the following assumption is made to ensure an interior solution:

*Assumption 1 : The interest elasticity of the monetary base,  $\varepsilon^{hi}$ , is assumed to be always less than  $1/(1+i)$*

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$$\begin{aligned} &= \frac{h}{(1+\pi)^2} \left[ 1 + \frac{(1+\pi)\pi}{h} \frac{\partial h}{\partial \pi} \right] \\ &= \frac{h}{(1+\pi)^2} \left[ 1 + \frac{(1+i)\pi}{i} \left( \frac{\partial h}{\partial i} \frac{i}{h} \right) \right] \quad \text{since } \frac{\partial h}{\partial \pi} = \frac{\partial h}{\partial i} \frac{di}{d\pi} = \frac{\partial h}{\partial i} (1+r) \\ &= \frac{h}{(1+\pi)^2} \left[ 1 + \frac{(1+i)\pi}{i} \varepsilon^{hi} \right] \end{aligned}$$

<sup>11</sup>Note that equation (1) is used here. See Appendix B for further details.

in absolute value, i.e.

$$0 < \varepsilon^{hi} \equiv -\frac{\partial h}{\partial i} \cdot \frac{i}{h} < \frac{1}{1+i}$$

This assumption ensures that the second term is positive; if this is not satisfied then both lefthand-side terms will be negative and (16) will not be satisfied. Assumption 1 also ensures that  $\partial s/\partial \pi$  is positive, since from (14) the positivity of this derivative is satisfied when  $\varepsilon^{hi} < \left(\frac{i}{1+\pi}\right) \left(\frac{1}{1+i}\right)$ , the right-hand side which is less than  $1/(1+i)$ .<sup>12</sup>

With assumption 1, equation (16) may be interpreted as follows. It requires that the marginal cost of an increase in the nominal interest rate is equal to the associated marginal benefit. The cost of a rise in the interest rate comes from the fall in consumption, as is reflected in equation (13), and which was also discussed in the previous section of this paper. The benefit of increasing the nominal interest rate is derived from the corresponding increase in seigniorage revenues, since that may be used to finance a higher level of real government spending.

Now consider the effect of relaxing entry barriers to the banking sector, which increases competition with a larger number of banks  $n$ . The following proposition considers the impact on the budget deficit of the government:

*Proposition 1 : If the interest elasticity of the monetary base does not decline excessively as bank competition rises, this increase in bank competition lowers government borrowing, i.e. reduces the budget deficit.*

*Proof :* Let  $H(i, b; n) \equiv V'(g_t) - V'(g_{t+1})$  and  $F(\pi, b; n)$  be defined as the left-hand side of (16). Hence the first-order conditions to the government's maximization problem in (15) and (16) correspond to  $H(\pi, b; n) = 0$  and  $F(\pi, b; n) = 0$  respectively. Using the implicit function theorem yields

$$\frac{\partial H}{\partial n} + \frac{\partial H}{\partial \pi} \frac{d\pi}{dn} + \frac{\partial H}{\partial b} \frac{db}{dn} = 0 \quad (17)$$

$$\frac{\partial F}{\partial n} + \frac{\partial F}{\partial \pi} \frac{d\pi}{dn} + \frac{\partial F}{\partial b} \frac{db}{dn} = 0 \quad (18)$$

First consider equation (18). The term in parentheses (i.e.  $\partial F/\partial i$ ) is the partial derivative of  $F$  with respect to  $i$  while holding  $b$  (and of course  $n$ ) constant. This term has to be negative to satisfy the second-order condition associated with the government's maximization problem with respect to  $i$ , i.e.  $F(\pi, b; n) = 0$ . Next consider  $\partial F/\partial b = V''(g_t)\partial g_t/\partial b_t = V''(g_t) < 0$ .<sup>13</sup> Taken together, it therefore follows that  $\partial F/\partial b < 0$ . In addition, straightforward (partial) differentiation of  $F(\pi, b; n)$  with respect to  $n$  shows that  $\partial F/\partial n$  is ambiguously signed, as shown in Appendix C. Now consider equation (17). The derivative  $\partial H/\partial n = V''(g_t)(\partial g_t/\partial n) - V''(g_{t+1})(\partial g_{t+1}/\partial n)/(1+\pi)$ , which is negative since

$$\frac{\partial g_{t+1}}{\partial n_t} = \frac{\partial h_{t+1}}{\partial n_t} - \frac{1}{1+\pi} \frac{\partial h_t}{\partial n_t}$$

<sup>12</sup>Note that from Fisher's equation  $1+i = (1+\pi)(1+r) = (1+\pi)/\beta$ . This implies that  $\partial h/\partial \pi = (\partial h/\partial i) \cdot (\partial i/\partial \pi)$ .

<sup>13</sup>Recall that  $\partial g_t/\partial b_t = 1$  from equation (1).

$$= \frac{\pi}{1 + \pi} \frac{\partial h_t}{\partial n_t} < \frac{\partial h_t}{\partial n_t} = \frac{\partial g_t}{\partial n_t}$$

As for  $\partial H/\partial \pi$ , it is given by  $V''(g_t)(\partial g_t/\partial \pi_t) - V''(g_{t+1})(\partial g_{t+1}/\partial \pi_t)$ . Note that  $\partial g_{t+1}/\partial \pi_t$  and  $\partial g_t/\partial \pi_t$  are both positive, since higher inflation raises financing through greater seigniorage revenues. Also,  $\partial g_{t+1}/\partial \pi_t = \pi_{t+1}(1 + \pi_{t+1})^{-1}(\partial h_t/\partial \pi_t) < \partial h_t/\partial \pi_t = \partial g_t/\partial \pi_t$ . Thus  $\partial H/\partial \pi$  is positive. Lastly  $\partial H/\partial b = V''(g_t) + V''(g_{t+1})(1 + r) < 0$ .

Substituting (17) into (18) for  $d\pi/dn$  yields

$$\frac{db}{dn} = \frac{(\partial F/\partial n)(\partial H/\partial \pi) - (\partial F/\partial \pi)(\partial H/\partial n)}{(\partial F/\partial \pi)(\partial H/\partial b) - (\partial F/\partial b)(\partial H/\partial \pi)} \quad (19)$$

The denominator of the right-hand side term is positive, but the numerator may be positive or negative. If the condition given in the proposition holds, then the numerator is positive and  $db/dn$  is negative. In this case, when the number of banks increases the amount of government borrowing falls. 2

Intuitively, greater bank competition affects the budget deficit because of the following. When the number of banks increases, the deposit interest rate also rises, thus attracting more deposits. The expansion of the deposit base raises the monetary base as well, leading to an increase in seigniorage revenues. However, greater bank competition may increase the (absolute) value of the interest elasticity of the monetary base as well, causing inflation to become a more distortionary method of financing. If this is true, inflation may decline to the extent that seigniorage revenues actually fall. If on the other hand the net impact on seigniorage is positive, then the government will be able to buy back bonds (i.e. reduce the budget deficit). Otherwise, the government will have to increase borrowing instead, leading to a larger budget deficit.

*Proposition 2 : As long as the interest elasticity of the monetary base does not decline excessively as bank competition rises, greater bank competition lowers the inflation rate.*

*Proof :* Equation (18) may also be written as

$$\frac{d\pi}{dn} = \frac{(\partial F/\partial b)(\partial H/\partial n) - (\partial F/\partial n)(\partial H/\partial b)}{(\partial F/\partial \pi)(\partial H/\partial b) - (\partial F/\partial b)(\partial H/\partial \pi)} \quad (20)$$

The denominator of the right-hand side term is again positive, as in (19). Due to the negativity of  $\partial F/\partial b$  and  $\partial H/\partial n$ , the first term of the numerator is positive. Since  $\partial F/\partial n$  may be either positive or negative, the numerator is ambiguously signed. The sign of  $d\pi/dn$  is therefore also ambiguous. Since inflation is positively related to the nominal interest rate  $i$ , inflation may rise or fall when the number of banks increases. If the condition specified in the proposition holds, then the numerator becomes negative, as does  $d\pi/dn$ . To show this substitute for  $db/dn$  in equation (20) using (19) and the proposition follows after some simplification. Inflation in this case decreases when bank competition rises. 2

The intuition behind the ambiguity on inflation is as follows. When bank competition rises, the expansion of the deposit and monetary base raises seigniorage revenues. As pointed out earlier, bank competition may also reduce the interest elasticity of the monetary base, which increases the distortionary impact of inflation. If this second effect is sufficiently strong, then optimal inflation will fall.

*Proposition 3 :* *The impact of greater bank competition on public spending  $g$  is negatively related to the impact of the increased competition on the interest elasticity of the monetary base. If this elasticity is not overly responsive to bank competition in the upward direction, then public spending rises when competition intensifies in the banking sector. Otherwise the optimal provision of the public good will decline when the number of banks increases.*

*Proof :* The provision of the public good  $g_t$  may be obtained from equation (1). Since government spending is constant after the exogenous increase in bank competition, we will focus on its steady-state value as given by  $g = s - rb$  (note time subscripts are dropped since all variables here are constant in the steady state). The variable  $s$  is seigniorage as given by  $\pi h/(1 + \pi)$ . Differentiating  $g$  with respect to  $n$  yields

$$\frac{dg}{dn} = \frac{\partial s}{\partial n} + \frac{\partial s}{\partial \pi} \frac{\partial \pi}{\partial n} - r \frac{db}{dn}$$

The derivative  $\partial s/\partial n$  is positive since greater bank competition increases the deposit and monetary base, thereby raising seigniorage revenues.<sup>14</sup> Note that  $\partial s/\partial \pi$  is also positive since the economy is on the upward-sloping portion of the Laffer curve. In the interval  $v_2 < \partial F/\partial n < v_1$ , the optimal inflation rises but the optimal budget deficit falls. The level of government spending  $g$  therefore unambiguously increases, since seigniorage revenues are higher while the burden of financing interest on the public debt falls. When  $\partial F/\partial n = v_2$ , the optimal inflation rate remains unchanged but the optimal budget deficit falls through the buy-back of government bonds. The corresponding level of government spending therefore has to decline. On the other hand, when  $\partial F/\partial n = v_1$ , the optimal budget deficit remains unchanged before and after the increase in bank competition. There is therefore no need to finance a buy-back of government bonds. At the same time, optimal inflation rises, which makes it possible to raise public spending. For values of  $\partial F/\partial n$  above  $v_1$ , the optimal budget deficit is increased together with inflation, making even more spending by the government possible. This proves that government spending will rise as long as  $\partial F/\partial n > v_2$ . Since  $\partial \varepsilon^{hi}/\partial n$  and  $\partial F/\partial n$  are opposite in sign, this condition is equivalent to  $\partial \varepsilon^{hi}/\partial n$  being not too large, i.e.  $\varepsilon^{hi}$  does not increase excessively with greater bank competition. 2

The intuition behind Proposition 3 is related to the change in the costs of inflation when bank competition rises. The greater the increase in the elasticity  $\varepsilon^{hi}$ , the larger the distortionary effect of inflation will become. This is because a higher elasticity implies that the households' decisions on money and deposit holdings are more sensitive to inflation, and hence the distortion caused by inflation will be higher. The increase in the

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<sup>14</sup>See Appendix D for details.

costs of raising funds through inflation in turn reduces the optimal level of government spending.

## 4 Effects of Deregulation on Welfare

This part of the paper examines the welfare effects of lowering entry barriers in the banking sector. The previous section has already examined the effect of such deregulation on various policy variables. It is however not clear how changes in the levels of these instruments, together with greater bank competition, affect the welfare of the representative household. Assuming that the government is a benevolent one which adopts the optimal policies derived earlier, the welfare impact is given formally in the following proposition:

*Proposition 4 : Increasing bank competition unambiguously improves welfare.*

*Proof :* Consider the utility function of the representative agent in equation (2). Let the indirect utility function be represented by  $\mathcal{W}$ . Then the impact of greater bank competition on welfare is given by (using envelope theorem)

$$\begin{aligned} d\mathcal{W}/dn &= -\lambda_t \Psi' + V'(g_t) \partial h_t / \partial n - \frac{V'(g_{t+1})}{1 + \pi_{t+1}} \frac{\partial h_t}{\partial n} \\ &= -\lambda_t \Psi' + V'(g_t) \frac{\partial h}{\partial n} \frac{\pi_{t+1}}{1 + \pi_{t+1}} > 0 \end{aligned} \quad (21)$$

where the second equality follows from (15). Note that  $\partial h / \partial n$  is positive (see Appendix). Hence welfare unambiguously improves.  $\square$

The impact is intuitively as follows. The welfare of the representative agent depends on the consumption good as well as the public good. The direct impact of increasing bank competition on consumer is through the increase in his deposit holdings. When a larger number of banks, the deposit interest rate rises, thereby increasing deposits. From an aggregate perspective, this rise in the deposit interest rate merely represents a transfer from banks to depositors, and hence has no aggregate impact. However, the increased deposit holdings reduce the transaction costs incurred in consumption; since these transaction costs are deadweight losses to the economy, a reduction in such costs raises welfare. This is represented by the term  $-\lambda_t \Psi'$  in (21). As for the other term, that simply represents the increase in welfare with higher provision of the public good, made possible by the higher seigniorage revenues from an enlarged deposit and monetary base. This second effect therefore captures the increase in welfare with greater consumption of the public good.

The welfare gain therefore arises from the increased consumption of both the private consumption good and public good. Since the welfare gain relies on the increase in deposits in both of these cases, this gain is positively related to the sensitivity of deposits to the higher deposit interest rate that comes with a larger number of banks. This is stated formally in the proposition below.

*Proposition 5 : The greater is the elasticity of deposits with respect to the interest differential  $r^d - r$ , the higher will be the welfare gain when bank competition rises.*

#### 4.1 Substitutability Between Money and Deposits

The result that welfare improves critically hinges on the positivity of the direct impact that a larger number of banks has on seigniorage. As noted earlier, rising bank competition has no direct effect on money holdings. The money demand equation implicit in equation (4) simply states that real money demand is a function of consumption and the nominal interest rate, and not the degree of banking competition. On the other hand, the number of banks directly affects deposit holdings, as may be seen in equation (10).

In the present model, there is zero substitutability between money and deposits. This is because of the formulation that a fixed fraction of all purchases are conducted in money, while the rest are performed using checking deposits. The effect of relaxing this assumption is considered in the proposition below.

*Proposition 6 : If substitutability exists between money and deposits, then Proposition 4 is altered in that welfare may either improve or deteriorate when bank competition increases. In this case the welfare change is negatively related to the degree of substitutability between these two groups of monetary assets.*

The key to this result is the effect that greater bank competition has on seigniorage. In the previous case with no substitutability, an increase in the number of banks has no effect on money holdings. All that happens is a shift from government bonds to bank deposits. However if money and deposits are substitutable, then an increase in attractiveness of deposits due to higher deposit rates will cause the household to shift from money to deposits as well. When that happens, seigniorage may fall because while 100% of all money count toward the monetary base  $m + \rho d$ , only a fraction  $\rho$  of all deposits does ( $\rho$  is the required reserves ratio). The greater the degree of substitutability between these two assets, the lower will be the monetary base and seigniorage revenues. When this substitutability is above a threshold level, seigniorage will actually fall, as will government spending and welfare.

This result that the impact of a larger number of banks on seigniorage is ambiguous is not new. Using a specific functional form, Baltensperger and Jordan (1997) has shown that relative parameter values determine if a larger number of banks will increase or decrease seigniorage. In the present context, this ambiguity carries a new significance because of its implications for welfare; the ambiguity in seigniorage causes the welfare change to be also ambiguous. Therefore deregulation may not necessarily improve welfare if money and deposits may be used interchangeably in transactions.

The substitutability of money and deposits would also have implications on the optimal levels of inflation and budget deficit derived in section 3. Again, the key is the inverse relationship between this substitutability

and the level of seigniorage collected. When money and deposits are substitutable, the resultant decline in seigniorage increases the financing burden, thus leading to higher budget deficits and inflation. Proposition 7 below provides a summary:

*Proposition 7 : The greater the substitutability between money and deposits, the higher will be the optimal levels of inflation and budget deficit.*

## 5 Concluding Remarks

The recent currency crisis is likely to cause bank deregulation to occur in many parts of Asia. One important aspect of deregulation is the lowering of entry barriers that leads to greater competition among banks. Past studies that examine the impact of increasing competition in banking tend to be based on partial equilibrium models that focus on the banking sector. As a result, they are not built for analyzing the effect of greater bank competition on the aggregate economy. This paper makes a contribution by developing a dynamic, general equilibrium model that provides a framework to examine the effects of lower entry barriers from a macroeconomic angle.

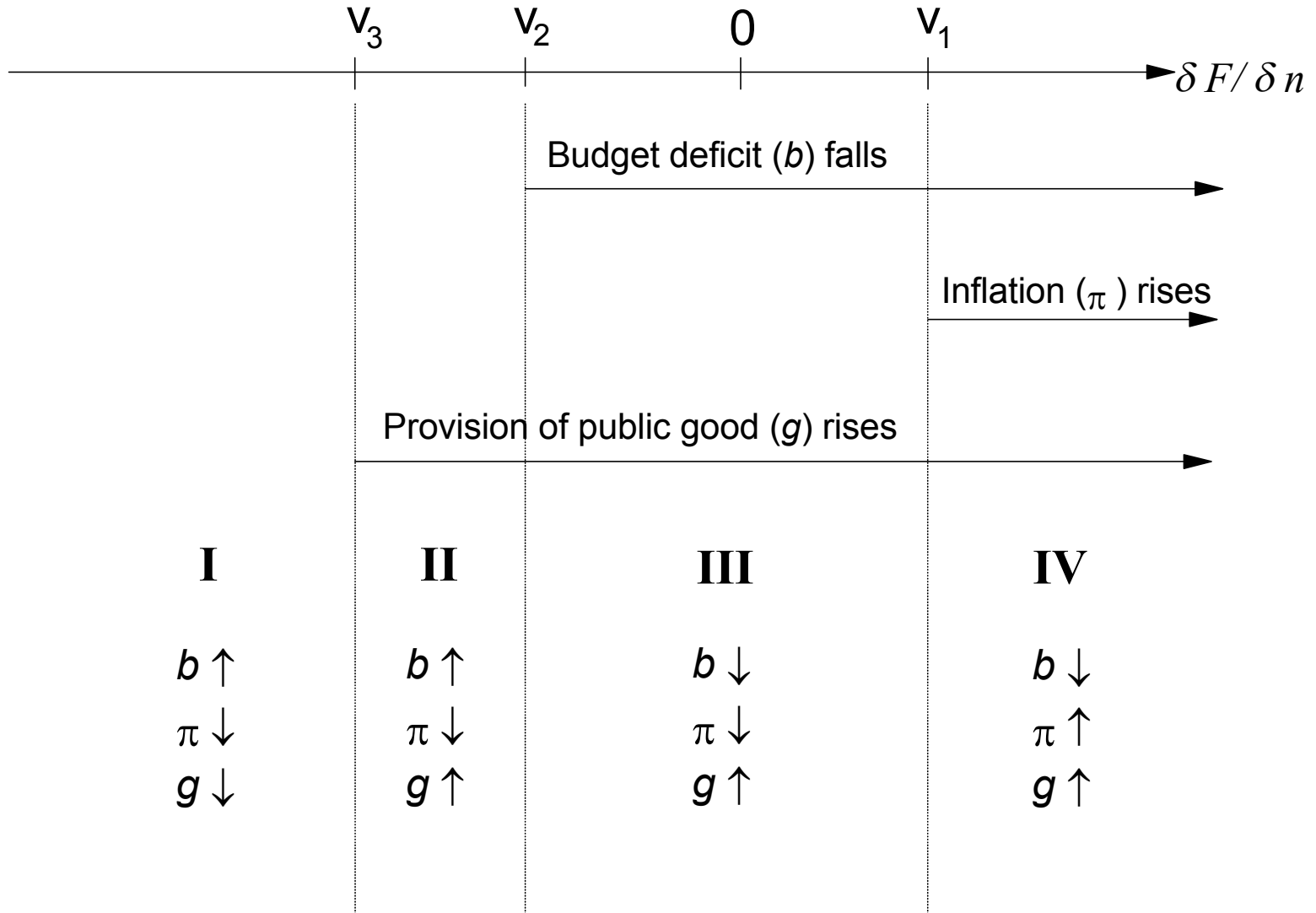
An important component of this aggregative analysis is the effect of deposits or checking accounts on the transaction costs of conducting purchases of the consumption good. Having more deposits reduces such transaction costs because of the standard arguments on the role of bank checks as a medium of exchange. The optimal response of various policy instruments to greater bank competition may also be derived through standard maximization procedures. It was found that the optimal levels of these policy variables may rise or fall, depending on the degree to which deposits help to reduce transaction costs at the margin.

The past literature on the effects of bank deregulation have found important results that usually emphasize the welfare for different groups of agents. For instance, the study by Besanko and Thakor (1992) shows how borrowers and savers benefit at the expense of bank shareholders. This study differs in focusing on the welfare impact for the economy as a whole. Interestingly, the effect of greater bank competition on welfare may be either positive or negative. It was found that the elasticity of substitution between deposits and money holdings is an important determinant of this welfare impact. Specifically, welfare is higher when greater substitutability exists between these assets.

Lastly, there are a number of ways in which the present paper may be extended. One possible extension is to allow financial innovation to become endogenous such that the fraction of transactions that are conducted in money is variable. When bank competition is increased, financial innovation may then raise the ratio of transactions that are conducted using checking deposits. Another way to extend the present paper is to relax the assumption that the government is benevolent by introducing political economy considerations. In particular, the monetary and fiscal authorities may be modelled as separate decision-making units. In



this case, the game-theoretic interaction between the authorities may generate outcomes different from those presented in the current paper. Such and other extensions should, therefore, provide possibilities for future research.



## Appendix A : The Maximization Problem of the Household

Consider the maximization problem of the representative household, which is to maximize utility (2) subject to the budget constraint (3). Let  $\{\lambda_t\}_{t=0}^{\infty}$  be the Lagrange multipliers associated with the budget constraint. The necessary and sufficient conditions for a solution to this problem are

$$U'(c_t) = \lambda_t \left[ 1 + \Phi + \Psi - \frac{m_t}{c_t} \Phi' - \frac{d_t}{c_t} \Psi' \right] \quad (\text{A-1})$$

$$\lambda_t(1 + \Phi') = \beta \lambda_{t+1} / (1 + \pi_{t+1}) \quad (\text{A-2})$$

$$\lambda_t(1 + \Psi') = \beta \lambda_{t+1} (1 + r_t^d) \quad (\text{A-3})$$

together with the budget constraint (3). These three conditions are derived from differentiation with respect to  $c$ ,  $m$  and  $d$  respectively. The last equation is the transversality condition that ensures that the representative household does not borrow indefinitely into the future.

Consider an steady state in which all real variables are constant. Given equation (A-1), constant real levels of consumption ( $c_t$ ), money holdings ( $m_t$ ) and deposits ( $d_t$ ) imply that  $\lambda_t$  is also constant in the steady state. Since the discount factor  $\beta$  is equal to  $1/(1+r)$ , replacing  $\beta$  in the first-order conditions (A-2) and (A-3), and using the Fisherian equality  $1+i_t = (1+r)(1+\pi_{t+1})$ , leads to equations (4) and (5) in the text.

## Appendix B: Government's Maximization Problem

To derive the optimal policies for the government, we maximize the indirect utility function of the representative household subject to the government's budget constraint. This implies maximizing the Langrangian

$$\mathcal{L} = U(c_t) + V(g_t) + \lambda_t \left\{ \bar{y} + (1-\rho)r_{t-1}d_{t-1} - \frac{\rho\pi_{t-1}}{1+\pi_{t-1}}d_{t-1} + \frac{m_{t-1}}{1+\pi_t} - \left[ 1 + \Phi\left(\frac{m_t}{c_t}\right) + \Psi\left(\frac{d_t}{c_t}\right) \right] c_t - m_t - d_t \right\} \quad (\text{A-4})$$

subject to the government's constraint  $g_t = h_t - h_{t-1}/(1+\pi_t) + b_t - (1+r)b_t$ . In the maximization process, this constraint may be directly substituted into the Langrangian for  $g_t$ . Note the the  $\lambda_t$  in (A-4) is the Langrange multiplier in the household's problem (see Appendix A) while the its associated constraint is obtained from substituting for  $\Pi_t$  in (3) using (7). The government's problem is to choose  $\{\pi_t, b_t\}_{t=0}^{\infty}$  to maximize the Langrangian in (A-4). The first-order condition with respect to  $b_t$  is obtained (with the envelope theorem) as

$$V'(g_t) - \beta V'(g_{t+1}) \cdot (1+r) = 0 \quad (\text{A-5})$$

Since the discount rate  $\beta = 1/(1+r)$ , equation (15) in the text follows. As for differentiating with respect to  $\pi_t$ , the corresponding first-order condition is

$$\begin{aligned}
-\lambda_t \frac{h_{t-1}}{(1+\pi_t)^2} + V'(g_t) \left[ \frac{h_{t-1}}{(1+\pi)^2} - \frac{\partial h_{t-1}/\partial \pi_t}{1+\pi_t} \right] \\
+ \beta^{-1} V'(g_{t-1}) \left[ \frac{\partial h_{t-1}}{\partial \pi_t} \right] = 0
\end{aligned} \tag{A-6}$$

From the previous first-order condition,  $V'(g_t) = V'(g_{t-1})$ . Thus (A-6) may be rewritten as

$$-\lambda_t \frac{h_{t-1}}{(1+\pi_t)^2} + V'(g_t) \left[ \frac{h_{t-1}}{(1+\pi)^2} - \frac{i_{t-1}}{1+\pi_t} \frac{\partial h_{t-1}}{\partial \pi_t} \right] = 0$$

Dividing both sides by  $h_{t-1}/(1+\pi_t)^2$ , and noting that  $\partial h_{t-1}/\partial \pi_t$  equals  $(1+r)\partial h_{t-1}/\partial i_{t-1}$ , yields (16) in the text.

## Appendix C: To derive $\partial F/\partial n$

Since  $F(i, b; n)$  is defined as the left-hand side of equation (16), it is written as

$$F(i, b; n) \equiv -\lambda_t + V'(g_t) [1 - (1+\pi)\epsilon] \tag{A-7}$$

From equation (A-2),  $\lambda_{t+1}/\lambda_t = (1+\pi_{t+1})(1+\Phi')/\beta$ , where  $\Phi'$  is simply a function of the ratio  $m/c$ . An increase in the number of banks  $n$ , keeping inflation  $\pi$  constant, has no effect on the right-hand side and therefore does not affect  $\lambda$ . Hence  $\partial \lambda_t/\partial n = 0$ . As for the second term  $V'(g_t)[1 - (1+i)\epsilon^{hi}]$ , its partial derivative with respect to  $n$  while keeping  $\pi$  constant is  $[1 - (1+i)\epsilon^{hi}]V''(g_t)\frac{\partial g}{\partial n} - V'(g_t)(1+i)(\partial \epsilon^{hi}/\partial n)$ . The first of these two terms is negative but the second is ambiguous due to the ambiguity of  $\partial \epsilon^{hi}/\partial n$ . Hence it follows that  $\partial F/\partial n$  is also ambiguous. It is important to note that  $\partial F/\partial n$  and  $\partial \epsilon^{hi}/\partial n$  are negatively correlated.

## Appendix D : To derive the signs of $\partial c/\partial n$ , $\partial d/\partial n$ , $\partial m/\partial n$ , $\partial h/\partial n$ and $\partial s/\partial n$ .

From equation (11), one may show that

$$[1 + \Phi + \Psi - \Phi' m/c - \Psi' d/c]dc = -\Psi' dd - \Psi' dm - \frac{\pi}{1+\pi} dm + \left[ r(1-\rho) - \frac{\pi\rho}{1+\pi} \right] dd \tag{A-8}$$

where  $d\pi$  is set to zero since  $\pi$  is held to be held constant here. Note that  $dm$  may be substituted with  $(m/c)dc$  from equation (4) since the nominal interest rate held constant (i.e.  $di = 0$ ). From equation (10), we may obtain another equation in  $dc$  and  $dd$  (also with  $d\pi = di = 0$ ):

$$\begin{aligned}
\frac{dd}{d} - \frac{dc}{c} &= -\frac{(\beta\rho r + \Psi')c}{d} \frac{dn}{\mathcal{A}} \quad \left( \text{where } \mathcal{A} = \frac{(n+1)\Psi''c + \Psi'''d}{c} \right) \\
&= \frac{\Psi''}{\mathcal{A}} \frac{dn}{n}
\end{aligned} \tag{A-9}$$

where the second equality follows from the first-order condition in (8) and where  $\mathcal{A}$  is positive from the inequality in (9). This last equation indicates that the percentage change in deposits exceed that for con-

sumption (i.e.  $dd/d \geq dc/c$ ), which implies that  $d/c$  rises as  $n$  increases. Substituting for  $dd$  in equation (A-8) using (A-9) leads to

$$\frac{\partial c}{\partial n} = \left[ r(1 - \rho) - \frac{\pi\rho}{1 + \pi} - \Psi' \right] \frac{\Psi''d}{n\mathcal{A}} / \left\{ 1 + \Phi + \Psi + \frac{\pi m}{(1 + \pi)c} - \left[ (1 - \rho)r - \frac{\pi\rho}{1 + \pi} \right] \frac{d}{c} \right\} > 0$$

where the partial derivative indicates that the vector  $(i, b)$  is held constant. In addition, since the deposit-consumption ratio  $d/c$  increases as  $n$  increases (from (A-9)), it follows that  $\partial d/\partial n$  is positive as well. In addition, because the money-consumption ratio  $m/c$  is constant if inflation  $\pi$  is, it is also true that  $\partial m/\partial n > 0$ . With high-powered money  $h$  being equal to  $m + \rho d$ , we also have  $\partial h/\partial n > 0$ . It therefore also follows that  $\partial s/\partial n = (\pi/(1 + \pi))\partial h/\partial n > 0$ .

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