Wealth Distribution, Moral Hazard, and International Borrowing

Sanjay Banerji[a] **and Ngo Van Long**[b]

date : 14 December 99. Filename: sanj9d.tex

.

Abstract

This paper demonstrates how changes in the weath distribution (while keeping the mean level of wealth constant) within a given country can lead to a switch in its international net indebtness position: a debtor country can become a creditor country. The model relies on three key factors: risk aversion, wealth distribution, and moral hazard. We show that, under certain assumptions, only agents in the middle range of the wealth distribution choose to be entrepreneurs, while very wealthy agents (i.e., those at the high end of the wealth distribution) and agents at the low end of the wealth distribution find it optimal to be lenders, and avoid being entrepreneurs. Only entrepreneurs invest in risky projects, and they borrow to partially finance their investments. Thus, two countries with the same population size and the same per capita wealth may behave differently from each other, even when individuals have identical preferences (i.e., identical utility functions), because of the difference in the distribution of wealth.

.

a) Faculty of Management, McGill University. Email: sanjayb@management.mcgill.ca. Telephone: (514) 398 4062

b) Department of Economics, McGill University, 855 Sherbrooke St west, Montreal, Quebec, H3A 2T7, Canada. Email: longn@cirano.umontreal.ca. Telephone: (514) 398 4844

1 Introduction

This paper demonstrates how changes in the weath distribution (while keeping the mean level of wealth constant) within a given country can lead to a switch in its international net indebtness position: a debtor country can become a creditor country. The model relies on three key factors: risk aversion, wealth distribution, and moral hazard. We show that, under certain assumptions, only agents in the middle range of the wealth distribution choose to be entrepreneurs, while very wealthy agents (i.e., those at the high end of the wealth distribution) and agents at the low end of the wealth distribution find it optimal to be lenders, and avoid being entrepreneurs. Only entrepreneurs invest in risky projects, and they borrow to partially finance their investments. Thus, two countries with the same population size and the same per capita wealth may behave differently from each other, even when individuals have identical preferences (i.e., identical utility functions), because of the difference in the distribution of wealth.

We also study the effect of the policy of bailing out firms that have financial difficulties. If this bailing out is financed by taxing successful firms, the net effect of the policy is an increase in the number of entrepreneurs at any given market rate of interest. To restore equilibrium, the rate of interest must rise. For a two-country world, this analysis implies that, ceteris paribus, the country that has such a bail-out policy will become a net borrower.

An interesting feature of the paper is the endogenous determination of the entrepreneur (as opposed to non-entrepreneur) status. Individuals with identical utility functions but different wealth endowments self-select to be (or not to be) entrepreneur. In our model, the outcome of any investment project depends on the effort level of the entrepreneur in charge of the project. This effort level is not observable by lenders or financial intermediaries. Contracts are designed to give entrepreneurs incentive to exert effort. While these contracts mitigate against opportunism by entrepreneurs, they cannot replicate the outcome that would be obtained under symmetric information.An important implication of our model is that a well-designed redistribution of wealth may stimulate risk-taking activities and result in a higher growth rate.

Our paper builds on, and extends, earlier contributions to the literature that connects income distribution to occupational choice in the context of capital market imperfections. It is useful to offer here a brief comparison of assumptions and results. Galor and Ziera (1993) consider a model with irreversible investment involving fixed costs. They assume that agents borrow to finance their investment in skill acquisition, and bequeath some of their wealth. The lending rate is higher than the borrowing rate, and the cost of borrowing is higher for borrowers with low initial wealth. The amounts these borrowers bequeath are also smaller. The authors show that agents whose wealth lies below a threshold level find it optimal to choose unskilled jobs, with low wages. As a result, their descendents will also choose to be unskilled workers due to their low level of inherited wealth. A similar model is studied by Banerji and Newman (1993) who assume that borrowers need to offer collaterals to lenders. Children of poor individuals do not inherit much, and therefore cannot offer sufficient collaterals to potential lenders. They are thus forced to choose not to be entrepreneurs. de Meza and Webb (1999) show that lack of information on the part of banks may lead to an over-provision of loans that encourages entry into entrepreneurship. If the associated incentive effects are strong, then there will be a positive relationship between wealth and entrepreneurial activities.

Aghion and Bolton (1997) formulate a model similar to ours, but they assume that all agents are risk-neutral. The probability of success of a project is dependent on the effort level chosen by the entrepreneur. In their model, individuals in the right-hand tail of the wealth distribution are entrepreneurs who do not borrow, while individuals in the middle section of the wealth distribution need to borrow to be entrepreneurs, because they do not have sufficient wealth to pay for the lumpy investment. This is in marked contrast to our result that, in some cases, very wealthy individuals may find it optimal not to be entrepreneur. The difference is partly due to the fact that we assume (a) risk aversion and (b) bankruptcy cost is non-zero, while Aghion and Bolton assume risk neutrality and strictly limited liability (zero payments to banks in the event of project failure).

Newman (1995) assumes risk aversion, and find that, under moral hazard, the poorer individuals tend to be risk-takers and wealthy individuals are risk-averters. This may be explained as follows. Optimal contract under moral hazard serve to resolve the tension between consumption- smoothing across states of nature, and efficient deployment of effort. We know that wealthier individuals tend to need less insurance at the margin. Optimal contracts prompt them to bear more risk at any given effort level. Individuals with greater wealth find it too costly (in terms of effort) to bear risks, as designed in the contracts.

All the models cited above share a common assumption: there is no transfer of resources from the entrepreneurs to the lenders in the event of project failure. This may be called the "strong limited liability" assumption. Our model allow a weaker version of limited liability: we assume that part of an entrepreneur's private savings must be used to pay debts in the event of project failure. In addition, we assume the existence of a small real resource cost in the settling of a bankruptcy case. These twin assumptions play an important role in our model. We find that the very poor do not take risks, because the marginal utilty of wealth in the event of bankruptcy is very high. We also find that the very rich do not become entrepreneurs, because the effort is very costly.

The plan of the paper is as follows. In section 2, we develop a model of a closed economy, and state the assumptions and notation. In Section 3, we characterize equilibrium loan contracts under moral hazard, and show how the equilibrium riskless rate of interest is determined. Section 4 deals with international borrowing.

2 The Basic Model: A Closed Economy With Moral Hazard

2.1 Assumptions and Notations

We begin by considering a model of a closed economy. There is a continuum of individuals in this economy. They differ in their initial wealth, denoted by w. The (cumulative) distribution of initial wealth is $F(w)$, and $f(w)$ denotes the corresponding density function. It is assumed that there is a closed interval $[w_L, w_H]$ over which $f(w)$ is strictly positive, and that $f(w) = 0$ for all w outside this interval. We assume $0 < w_L < w_H$. Then

$$\int_{w_L}^{w_H} f(w)dw = 1$$

and the per capita wealth is

$$\bar{w} = \int_{w_L}^{w_H} wf(w)dw.$$

There are only two periods. For simplicity, we assume that there is no consumption in period one[1]. In period one, each individual may choose

[1]This assumption is also made by Gertler and Rogoff (1990), but in their model, unlike ours, entrepreneurship is not endogenous. Aghion and Bolton (1997) also assume that consumption takes place after the realization of the investment.

to be an entrepreneur (in which case he would carry out a risky business activity, using part of his own wealth, plus some additional borrowing, to finance his investment project), or he may choose not to be an entrepreneur (in which case he would keep all his wealth in the form of a deposit in a financial institution, which offers him the safe market gross rate of return $r > 1$). This rate is endogenous, and its determination will become clear as we proceed. We assume that wealth, in its physical form, cannot be stored. It follows that all wealth must eventually be lent (via the financial institutions) to entrepreneurs, who use them as input in their investment projects. We adopt the standard assumption that financial institutions are risk neutral and perfectly competitive (their expected profits are zero).

Each entrepreneur can carry out only one investment project. Each project requires $\bar{k}$ units of wealth (one may think of this as a fixed cost, and that there is no variable cost). Each can turn out to be a success or a failure. In the case of success, the pay-off of the project is $a > \bar{k}$. In the case of failure, the pay-off is zero. The probability of success is denoted by $\pi(e)$, where e is the effort level of the entrepreneur, which is not observable by anyone, except the entrepreneur himself. For simplicity, we assume that e can take only two possible values, 0 or 1. We write $p = \pi(1) > q = \pi(0)$. This indicates that the model exhibits the moral hazard property: the entrepreneur, who is a net borrower, may have an incentive to work at an effort level that is lower than what would be efficient in a world of perfect information. In what follows, we assume that $p - q$ is sufficiently great, so that the equilibrium contracts have the following property: entrepreneurs are sufficiently rewarded for success that they have an incentive to set $e = 1$ even though e is not observable.We make the following assumption

ASSUMPTION A1:

$$\bar{k} > \bar{w}. \tag{1}$$

Since each entrepreneur uses $\bar{k}$ units of capital as input, this assumption implies that in equilibrium, the endogenous number of entrepreneurs, N, is less than the number of individuals in this economy, M.

Each individual has the following utility function

$$U = U(y, e) = v(y) - \mu e, \qquad \mu > 0$$

where y denotes his wealth in period 2, e is his effort level, $e \in \{0, 1\}$, and μ is a measure of effort cost. The function $v(y)$ is increasing and strictly concave, indicating risk aversion. We assume that

$$v'(0) = \infty.$$

This assumption implies that each individual will invest some wealth in the riskless asset, to avoid having zero wealth in period 2.

An entrepreneur can choose e to be 0 or 1. Non-entrepreneurs need not expend any effort, thus their e is 0. (Note that in what follows, we normalize by setting $\mu = 1$). We also make the following assumption, which implies that even in the absence of financial markets, it would pay for wealthy individuals to invest in the risky asset and exert full effort:

ASSUMPTION A2:

$$pv(a) + (1-p)v(0) - 1 > qv(a) + (1-q)v(0)$$

that is, in the absence of financial markets, if an individual with wealth equal to $\bar{k}$ does not invest in the risky project, his utility will be $v(0)$, (because by assumption wealth is not storable), which is less than the expected utility of investing, with effort $e = 0$, which in turn is less than the expected utility of investing, with effort $e = 1$.

Assumption A2 can be stated as

$$(p-q)\left[v(a) - v(0)\right] > 1. \tag{2}$$

While the outcome of any given project is uncertain, we assume the probability of success of any given project is independent of those of other projects, and that the number of projects is large enough so that the law of large number applies. Thus, for the economy as a whole, if all entrepreneurs choose $e = 1$, aggregate output is paN where N is the measure of the set of individuals who, in equilibrium, choose to be entrepreneur. (Recall that the population is M, and assumption A1 ensures that $N \leq M$).We also assume that financial institutions that try to recover part of their loans to entrepreneurs who turn out to fail must incur some cost of settling the bankruptcy of these entrepreneurs. In equilibrium, since all physical wealth must be invested, we have

$$N\bar{k} = M\bar{w} \tag{3}$$

For the economy as a whole, the average gross rate of return on capital is

$$\rho = \frac{paN - (1-p)BN}{N\bar{k}} = \frac{pa - B}{\bar{k}} \tag{4}$$

where B is the average bankruptcy cost per entrepreneur, which depends (as will become clear later) on how much each entrepreneur borrows. It follows that

$$pa > \rho \bar{k}. \tag{5}$$

Let r denote equilibrium riskless rate of interest that financial intermediary firms offer to depositors. Clearly, in equilibrium, ρ is greater than r because the equilibrium contract between a financial intermediary and a representative entrepreneur with wealth w must compensate for his risk bearing and his effort e. In view of this observation and (5), we expect that the equilibrium interest rate r is such that

$$r\bar{k} < pa + (1-p)0 \tag{6}$$

2.2 The First Benchmark Case: Observable Effort and No Bankruptcy Cost

If the effort level of each entrepreneur is observable, and if there is no bankrupcy cost, then we are in the first-best world. Since there is a continuum of individuals and a continuum of investment projects, in the aggregate there is no risk, and a perfect insurance market implies that all individuals are perfectly insured, given that their function $v(y)$ is strictly concave. If the effort cost parameter μ is sufficiently small, contracts will specify that entrepreneurs must exercise full effort ($e = 1$), and get compensated for it.

The proof is straightforward. Assume it is optimal to choose full effort. The appropriate maximization problem is to choose, for any given w, the amount $\bar{k} - k$ that the entrepreneur with with w is to borrow, and the payments R and R_2 that he must make in the events of success and failure respectively, so as to maximize his expected utility subject to zero expected profits

$$\max pv\left[(w-k)r + a - R\right] + (1-p)v\left[(w-k)r - R_2\right] - 1$$

subject to

$$pR + (1-p)R_2 = r(\bar{k} - k)$$

The solution yields constant utility across state of nature

$$(w-k)r + a - R = (w-k)r - R_2$$

$$R = (1-p)a + r(\bar{k} - k)$$

and

$$R_2 = R - a$$

A more interesting case is full information, but in the presence of bankruptcy cost. We will look at this case after analyzing the moral hazard case.

2.3 The Moral Hazard Case: Unobservable Effort

Now we turn to the case of unobservable effort levels. We assume that each entrepreneur's wealth is *known* to the financial intermediaries. This, and the assumption that individuals have the same utility function and identical ability means that there is no adverse selection problem: no one can lie about his wealth or his utility function. The only problem is moral hazard: if how much an entrepreneur must pay back to the financial intermediaries is independent of his effort level, then he may have an incentive to exert no effort. Contracts must therefore be designed to provide sufficient incentive for enterpreneurs to choose $e = 1$. (This is of course based on the assumption that $p - q$ is sufficiently great to justify the choice $e = 1$.)

We now describe a contract for an entrepreneur with wealth level w. This contract says that "if your wealth is w and you contribute an amount $k \leq \min\{\bar{k}, w\}$ as your "equity" in your investment project (so that your borrowing from your financial institution is $\bar{k} - k \geq 0$, and you deposit your remaining $w - k$ at a financial institution that gives you the safe rate of return r), then you must pay back to your lender (the financial institution, or FI for short) an amount which depends on the outcome of your project. If the outcome is "success", your investment yields the gross return a, and you must pay an amount R to your FI; if your outcome is "failure" (the investment yields a gross return of zero), then, with your period two wealth $(w-k)r$, you must pay back to the FI an amount $\alpha(\bar{k} - k) > 0$ where α is a small positive number". (In what follows, we assume $\alpha < r$.)

It follows that if the entrepreneur exerts effort (i.e., $e = 1$), then his expected utility is

$$pv\left[(w-k)r + a - R\right] + (1-p)v\left[(w-k)r - \alpha(\bar{k} - k)\right] - 1 \equiv F(w, k, R, 1)$$

and if he does not exert effort (i.e., $e = 0$), then his expected utility is

$$qv\left[(w-k)r + a - R\right] + (1-q)v\left[(w-k)r - \alpha(\bar{k} - k)\right] \equiv F(w, k, R, 0)$$

We assume also that when a bankrupt entrepreneur pays the amount $\alpha(\bar{k}-k) > 0$ to his FI, the latter only gets a fraction β of it. In other words, the FI incurs a real cost $(1-\beta)\alpha(\bar{k}-k)$ in collecting $\alpha(\bar{k}-k)$ from the failed entrepreneur.

Thus an FI that lends the amount $\bar{k}-k$ to an entrepreneur who does exert effort can expect to get

$$pR + (1-p)\alpha\beta(\bar{k}-k)$$

On the other hand, the FI takes r as given, and must pay the amount $r(\bar{k}-k)$ to depositors. In equilibrium, we have the following zero expected profit condition (if $e = 1$):

$$pR + (1-p)\alpha\beta(\bar{k}-k) = r(\bar{k}-k) \quad (7)$$

Competition among the FIs imply that, for given r, the FIs will offer to any entrepreneur with wealth w a contract that maximizes his expected utility, subject to the zero profit condition. We also assume that the contract provides enough incentive for the entrepreneur to exert effort ($e = 1$). More formally, FI j will offer to the entrepreneur with wealth w a pair of numbers (R, k) that maximizes $F(w, k, R, 1)$ subject to the *incentive compatibility constraint*

$$F(w, k, R, 1) \geq F(w, k, R, 0) \quad (8)$$

and the zero profit constraint (7).

3 Properties of the Equilibrium Contracts

We now turn to a fuller characterization of equilibrium contracts. We proceed as follows. First, we take the interest rate r as given, and show how the incentive compatibility constraint and the zero profit condition determine the contract for entrepreneurs at each wealth level. Then we show how the the interest rate r is determined endogenously.

3.1 Equilibrium loan contracts for entrepreneurs, given the interest rate on the safe asset

In equilibrium, profit will be zero, and the incentive compatibility constraint binds for each entrepreneur. The participation constraint also binds, but only in the sense that $\phi(w)$ will be "bid up" so that it is the equilibrium expected utility level of the entrepreneur with wealth w. It follows that, for given r, conditions (7) and (8) determine the equilibrium contract for entrepreneurs with wealth w. The following lemma characterizes the equilibrium contract, under the assumption that α is small:

LEMMA 1: (Equilibrium contract) In equilibrium, the entrepreneur with wealth w will be lent the amount $z(w)$ and will be asked to pay the amount $R(w)$ in the event of success, and $\alpha z(w)$ in the event of failure, where $z(w) = \bar{k} - k(w)$, and the pair $(R(w), k(w))$ satisfy the following equations

$$R = \frac{(\bar{k} - k)}{p}\left[r - (1-p)\alpha\beta\right] \text{ for } k \leq \bar{k} \tag{9}$$

and

$$v\left[(w-k)r + a - R\right] - v\left[(w-k)r - \alpha(\bar{k} - k)\right] = \frac{1}{p-q} \tag{10}$$

Proof: See the Appendix.

REMARK 1: Equation (9) implies that the repayment to the FI (in the event of success) per dollar borrowed is a constant, independent of the amount borrowed:

$$r_s \equiv \frac{R}{\bar{k} - k} = \frac{1}{p}\left[r - (1-p)\alpha\beta\right].$$

(Note that the repayment received by the FI, in the event of failure, is $\alpha\beta$ per dollar borrowed.)

REMARK 2: Equation (9) can be represented by the downward sloping curve KK in the (k, R) space, with the vertical intercept$[r - (1-p)\alpha\beta]\ \bar{k}/p$. If $p - q$ is sufficiently close to one, and α is sufficiently small, then the intercept of the second curve (defined by (10) and denoted as the curve VV) is below $[r - (1-p)\alpha\beta]\ \bar{k}/p$ and hence the two curves will have an intersection $(k(w), R(w))$ with $k(w) > 0$ and $R(w) > 0$, if w is not too small.

LEMMA 2: Entrepreneurs with greater w will borrow less (i.e. $\bar{k} - k(w)$ decreases with wealth) and hence *invest more (put more equity) in the risky project*, i.e. $k'(w) > 0$, given that contracts must satisfy (9) and (10).

Proof: Note that the curve KK is independent of w. For any given k, an increase in w will shift the curve VV down, resulting in an intersection to the right of the former intersection. Hence k^* increases, and R^* decreases. This means the individual borrows less when his wealth increases.

REMARK 3: One of the standard text-book results is that if the payoff per dollar invested a risky asset in each state of nature is given (i.e., *independent* of the amount invested in the risky asset), then the amount an individual invests in the risky asset is an increasing function of his wealth if and only if his *absolute risk aversion is a decreasing function* of wealth. Our result in Lemma 2 is different, because the pay-off per dollar of equity in the event of success is

$$\delta_s = \frac{a-R}{k} = \frac{a}{k} - \frac{1}{pk}\left[r-(1-p)\alpha\beta\right](\bar{k}-k)$$

which is *dependent* on k (*decreasing* in k). (And similarly, the pay-off per dollar of equity in the event of failure increases in k.) Lemma 1 states that, with a concave utility function, but independently of whether *absolute risk aversion is a decreasing or increasing function*, entrepreneurs with greater wealth will contribute more equity in the project.

LEMMA 3: Given that contracts must satisfy (9) and (10), entrepreneurs with greater w will invest MORE in the riskless asset, (i.e., $w-k^*(w)$ increases with w), and at the same time putting more equity in the risky project, *if and only if* the following condition holds

$$v'(s)(r-\alpha\beta) > \left[v'(u)-v'(s)\beta\right]\alpha p \tag{11}$$

where u is the wealth in the failure state,

$$u \equiv u(w) \equiv (w-k(w))r - \alpha(\bar{k}-k(w)) \tag{12}$$

and s is the wealth in the success state,

$$s \equiv s(w) \equiv (w-k(w))r + a - R(w) \tag{13}$$

Proof: From (9) and (10), we get

$$\frac{dk}{dw} = \frac{\left[v'(u)-v'(s)\right]pr}{\left[v'(u)-v'(s)\right]pr + v'(s)(r-\alpha\beta) - \left[v'(u)-v'(s)\beta\right]\alpha p} \tag{14}$$

which is less than 1 if and only if (11) holds.

REMARK: Condition (11) holds if α is small, which is assumed here.

3.2 An example

Let

$$v(y) = \ln y$$

then we obtain from (10)

$$\ln\left[\frac{(w-k)r + a - R}{(w-k)r - \alpha(\bar{k}-k)}\right] = \frac{1}{p-q}$$

hence

$$wr - kr + a - (\bar{k}-k)r_s = \left[wr - kr - \alpha(\bar{k}-k)\right] e^{1/(p-q)}$$

Thus

$$\left[r_s + (r-\alpha)e^{1/(p-q)} - r\right] k = \left[r_s - \alpha e^{1/(p-q)}\right] \bar{k} - a + rw \left[e^{1/(p-q)} - 1\right] \quad (15)$$

i.e.,

$$k(w) = \frac{\left[r_s - \alpha e^{1/(p-q)}\right] \bar{k} - a + rw\left[e^{1/(p-q)} - 1\right]}{\left[r_s + (r-\alpha)e^{1/(p-q)} - r\right]} \quad (16)$$

which is positive if w is not too small, and

$$\frac{dk}{dw} = \frac{\left[e^{1/(p-q)} - 1\right] r}{\left[e^{1/(p-q)} - 1\right] r + \left[r_s - \alpha e^{1/(p-q)}\right]}$$

which is positive and less than 1 provided α is small. This is consistent with Lemmas 2 and 3.

3.3 To be or not to be an entrepreneur

So far, we have characterized contracts that the FIs offer to potential entrepreneurs with wealth w, which would make him choose $e = 1$, *assuming* that he does want to be an entrepreneur. But depending on his wealth, an individual may find that the utility of being a lender, $v(rw)$, may exceed the expected utility of being an entrepreneur, i.e., it is possible that, for some w, we have

$$v(rw) > EU \equiv pv(s) + (1-p)v(u) - 1 \quad (17)$$

Clearly, if w is close to zero, condition (17) will be satisfied. Thus the curve $v(rw)$ lies above the curve EU when w is small. At some $\widehat{w} > 0$, the curve EU cuts the curve $v(rw)$ from below (here, some assumptions may be needed to ensure that such a crossing exists.) The question is whether there exists some value $\widetilde{w} > \widehat{w}$ such that the curve $v(rw)$ again overtakes the curve EU. The answer turns out to depends on the convexity of the following function

$$\psi(y) \equiv \frac{1}{v'(y)}$$

which measures the marginal rate of substitution between effort and income:

$$MRS_{ey} = -\frac{U_e}{U_y} = \frac{1}{v'(y)}$$

LEMMA 5:
If $\psi(y)$ is concave, then

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} \leq \frac{1}{v'(ps + (1-p)u)} \tag{18}$$

and if $\psi(y)$ is convex, then

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} \geq \frac{1}{v'(ps + (1-p)u)} \geq \frac{1}{v'(rw)} \tag{19}$$

Proof: The first inequality in (19) follows from Jensen's inequality.The second inequality in(19) holds if and only if

$$rw < ps + (1-p)u \tag{20}$$

Now $ps + (1-p)u = rw - rk(w) + pa - (\bar{k} - k(w))[r - (1-p)\alpha(\beta - 1)] \simeq rw + pa - r\bar{k} > rw$ because of (6).

LEMMA 6: If $\psi(y)$ is convex, then the curve EU may cut the curve $v(rw)$ from below at some value $\widehat{w} > 0$, and then from above, at some value $\widetilde{w} > \widehat{w}$.

Proof: see the appendix.

REMARK: $\psi(r)$ is convex if and only if the coefficient of prudence is smaller than twice the coefficient of absolute risk aversion:

$$-\frac{2v''}{v'} \geq -\frac{v'''}{v''} \tag{21}$$

(the right-hand side of (21) is called the coefficient of prudence by Kimball, 1990). Note that decreasing absolute risk aversion holds iff

$$-\frac{v'''}{v''} > -\frac{v''}{v'}$$

which is not inconsistent with (21).

Proposition 1: if the coefficient of prudence is smaller than twice the coefficient of absolute risk aversion, then it is possible that, given r, there are two values $\widehat{w}(r) < \widetilde{w}(r)$ such that only individuals whose wealth lies between them will choose to be entrepreneurs. Individuals with wealth exceeding $\widetilde{w}(r)$, and those with wealth below $\widehat{w}(r)$ will choose to invest in the safe asset (i.e., lend to financial intermediaries).

REMARK: Proposition 1 may be explained as follows: given that the marginal rate of substitution between effort and incone is convex in y, individuals with very low wealth do not want to become entrepreneur because they are not willing to take risks, and individuals who are very wealthy do not want to become entrepreneur because they do not want to exert effort.

We now show how the two critical values $\widehat{w}(r)$ and $\widetilde{w}(r)$ change when r increases.

Proposition 2: $\widehat{w}'(r) > 0$ and $\widetilde{w}'(r) < 0$ if

$$\frac{\bar{k} - w_H}{w_H - k(w_H)} > \gamma(1-p)$$

where $\gamma \equiv \{(r-\beta\alpha)/(r-\alpha)\} - 1 \geq 0$

Proof: see the appendix.

EXAMPLE (CONTINUED)

The critical values of w which makes an individual with wealth w indifferent between being or not being an entrepreneur is

$$\ln(wr) = p \ln\left[(w - k(w))r + a - r_s(\bar{k} - k(w))\right] +$$

$$(1-p)\ln\left[(w-k(w))r - \alpha(\bar{k} - k(w))\right] - 1 \tag{22}$$

where $k(w)$ is linear in w, and is given by (16) above. This equation can have several roots. For example, if $p = 0.5$ then (22) may be written as (using the fact that $\ln e = 1$)

$$(ewr)^2 = \left[(w - k(w))r + a - r_s(\bar{k} - k(w))\right]\left[(w - k(w))r - \alpha(\bar{k} - k(w))\right] \quad (23)$$

This is a quadratic equation in w, and under suitable assumptions will have two positive real roots.

3.4 Endogenous determination of the interest rate on the safe asset

So far, we have taken the interest rate r as given. Now we turn to its determination. This is given by the condition that the the interest rate must equate the aggregate lending by non-entrepreneurs (to the financial institutions) to the aggregate borrowing by entrepreneurs (from the financial institutions). The former is given by

$$L(r) = \int_{w_L}^{\widehat{w}(r)} wf(w)dw + \int_{\widetilde{w}(r)}^{w_H} wf(w)dw$$

and the latter is

$$B(r) = \int_{\widehat{w}(r)}^{\widetilde{w}(r)} (\bar{k} - w)f(w)dw$$

The excess demand (function) for fund is

$$D(r) = B(r) - L(r) = \bar{k}\int_{\widehat{w}(r)}^{\widetilde{w}(r)} f(w)dw - \int_{w_L}^{w_H} wf(w)dw = \bar{k}\int_{\widehat{w}(r)}^{\widetilde{w}(r)} f(w)dw - \bar{w}$$

Clearly, if r is very high, then $D(r) < 0$, and if if $r = 0$, then $D(r) < 0$ (recall that $\bar{k} > \bar{w}$). By continuity, there exists a value r^* such that $D(r^*) = 0$. Furthermore, such r^* is unique, because $D'(r) < 0$. Thus we obtain the following result:

Proposition 3: There exists a unique $r^* > 0$ such that the loan market clears. Individuals with wealth exceeding $\widetilde{w}(r^*)$ lend all their wealth to the financial institutions, at the safe interest rate r^*, and so do individuals with wealth below $\widehat{w}(r^*)$. Individuals whose wealth lies between these two critical values will be entrepreneurs, and they invest a fraction of their wealth in the safe asset, the remaining fraction being used as equity capital, which is always less than the total amount of capital invested in each risky project. The equilibrium number of entrepreneurs is

$$N = \frac{M\bar{w}}{\bar{k}} = \int_{\widehat{w}(r)}^{\widetilde{w}(r)} f(w)dw.$$

4 Comparison with the Second Benchmark Case: Full Information, with Bankruptcy Cost

Now we return to the full information case, but we add the assumptions that (i) in the event of failure, the entrepreneur must pay back to the financial intermediary (FI) a fraction of the amount he borrows, $\alpha(\bar{k} - k)$, and (ii) due to bankruptcy cost, the FI only obtains a fraction $\beta < 1$ of this amount. Then the appropriate maximization problem is to choose, for any given w, the pair (R, k) that maximizes

$$pv\left[(w-k)r + a - R\right] + (1-p)v\left[(w-k)r - \alpha(\bar{k} - k)\right] - 1$$

subject to

$$pR + (1-p)\alpha\beta(\bar{k} - k) = r(\bar{k} - k)$$

After inserting the zero profit condition into the objective function, we get the first order condition

$$\frac{v'(u)}{v'(s)} = \frac{r - \alpha\beta}{r - \alpha} > 1 \tag{24}$$

where

$$s \equiv (w-k)r + a - R$$

and

$$u \equiv (w-k)r$$

(Note that α is exogenously specified).Thus, with the bankruptcy cost factor $\beta < 1$, marginal utilities are not equalized across states of nature.

In this case, the condition for an individual to prefer being an entrepreneur to being a pure lender is

$$EU(w) \equiv pv(s) + (1-p)v(u) - 1 > v(wr)$$

Again, we must compare the slope of the EU curve with the slope of the right-hand side (with respect to w). The slope of the EU curve is

$$pv'(s) + (1-p)v'(u) = [1+\kappa]\, v'(s)r$$

where

$$\kappa \equiv \frac{\alpha(1-\beta)(1-p)}{r-\alpha}$$

If, given r, at any w^* such that $EU(w^*) = v(w^*r)$, the following condition holds

$$\frac{v'(s)}{v'(w^*r)} > \frac{1}{1+\kappa} \tag{25}$$

then we can conclude that all individuals with wealth below w^* will be lenders and all other individuals will be entrepreneurs. Given our assumption, condition (25) is likely to be met.

The above analysis shows that under full information with bankruptcy cost (and the assumption that failed entrepreneurs must pay $\alpha(\bar{k}-k)$), the pattern of entrepreneurial choice is quite different from that under moral hazard: there is no "re-switching."

5 International Borrowing Under the Moral Hazard Case

Now consider two countries with the same population size and the same level of per capita wealth, but different distributions of wealth. We will look at their autarkic equilibria, and examine the incentive for international borrowing. Let us assume there are two countries, A and B. The density functions of the two wealth distributions are respectively $f_A(w;\theta_A)$ and $f_B(w;\theta_B)$, where θ_A and θ_B are shift parameters. Now consider the autarkic equilibrium of country A, and let r_A be the autarkic interest rate. Consider the following thought experiment: Suppose the distribution $f_A(.)$ undergoes a change: θ_A

now becomes θ_A^* , which makes $f_A(w;\theta_A^*) < f_A(w;\theta_A)$ for all w in the interval $[\widehat{w}(r_A), \widetilde{w}(r_A)]$. Then clearly at the value r_A , the number of willing entrepreneurs will be less than N. To restore equilibrium, r must be lower.

It follows from the above reasoning that if the density function $f_B(w;\theta_B)$ differs from $f_A(w;\theta_A)$ in that

$$f_B(w;\theta_B) < f_A(w;\theta_A) \text{ for all } w \in [\widehat{w}(r_A), \widetilde{w}(r_A)]$$

then country B will have a higher autarkic gross rate of return: $r_B > r_A$Under these conditions, the opening of world financial markets will result in capital flow from A to B. We obtain the following proposition:

Proposition 4: Two countries with identical per capita wealth and identical individual utility function may engage in intertemporal trade if the distributions of wealth are different.

6 Bailouts

In this section, we set up a framework for studying the implications of government policies that seek to help bankrupt entrepreneurs. Suppose the government taxes successful entrepreneurs, collecting from them amount T each, and pay the unsuccessful ones an amount η. Balanced budget requires that

$$pT \int_{\widehat{w}(r)}^{\widetilde{w}(r)} f(w)dw = \eta(1-p) \int_{\widehat{w}(r)}^{\widetilde{w}(r)} f(w)dt \tag{26}$$

where $[\widehat{w}(r), \widetilde{w}(r)]$ are solutions of the equation

$$v(rw) = pv\left[(w-k)r + (a-R) - T\right] + (1-p)v\left[(w-k)r - \alpha(\bar{k}-k) + \eta\right] - 1$$

Does this bailout policy result in a lower or higher equilibrium interest rate? What is its implication on the country's net borrowing?

First, note that (26) implies that

$$\frac{dT}{d\eta} = \frac{1-p}{p}$$

The contract between the financial intermediary (FI) and the entrepreneur with wealth w must now satisfy

$$(p-q)\left[v(s) - v(u)\right] = 1 \tag{27}$$

and

$$pR = \left(\bar{k} - k\right)[r - (1-p)\alpha\beta] \tag{28}$$

where now

$$s \equiv (w-k)r + (a-R) - T$$

and

$$u \equiv (w-k)r - \alpha(\bar{k} - k) + \eta$$

From (27) and (28), we have

$$\frac{\partial k}{\partial T} = \frac{pv'(s)}{pv'(u)(r-\alpha) + (1-p)v'(s)(r-\alpha\beta)}$$

and

$$\frac{\partial k}{\partial \eta} = \frac{pv'(u)}{pv'(u)(r-\alpha) + (1-p)v'(s)(r-\alpha\beta)}$$

Hence

$$\frac{dk}{d\eta} = \frac{\partial k}{\partial T}\frac{dT}{d\eta} + \frac{\partial k}{\partial \eta}$$

$$= \frac{(1-p)v'(s) + pv'(u)}{pv'(u)(r-\alpha) + (1-p)v'(s)(r-\alpha\beta)}$$

To find the respose of the intersection points $\widetilde{w}(r)$ and $\widehat{w}(r)$ with respect to η, we use the equation of indifference, which is now

$$\frac{q}{p-q} + v\left[(w-k^*)r - \alpha(\bar{k} - k^*) + \eta\right] = v(wr) \tag{29}$$

where $k^* = k^*(w, T, \eta(T))$. From (29) we get

$$\frac{dw}{d\eta} = \left(\frac{1}{\Delta}\right)\frac{(1-p)v'(u)v'(s)\alpha(\beta-1)}{(1-p)v'(s)(r-\alpha\beta) + pv'(u)(r-\alpha)} \tag{30}$$

where

$$\Delta = v'(s)\left[r - (r-\alpha)\frac{\partial k}{\partial w}\right] - v'(NE)r \tag{31}$$

which is positive at $\widehat{w}(r)$ and negative at $\widetilde{w}(r)$. Hence, for a given r, $\widehat{w}(r)$ increases and $\widetilde{w}(r)$ decreases when there is an increase in η. This implies that the number of entrepreneurs increases, leading to an increases in the demand for fund. To restore equilibrium, the rate of interest r must increase.

Proposition 5: Bail-out policies of the type described above will lead to an increase in the equilibrium rate of interest. In a two-country world, this means that the country with such bail-out policies will become the debtor country.

Remark: The role of the bankuptcy cost factor $\beta < 1$ is crucial here. If $\beta = 1$, then bail-out has no effect on the equilibrium interest rate.

7 Concluding remarks

We have set up a model to show that a country's wealth distribution influences the occupational choice of individuals: to be or not to be an entrepreneur. We have also showed that countries that have the same level of per capita wealth and the same individual utility function may engage in mutual intertemporal trade, as long as they have different wealth distributions. The key element in our model is the tension between consumption smoothing across states of nature on the one hand, and contract design to overcome moral hazard on the other hand.

There are several directions of generalization, which we wish to pursue in our future work. An obvious extension is the process of capital accumulation. One would then be able to see how moral hazard and initial wealth distributions influence growth rates, and to determine conditions under which cycles may occur. Taxation policies, including transfers, may be studied in the context of moral hazard and endogenous choice of occupation.

APPENDIX

PROOF OF LEMMA 1

Consider the problem

$$\max_{R,k} F(R,k,w,1) = pv((w-k)r + a - R) + (1-p)v((w-k)r - \alpha(\bar{k}-k)) - 1$$

subject to

$$v((w-k)r + a - R) - v((w-k)r - \alpha(\bar{k}-k)) \geq \frac{1}{p-q} \tag{32}$$

and

$$pR - (\bar{k}-k)\left[r - (1-p)\alpha\beta\right] \geq 0 \tag{33}$$

In the (k, R) space, the feasible set is the intersection of area above the line

$$R = \frac{(\bar{k}-k)}{p}\left[r - (1-p)\alpha\beta\right] \tag{34}$$

with the area below the (positively sloped) curve defined by (32). Now it is easy to verify that the objective function $F(R,k,w,1)$ is strictly concave and *decreasing* in (k, R). This means that the iso-expected utility curves are concave in the (k, R) space, with negative slope given by

$$\frac{\partial R}{\partial k} = \frac{-F_k}{F_R}$$

which can be shown to be steeper than the slope of the line (34) if α is small. It follows that the maximum occurs at the point where both constraints hold with equality.

PROOF OF LEMMA 5

We find conditions for the slope of EU to be greater or smaller than the slope of $v(NE)$ at their intersection.

Proof:

Recall that from LEMMA 2 and 3, k is an increasing function of w, and

$$\frac{\partial k}{\partial w} = \frac{pr\left[v'(u) - v'(s)\right]}{pv'(u)(r-\alpha) + (1-p)v'(s)(r-\beta\alpha)} \tag{35}$$

Now at any intersection between EU and $v(rw)$, we must have

$$pv(s) + (1-p)v(u) = v(rw)$$

This and (10) give

$$\frac{q}{p-q} + v(u) = v(rw) \equiv v(NE) \tag{36}$$

The slope (with respect to w) of the LHS of (36) is greater than the slope of the RHS of (36) iff

$$G \equiv v'(u)\left[r - (r-\alpha)\frac{\partial k}{\partial w}\right] - rv'(NE) > 0 \tag{37}$$

which is equivalent to

$$r - (r-\alpha)\frac{\partial k}{\partial w} > \frac{rv'(NE)}{v'(u)}$$

i.e.,

$$\left[1 - \frac{v'(NE)}{v'(u)}\right] r > (r-\alpha)\frac{\partial k}{\partial w}$$

i.e.,

$$\left[\frac{v'(u) - v'(NE)}{v'(u)}\right] > (r-\alpha)\frac{p\left[v'(u) - v'(s)\right]}{pv'(u)(r-\alpha) + (1-p)v'(s)(r-\beta\alpha)}$$

i.e.,

$$\left[\frac{v'(u) - v'(NE)}{v'(u)}\right] > \frac{p\left[v'(u) - v'(s)\right]}{pv'(u) + (1-p)v'(s)\{(r-\beta\alpha)/(r-\alpha)\}} \tag{38}$$

Let $1+\gamma = \{(r-\beta\alpha)/(r-\alpha)\} \geq 1$. If (38) holds, then

$$\left[\frac{v'(u) - v'(NE)}{v'(u)}\right] > \frac{p\left[v'(u) - v'(s)\right]}{pv'(u) + (1-p)v'(s)(1+\gamma)} \tag{39}$$

Now (39) holds iff

$$\left[\frac{pv'(u) + (1-p)v'(s)(1+\gamma)}{v'(u)v'(s)}\right]\left[v'(u) - v'(NE)\right] > \frac{p\left[v'(u) - v'(s)\right]}{v'(s)} \tag{40}$$

Let

$$Z = \frac{pv'(u) + (1-p)v'(s)}{v'(u)v'(s)} \tag{41}$$

.Then (40) is

$$1 + \gamma(1-p)\left[1 - \frac{v'(NE)}{v'(u)}\right] > zv'(NE) \tag{42}$$

So we require, for EU to cut $v(NE)$ from below, that

$$Zv'(NE) - \gamma(1-p)\left[1 - \frac{v'(NE)}{v'(u)}\right] < 1 \tag{43}$$

A sufficient condition for the slope of EU to be greater than the slope of $v(NE)$ is $zv'(NE) < 1$. Now $zv'(NE) < 1$ if and only if

$$\frac{p}{v'(s)} + \frac{1-p}{v'(u)} < \frac{1}{v'(NE)} \tag{44}$$

i.e. if $\psi(w)$ is strictly concave. It follows that (a) if $\psi(w)$ is strictly concave, then the EU curve cannot cut the $v(NE)$ curve from above, and (b) if $\psi(w)$ is convex, then the EU curve may cut the $v(NE)$ curve from above, at some $\widetilde{w} > \widehat{w}$.

PROOF OF PROPOSITION 2

From (9) and (10), we have

$$\frac{\partial k}{\partial w} = \frac{1}{\Delta}\left[v'(u) - v'(s)\right] pr > 0$$

and

$$\frac{\partial k}{\partial r} = \frac{1}{\Delta}\left[\{v'(u) - v'(s)\}(w-k)p + v'(s)(\bar{k} - k)\right]$$

where

$$\Delta = pv'(u)(r-\alpha) + (1-p)v'(s)(r-\alpha\beta) > 0$$

Differentiating the condition for indifference (equation (36)) with respect to r, we get

$$\frac{dw}{dr} = \frac{1}{G}\left[wv'(NE) - v'(u)(w-k) + (r-\alpha)v'(u)\frac{\partial k}{\partial r}\right] \tag{45}$$

where G, defined in (37) above, is positive at $\widehat{w}(r)$ and negative at $\widetilde{w}(r)$. The numerator in(45) must now be signed. It can be written as

$$wv'(NE) + v'(u)\left[\frac{p(w-k)\{v'(u) - v''(s)\} + v'(s)(\bar{k} - k)}{pv'(u) + (1-p)v'(s)(1+\gamma)} - (w-k)\right]$$

The term inside the square brackets can be written as

$$v'(s)(w-k)\left[\frac{\bar{k}-k}{w-k}-1-\gamma(1-p)\right]$$

or

$$v'(s)(w-k)\left[\frac{\bar{k}-w}{w-k}-\gamma(1-p)\right]$$

Now, since $w-k(w)$ is increasing in w, we have

$$\frac{\bar{k}-w}{w-k(w)} > \frac{\bar{k}-w}{w_H-k(w_H)} > \frac{\bar{k}-w_H}{w_H-k(w_H)}$$

which is positive if $\bar{k} > w_H$ and γ is sufficiently small.

References

Aghion, P. and P. Bolton, 1997, A Theory of Trickle-Down Growth and Development, Review of Economic Studies 64, pp151-172.

Banerji, A. and A. Newman, 1993, Occupational Choice and the Process of Economic Development, Journal of Political Economy 101, pp. 274-98

de Meza, D. and D. Webb, 1997, Wealth, Enterprise and Credit Policy, Economic Journal 109, pp 133-65.

Galor, O. and J. Ziera, 1993, Income Distribution and Macroeconomics, Review of Economic Studies 60, pp. 35-52.,

Gertler, Mark and Kenneth Rogoff, 1990, North-South Lending and Endogenous Domestic Capital Market Inefficiency, Journal of Monetary Economics 26, pp 245-266.

Hartwick, J.M. , 1999, Insuring and $u'''(y)$, Economics Letters 65, 205-212.

Kimball, M. S., Precautionary saving in the small and in the large, Econometrica 58, 53-73.

Newman, A., 1995, Risk Bearing and Knightian Entrepreneurship, typescript.

Rogerson, W., 1985, Repeated Moral Hazard, Econometrica 62, pp 459-465.