The distribution of human capital, market structure and investment in technology

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Abstract: In a Schumpeterian growth model, the distribution of human capital affects the prices the capital good innovators may charge for their innovations and the number of such firms that may obtain market shares at any one time. Under a more dispersed distribution, the innovator may only charge a lower price to cover the entire market. The payoffs to innovation declines, causing investment to fall. When the dispersion has reached a critical level, the innovator will no longer finds it optimal to price out the incumbents, turning the capital good market into a natural oligopoly with firms selling different grades co-existing at any one time. This is despite of the fact that for efficiency, only the highest grade should be produced. Any further increases in dispersion now raises the payoffs to innovation, causing investment to increase.

Keywords: R&D, inequality, human capital, market structure, gradual obsolescence

JEL classification: L12, L13, L16, O30.

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1. INTRODUCTION

In affecting the returns to innovation, market structure plays a crucial role in Schumpeterian growth models and has rightfully attracted much attention.[1] Typically, the market structures in growth models, where the term usually refers to the markup a monopolistically competitive firm may charge,[2] are determined by some exogenous parameters on technologies. While technology is certainly an important factor, there are other important determinants too. In this paper, I explore how the distribution of human capital may also affect the structure of and the intensity of competition in the market for capital goods.

The analysis is based on the celebrated Schumpeterian growth model of Aghion and Howitt (1992).[3] The point of departure is that with the usual human capital and physical capital complementarity that high skill workers can make better use of high quality capital goods, firms that employ high skill workers will have greater demand for the quality of capital goods over firms hiring low skill workers. In turn, the heterogeneity in the willingness to pay for quality determines how much capital good innovators may charge for their innovations and the number of such firms that may obtain positive market shares at any one time, both of which affects the returns to innovation and growth.

Specifically, a capital good monopolist may only expand sales in a more disperse market as much as in a more concentrated one through a larger price cut. Equivalently, the elasticity of demand facing the capital good monopolist declines when there is greater dispersion in the distribution of human capital. And the firm may only cover the whole market by charging a lower price. In this way, greater inequal-

[1]See for example Aghion and Howitt (1998) and Peretto (1999).

[2]In contrast, Industrial Organization economists mostly refers market structure to the number of firms in a well-defined industry.

[3]See also the closely related model of Grossman and Helpman (1991).

ity in human capital endowment in the workforce makes it harder for capital good innovators to appropriate their R&D investment and growth suffers as a result.

This is only part of the story, however, because there is no guarantee that the innovator will choose to cover the whole market. In the standard quality ladder model with identical demand for quality among the buyers, it always pays the top firm in the quality ladder for a given variety of capital good to cover the entire market by pricing out firms that only produce the lower grades of the same variety. In other words, once a higher grade is available, all previous grades are discarded, a process commonly known as creative destruction. However, when the demand for quality becomes sufficiently disperse, this may no longer be the case. There is a sizable literature in Industrial Organization that analyzes how firms producing different grades of the same good can co-exist in a Bertrand-Nash equilibrium by virtue of there being sufficient heterogeneity in the demand for quality among buyers.[4] In these models, the firm producing the highest grade of the good in a sufficiently disperse market finds it most profitable to serve the high demand buyers only, leaving room for firms producing the lower grades to acquire some market share. The same result carries over to the present model. If the distribution of human capital is sufficiently unequal, the market for capital goods will turn into a natural oligopoly. And the obsolescence of capital goods is no longer immediate upon the availability of a higher grade but becomes gradual.

A static allocative inefficiency immediately creeps in. If the increase in output made possible through the use of the latest innovation in capital good exceeds the possibly higher cost of production, the social planner will clearly assign all workers with the highest grade capital good available at each point in time. But this will not happen in equilibrium when the dispersion in human capital endowment has turned

[4] The classic study is Shaked and Sutton (1983). See Anderson, De Palma and These (1992, chapter 8) for a brief survey.

the market for capital goods into a natural oligopoly in which only the higher skill workers are equipped with the highest grade capital good, whereas recently obsolete capital goods are continuously sold to firms that employ low skill workers.

There is also a dynamic inefficiency. In a natural oligopoly capital good market, the incumbent technological leader will not be ousted all together when the next innovation arrives. Instead, the firm may still sell to the low end of the market. During these times, its payoff no longer has a close link to the rate of improvement it makes over the previous grade as opposed to the times when the firm is the technological leader. This introduces a distortion in the innovator's choice of the targeted rate of improvement. In equilibrium, firms target a rate of improvement below the maximum attainable for any investment levels that represents an outright wastage. This is an extreme inefficiency for the average growth rate can be raised without any increases in investment at all.

When there is sufficient dispersion in demand to turn the market for capital goods into a natural duopoly,[5] the effect of inequality on investment incentives is reverse. Previously, the fall in the price elasticity caused by greater demand dispersion lowers the innovator's payoff because a greater price cut is necessary for the capital good monopolist to cover the entire market. Now greater inequality raises the payoff instead. Roughly speaking, this is due to the fall in the "intensity" of price competition among the capital good oligopolists in a more disperse market. More precisely, the oligopolists are better off under a lower price elasticity because a firm loses less of its market share to other firms for a given price increase. The conclusion then is that how the distribution of human capital affects long run macroeconomic performances through its effect on the market structure can only be ascertained conditional on how

[5] A natural duopoly results when there exists sufficient dispersion in demand so that the top firm in the quality ladder finds it optimal only to serve a fraction of the market, but not sufficient for more than two firms to enter so that exactly two firms will serve the market at any one time.

much inequality exists to begin with. With low initial inequality, greater inequality results in the capital good monopolist earning a lower profit and less investment follows. With high initial inequality, greater inequality results in the capital good oligopolists earning higher profits and greater investment follows in equilibrium.

The distribution of human capital and economic growth has been previously investigated in Galor and Tsiddon (1997), Galor and Zeira (1993) and Benabou (1996), among others. This strand of investigation focuses on models in which the engine of long run growth is the accumulation of human capital. In a recent paper, Grossman (1999) analyzes how greater inequality in human capital endowment may exacerbate the distortion in the sectoral allocation of workers caused by differences in compensation schemes among sectors. Gradual obsolescence in growth models is not new either. Notable in this strand of investigation includes Chari and Hopenhayn (1991), Caballero and Jaffe (1993), Caballero and Hammour (1996), Young (1993) and Lai (1998). In each one of these studies, there are economic reasons for a product to be gradually phrased out,[6] but not in the present paper in which gradual obsolescence is purely wasteful. In a related paper (Tse, forthcoming), I explore, in more microeconomic details on how dispersion in demand affects investment in quality improvements, focusing on the heterogeneity in firms' targeted rates of improvement under different degrees of demand dispersion.

The next section presents the model and explains how the market structure for capital goods is determined by the distribution of human capital. Section 3 shows how investment and growth differ under the different market structures. Section 4 is a systematic investigation of the effects of inequality (in human capital endowment) on long run macroeconomic performance. Section 5 explains how inequality inefficiently lowers investment and growth below the optimum. Section 6 concludes.

[6] Though the rates of obsolescence in equilibrium could be either excessive or suboptimal.

2. MODEL

A. Preferences

There are two types of goods in the economy : final and intermediate goods with the time τ final good serving as the numeraire. There is a continuum of production workers of measure one and specialized workers of measure S. Production workers may only be employed in final good production and specialized workers may only be employed in the R&D to improve the quality of the intermediate good.[7] Each has the same linear preference given by

$$U = \int_0^\infty e^{-r\tau} c(\tau)\, d\tau$$

where $c(\tau)$ is consumption in time τ and r is the subjective discount rate which will also be the interest rate.

B. Final good production

The final good sector is competitive. Production takes place in production units each of which hires one production worker and one unit of the intermediate good. The production workers are differentiated by their human capital endowment h, uniformly distributed over the interval $\left[\overline{h} - \delta, \overline{h} + \delta\right]$. The distribution has $\overline{h}$ as the mean and $\frac{1}{3}\delta^2$ as the variance. The output of a production unit hiring a worker with h units of human capital is equal to

$$y = hq \tag{1}$$

where q is the quality of the intermediate good the worker is equipped with.

[7] The analysis becomes considerably more complicated in allowing for workers to choose between working in the research or the production sector for there is the issue of selection to be confronted in a model where workers are heterogeneous. The question of who will be selected into research and production in Schumpeterian growth models is an interesting one and deserves a separate treatment.

The point of departure from the standard Schumpeterian growth model of Aghion and Howitt (1992) is that workers are heterogeneous in terms of their innate productivities. Besides, I rule out the usual quality–quantity substitution possibility in the use of the intermediate good in final good production in that a production unit may only hire one unit but no more nor less of the intermediate good. Both assumptions are necessary for different grades of the intermediate good to be used in equilibrium at the same point in time, to each I now turn.

C. Competition and market structure

It costs $c(q) = \eta q$ units of the final good to produce one unit of the intermediate good of quality q for some $\eta < \overline{h} - \delta$. The inventor of each grade of the intermediate good is granted a perpetual patent for the given grade. Say at a given point in time, there are some t firms offering different grades of the good. Denote the qualities of the goods offered by the firms as $q_t \geq q_{t-1} \geq q_{t-2} ... \geq q_0$. In maximizing profit, a production unit hiring worker h chooses which grade of the good to purchase according to

$$\max_j \{hq_j - p_j\}, \quad j = 1, ..., t$$

where p_j is the price charged by firm j.

Assume that the t firms engage in a Bertrand price competition. It is well known that were the buyers possessed identical demand for quality, the firm offering the highest grade of the good would always pick a price low enough to capture the entire market in the Bertrand-Nash equilibrium. Now that the buyers are heterogeneous in terms of their demand for quality, this may no longer be the case. There is a sizable literature in Industrial Organization that analyzes how different grades of the same good may be sold in equilibrium if there exists sufficient heterogeneity among buyers in their tastes for quality. In the present model, with h uniformly distributed over

$\left[\overline{h}-\delta,\overline{h}+\delta\right]$, Appendix A shows that the firm offering the highest grade, firm t, only finds it most profitable to price out others altogether to capture the entire market if[8]

$$\delta < \frac{1}{3}\left(\overline{h}-\eta\right). \tag{2}$$

The market is a natural monopoly under the above condition in the sense that the top firm in the quality ladder will set a price low enough to monopolize the market for any combinations of qualities offered by the firms. Specifically, under (2), firm t will set a price equal to

$$p_t = \left(\overline{h}-\delta\right)(q_t - q_{t-1}) + \eta q_{t-1} \tag{3}$$

in equilibrium. At this price, all production units will purchase the good from firm t unless firms offering the lower grades cut prices below unit production costs.

Conversely, if condition (2) is violated, it is not in the top firm's interest to charge a price as low as in (3). As a result, there will be room for firms offering the lower grades to enter. Intuitively, the top firm may only reach production units hiring lower h workers by charging a lower price. In particular, the top firm has a marginal buyer with its neighbor in the quality ladder firm $t-1$ defined by

$$h_t = \frac{p_t - p_{t-1}}{q_t - q_{t-1}}.$$

This implies the demand curve

$$z_t = \frac{1}{2\delta}\left\{\overline{h}+\delta - \frac{p_t - p_{t-1}}{q_t - q_{t-1}}\right\} \tag{4}$$

with the associated price elasticity

$$\varepsilon_t = \frac{p_t}{\left(\overline{h}+\delta\right)(q_t - q_{t-1}) - p_t + p_{t-1}}$$

[8]A special case of this condition with $\eta = 0$ is widely used and discussed in Industrial Organization, see Tirole (1988).

which is decreasing in δ. This means that when the distribution of h is more spread out, a greater price cut is necessary to expand sales. In other words, the marginal revenue schedule facing the top firm is steeper under a more unequal distribution of h. For the usual reason, it will not pay the firm to reach out to the lowest demand production units for a sufficiently steep marginal revenue schedule.

How many firms may be accommodated in equilibrium depends on how unequal the distribution of h is. Appendix A shows that under the condition[9]

$$\frac{1}{3}\left(\overline{h}-\eta\right) \leq \delta < \frac{3}{5}\left(\overline{h}-\eta\right) \tag{5}$$

exactly two firms, the top and the second firms in the quality ladder, will acquire positive market shares in equilibrium. More precisely, under the first inequality, the top firm charges a price above the level given in (3) in equilibrium, leaving room for others to enter. Under the second inequality, the second firm's best response to any prices charged by firms offering the lower grades is to charge a price low enough to price them all out. This turns the market for the intermediate good into a natural duopoly. Now only production units hiring better workers purchase the highest grade intermediate good while production units hiring workers with lesser human capital purchase a lower grade. If the heterogeneity in workers' productivities is still greater, a third firm may be accommodated in equilibrium. To keep the analysis manageable, I shall only deal with the cases of a natural monopoly and a natural duopoly in the following.

The assumption that there is no quality–quantity substitution in the use of the intermediate good in final good production not only serves to simplify, but is in fact necessary for firms producing the lower grades to acquire market shares. To see this, consider a production function for the final good that has the usual quality–quantity

[9] The condition is a special case of the condition derived in Anderson, de Palma and Thisse (1992, chapter 8) where the unit production cost is a general function of the quality of the good to be produced in place of the linear specification adopted in this paper.

substitution possibility. Let z be the quantity of the intermediate good used; then

$$y = h\left(qz\right)^{\alpha}$$

which implies the unit cost function

$$c\left(h, q\right) = \left(\frac{1}{h}\right)^{\frac{1}{\alpha}} \frac{p\left(q\right)}{q}$$

where $p(q)$ denotes the price charged by the firm selling the intermediate good of quality q. It follows from cost minimization that all production units, irrespective of the workers they hire, should purchase from the firm offering the lowest $p(q)/q$. No two different grades may then be sold in a Bertrand-Nash equilibrium. In contrast, under the production function in (1), a firm may not capture the entire market even if it offers the lowest $p(q)/q$. The reason is that with the assumed indivisibility in the use of the intermediate good, it may not pay for production units where the returns to quality are sufficiently low to purchase *as many as one* unit of the more expensive but higher quality intermediate good even it offers a lower $p(q)/q$.

D. The technology for quality improvements

The technology for quality improvements is the standard one. Now the subscript $t = 0, 1, ...$ is also used to denote the interval starting with the tth innovation that results in the invention of the intermediate good of quality q_t and ending just before the $t+1$th innovation. Assume that within each interval, all prices and quantities remain constant. There are two inputs to the production of blueprints : final goods and specialized labor. Remember that the supply of specialized labor is fixed at some S and whose only use is for improving the quality of the intermediate good.

If the currently highest grade is q_{t-1}, a firm investing xq_{t-1}units of the final good and hiring s units of specialized labor per unit of time innovates with the Poisson arrival rate $\lambda\left(g_t\right)\phi\left(x, s\right)$. The successful innovation will result in the invention of the intermediate good of quality

$$q_t = (1 + g_t)\, q_{t-1}.$$

I assume that

(A1a) $\phi(x, s)$ is twice continuously differentiable, constant returns, increasing and concave in the two arguments and satisfies the Inada condition, i.e. $\lim_{x \to 0} \phi_x = \infty$, $\lim_{x \to \infty} \phi_x = 0$, $\lim_{s \to 0} \phi_s = \infty$ and $\lim_{s \to \infty} \phi_s = 0$.

(A1b) $\lambda(g)$ is twice continuously differentiable, decreasing and tends toward 0.

(A1c) $g\lambda(g)$ is quasi-concave, first increasing then decreasing and approaching 0 and attains a maximum at some $g^* > 0$, i.e.

$$g\lambda'(g) + \lambda(g) \underset{<}{\overset{>}{}} 0 \Leftrightarrow g \underset{>}{\overset{<}{}} g^*.$$

Under (A1b), the arrival rate is decreasing in the targeted rate of improvement, meaning that it is more difficult to innovate when the research targets a greater improvement. To understand (A1c), notice that we can think of the expression

$$E[g] = g\lambda(g)\, \phi(x, s)$$

as the *instantaneous* expected rate of improvement, which attains a maximum at $g^* = \arg\max \{g\lambda(g)\}$ for any x and s by virtue of the multiplicative specification for the Poisson arrival rate. Then the term $g\lambda(g)$ in (A1c) is just the instantaneous expected rate of improvement per unit of $\phi(x, s)$.

For most countries, quality improvements in intermediate goods are the results of adopting advanced technologies invented abroad instead of from local research and development. I take that the model can be applied to this case too by interpreting the investment in research and development as the investment in adopting technologies originated from abroad to local conditions.[10] For this reason, I shall from now on refer to the investment necessary to introduce higher quality intermediate goods generically

[10] For a similar interpretation of the quality ladder growth model, see Rodriguez-Clare (1996).

as investment in quality improvements in lieu of the more commonly used term R&D investment. But I shall continue referring to firms investing in quality improvements as research firms, the introduction of the intermediate good of quality q_t as the tth innovation and the firm that does so as the tth innovator for lack of better names and because of their familiarity with readers.

3. INVESTMENT IN QUALITY IMPROVEMENTS AND MARKET STRUCTURE

A. Investment in a natural monopoly

Under a natural monopoly intermediate good market, the analysis of investment in the present model closely resembles the model of Aghion and Howitt (1992), with the defining difference being the use of an alternative demand function for the intermediate good as given in (4) in place of the commonly used iso-elastic demand curve.

By virtue of (3) and the fact that the firm is selling to the whole market of size one, the tth innovator earns a monopoly profit equal to

$$\pi_t = p_t - \eta q_t = \left(\overline{h} - \eta - \delta\right) q_{t-1} g_t$$

per unit of time by undercutting any firms offering the lower grades. The monopoly reign of the firm ends when the next innovation arrives. Denote the arrival rate of the $t+1$th innovation as a_{t+1}. The expected discounted payoff earned by the tth innovator is[11]

$$V_t\left(g_t, q_{t-1}\right) = \frac{\left(\overline{h} - \eta - \delta\right) q_{t-1} g_t}{r + a_{t+1}}. \tag{6}$$

Because of the constant returns assumption on the ϕ function, it is without loss of generality to assume that there is just one representative firm investing in quality improvements in each interval. In the $t-1$th interval, the firm chooses investment in terms of x_{t-1} and s_{t-1} per unit of time and the rate of quality improvement g_t to maximize expected return

$$\max_{\{x_{t-1}, s_{t-1}, g_t\}} \left\{\phi\left(x_{t-1}, s_{t-1}\right) \lambda\left(g_t\right) V_t\left(g_t, q_{t-1}\right) - q_{t-1} x_{t-1} - w^s_{t-1} s_{t-1}\right\}$$

where w^s is the wage rate for specialized labor. Define $\widehat{V}_t\left(g_t\right) = V_t\left(g_t, q_{t-1}\right)/q_{t-1}$. With the supply of and the demand for specialized labor equal in equilibrium, the maximization is simplified to

$$\max_{\{x_{t-1}, g_t\}} \left\{\varphi\left(x_{t-1}\right) \lambda\left(g_t\right) \widehat{V}_t\left(g_t\right) - x_{t-1}\right\}$$

where $\varphi\left(x_{t-1}\right) = \phi\left(x_{t-1}, S\right)$. Substituting in (6) and noting that in equilibrium $a_{t+1} = \varphi\left(x_t\right) \lambda\left(g_{t+1}\right)$, we have

$$\max_{\{x_{t-1}, g_t\}} \left\{\frac{\varphi\left(x_{t-1}\right)\left(\overline{h} - \eta - \delta\right) \lambda\left(g_t\right) g_t}{r + \varphi\left(x_t\right) \lambda\left(g_{t+1}\right)} - x_{t-1}\right\}. \tag{7}$$

This implies a choice of $g_t = g^*$ for all values of x_{t-1}. The first order condition for x_{t-1} is

$$\frac{\varphi'\left(x_{t-1}\right)\left(\overline{h} - \eta - \delta\right) \lambda\left(g^*\right) g^*}{r + \varphi\left(x_t\right) \lambda\left(g_{t+1}\right)} = 1. \tag{8}$$

[11] Following the convention adopted by previous authors, I take that all investment in innovations is undertaken by entrants and none by the incumbent. It can be shown that the efficiency effect whereby the incumbent tends to profit more from an innovation for it may charge a higher price by virtue of owning the patents for successive grades and the replacement effect whereby the entrant tends to gain more from the innovation because the incumbent is simply continuing its monopoly reign after the innovation exactly offset each other in the present model with a natural monopoly intermediate good market. Hence the identity of the innovator is both indeterminate and immaterial. The details are in an appendix available upon request.

The Inada condition on $\phi(x,s)$ ensures that (8) has a positive and finite solution for x_{t-1} for all $\varphi(x_t)\lambda(g_{t+1})$. Clearly, each generation of innovators will choose the same $g = g^*$. Substituting g^* for g_{t+1} in (8) yields a forward looking difference equation in x, which defines investment in the $t-1$th interval as a function of investment in the tth interval. It can be shown that x_{t-1} is declining in x_t and tends toward 0, which ensure the existence of a unique steady state defined by[12]

$$\frac{\varphi'(x)\left(\overline{h}-\eta-\delta\right)\lambda(g^*)g^*}{r+\lambda(g^*)\varphi(x)} = 1. \tag{9}$$

This implies that

Proposition 1 *When the market for the intermediate good remains a natural monopoly, steady state investment in quality improvements is increasing in $\overline{h}$ but decreasing in δ.*

The market for the intermediate good remains a natural monopoly so long as $\delta < \frac{1}{3}\left(\overline{h}-\eta\right)$. An increase in inequality within the specified range lowers the price the firm may charge without changing the quantity sold. This unambiguously lowers the rent for the innovator, causing investment to decline in equilibrium. However, greater dispersion in the distribution of human capital may not always lower investment once it is recalled that when δ rises beyond $\frac{1}{3}\left(\overline{h}-\delta\right)$, the market for the intermediate good becomes a natural oligopoly that has an entirely different set of payoff functions for the innovator. And it will be seen in the following that the effect of inequality is in fact reversed.

B. Investment in a natural duopoly

In a natural duopoly intermediate good market, the tth innovator only sells to production units hiring the good workers in the tth interval with the rest served by

[12] However, the stability of the steady state cannot in general be assured. See Aghion and Howitt (1992) for details.

the $t-1$th innovator. Appendix A shows that the firm's profit per unit of time is

$$\pi_t = \frac{\left(3\delta + \overline{h} - \eta\right)^2 (q_t - q_{t-1})}{18\delta}. \tag{10}$$

In contrast to where the market for the intermediate good is a natural monopoly, when the $t+1$th innovation arrives, the tth innovator will not be ousted from the market just yet; it may still sell to the low demand production units. Appendix A shows that the firm's profit per unit of time in the $t+1$th interval is equal to

$$\widehat{\pi}_{t+1} = \frac{\left(3\delta - \overline{h} + \eta\right)^2 (q_{t+1} - q_t)}{18\delta} \tag{11}$$

where the tilde in π is used to distinguish the variable from the profit of the $t+1$th innovator in the $t+1$th interval.

The expected discount payoff of the tth innovator may then be expressed as[13]

$$\begin{aligned} V_t\left(g_t, g_{t+1}, q_{t-1}\right) &= \frac{\pi_t}{r+a_{t+1}} + \frac{a_{t+1}}{r+a_{t+1}} \frac{\widehat{\pi}_{t+1}}{r+a_{t+2}} \\ &= \frac{q_{t-1} g_t}{18\left(r+a_{t+1}\right)} \times \\ &\quad \left\{ \frac{\left(\overline{h} - \eta + 3\delta\right)^2}{\delta} + \frac{\left(3\delta + \eta - \overline{h}\right)^2}{\delta} \frac{a_{t+1} g_{t+1}}{r+a_{t+2}} \left(\frac{1+g_t}{g_t}\right) \right\}. \end{aligned} \tag{12}$$

[13] The first line of (12) can be derived as follows. Denote L_t and L_{t+1} respectively as the lengths of the interval between the tth and $t+1$th innovations and between the $t+1$th and $t+2$th innovations, which are independent exponential variates with hazards a_{t+1} and a_{t+2}. The random payoff of the tth innovator is equal to

$$\int_0^{L_t} \pi_t e^{-r\tau} d\tau + \int_{L_t}^{L_t+L_{t+1}} \widehat{\pi}_{t+1} e^{-r\tau} d\tau.$$

The first line of (12) follows by taking expectation with respect to L_t and L_{t+1}. Note that I have also taken that all investment in innovations is undertaken by entrants as in the previous case of a natural monopoly. It can be shown that in present case the replacement effect dominates under assumption (A3) in appendix A so that in fact entrants stand to gain more from the innovation. In equilibrium, the incumbent will not invest in quality improvements since entrants will always outbid the incumbent in employing specialized labor. The details are in an appendix available upon request.

The representative research firm in the $t-1$th interval chooses investment in terms of s_{t-1} and x_{t-1} per unit of time and the rate of improvement g_t to maximize expected return

$$\max_{\{x_{t-1},s_{t-1},g_t\}} \left\{\phi\left(x_{t-1},s_{t-1}\right)\lambda\left(g_t\right)V_t\left(g_t,g_{t+1},q_{t-1}\right)-q_{t-1}x_{t-1}-w^s_{t-1}s_{t-1}\right\}.$$

Define $\widehat{V}_t\left(g_t,g_{t+1}\right)=V_t\left(g_t,g_{t+1},q_{t-1}\right)/q_{t-1}$. Set the two arrival rates a_{t+1} and a_{t+2} in (12) equal to $\varphi\left(x_t\right)\lambda\left(g_{t+1}\right)$ and $\varphi\left(x_{t+1}\right)\lambda\left(g_{t+2}\right)$ respectively, and with the equality of the demand for and the supply of specialized labor, the maximization simplifies to

$$\begin{aligned}
&\max_{\{x_{t-1},g_t\}}\left\{\varphi\left(x_{t-1}\right)\lambda\left(g_t\right)\widehat{V}_t\left(g_t,g_{t+1}\right)-x_{t-1}\right\}= \\
&\max_{\{x_{t-1},g_t\}}\left\{\frac{\varphi\left(x_{t-1}\right)\lambda\left(g_t\right)g_t}{18\left(r+\varphi\left(x_t\right)\lambda\left(g_{t+1}\right)\right)}\times\right. \\
&\qquad\left.\left(\frac{\left(\overline{h}-\eta+3\delta\right)^2}{\delta}+\frac{\left(3\delta+\eta-\overline{h}\right)^2}{\delta}\frac{\varphi\left(x_t\right)\lambda\left(g_{t+1}\right)g_{t+1}}{r+\varphi\left(x_{t+1}\right)\lambda\left(g_{t+2}\right)}\left(\frac{1+g_t}{g_t}\right)\right)-x_{t-1}\right\}.
\end{aligned} \tag{13}$$

The first order condition for g_t is

$$\left(\lambda'\left(g_t\right)g_t+\lambda\left(g_t\right)\right)\left(\frac{\overline{h}-\eta+3\delta}{3\delta+\eta-\overline{h}}\right)^2+\left(\lambda'\left(g_t\right)\left(1+g_t\right)+\lambda\left(g_t\right)\right)\frac{\varphi\left(x_t\right)\lambda\left(g_{t+1}\right)g_{t+1}}{r+\varphi\left(x_{t+1}\right)\lambda\left(g_{t+2}\right)}=0. \tag{14}$$

which, like the corresponding condition for a natural monopoly, is independent of x_{t-1}. Now I impose a further restriction on the $\lambda\left(g\right)$ function to guarantee the existence of a unique privately optimal g_t.

(A2) $\lambda'\left(g\right)\left(1+g\right)+\lambda\left(g\right)>0$ at $g=0$.

Lemma 1 *There exists a unique optimal $g_t>0$ for the research firm defined as the solution to* (14).

Proof. In appendix B.

Lemma 2 *The research firm in a natural duopoly chooses a rate of improvement below the level that maximizes the instantaneous expected rate of improvement, $g_t<g^*$.*

Proof. In appendix B.

This result has the following interpretation. From (10), the tth innovator in the tth interval earns a payoff proportional to the rate of improvement it makes over the previous grade,

$$\pi_t \propto q_t - q_{t-1} = g_t q_{t-1} \propto g_t.$$

This arises because as the top firm in the interval, it limits price to the improvement it makes over the previous grade according to the demand for quality of some marginal buyer.[14] In the absence of other forces affecting the firm's choice of g_t, its expected payoff would be maximized at

$$g^* = \arg\max_{g_t} \{\lambda(g_t) g_t\}$$

for any x_{t-1} by virtue of the multiplicative specification of the Poisson arrival rate, just like when the market for the intermediate good is a natural monopoly. But when the market is a natural duopoly, the tth innovator remains in the market in the $t+1$th interval as the low quality firm. Eq. (11) shows that the firm's payoff then is proportional to q_t,

$$\pi'_{t+1} \propto q_{t+1} - q_t = g_{t+1} q_t \propto q_t.$$

Since $q_t = (1+g_t) q_{t-1}$, the firm's expected payoff in the $t+1$th interval is maximized at some

$$\widehat{g} = \arg\max \{\lambda(g_t)(1+g_t)\} < g^* \qquad (15)$$

for any given q_{t-1}, which is below g^* because $\lambda' < 0$.[15] Intuitively, when the firm's payoff is proportional to the entire value of the innovation instead of merely proportional to the improvement the innovation makes over the previous grade, successes in

[14] See appendix A for details. The payoff of the firm is the product of the price-cost margin and the firm's market share. Appendix A shows that the firm's market share is independent of q_t and q_{t-1} so that the payoff function as shown in (10) is simply proportional to the price-cost margin.

[15] The existence of a positive $\widehat{g}$ is guaranteed by (A2).

innovation are valued more, which induces a more conservative choice of the targeted rate of improvement. Since the firm's payoff is the sum of payoffs in the two intervals, its optimal choice of g_t is accordingly a weighted average of $\widehat{g}$ and g^*.

Equation (14) defines g_t as a function of research investment (x_t, x_{t+1}) and the rates of improvement (g_{t+1}, g_{t+2}) in the next two intervals. The comparative dynamics are summarized in the following.

Lemma 3 *In equilibrium, g_t is decreasing in g_{t+1}, g_{t+2}, x_t but increasing in x_{t+1}.*

Proof. In appendix B.

These comparative dynamics operate through the effects of the variables concerned on the *relative* durations of tth and $t+1$th intervals. An increase in x_t speeds up the arrival of the $t+1$th innovation, thus shortening the tth interval. This weakens the force for choosing $g_t = g^*$ relative to that for choosing $g_t = \widehat{g}$, yielding a smaller optimal g_t for the research firm. An increase in x_{t+1} has the opposite effect. Similarly, an increase in g_{t+2} results in a smaller g_t because it delays the arrival of the $t+2$ innovation, lengthening the $t+1$ interval and strengthening the force for choosing $g_t = \widehat{g}$ relative to $g_t = g^*$. However, the converse is not true for an increase in g_{t+1}. Although the increase does delay the arrival of the $t+1$th innovation, strengthening the case for choosing $g_t = g^*$, an increase in g_{t+1} also raises the penalty of choosing a large g_t in the $t+1$ interval as evidenced in (14). In equilibrium with $g_{t+1} < g^*$, the latter effect dominates.

Now I turn to the firm's choice of investment. The first order condition of (14) for x_{t-1} is

$$\frac{\varphi'(x_{t-1})\lambda(g_t)g_t}{r+\varphi(x_t)\lambda(g_{t+1})}\left\{\frac{\left(\overline{h}-\eta+3\delta\right)^2}{18\delta}+\frac{\left(3\delta+\eta-\overline{h}\right)^2}{18\delta}\frac{\varphi(x_t)\lambda(g_{t+1})g_{t+1}}{r+\varphi(x_{t+1})\lambda(g_{t+2})}\left(\frac{1+g_t}{g_t}\right)\right\}=1. \tag{16}$$

The Inada condition on $\phi(x,s)$ ensures that the above has a positive and finite solution for x_{t-1}. The equilibrium is fully characterized by the two forward looking difference equations in x and g implicitly defined in (14) and (16).

Lemma 4 *In equilibrium, x_{t-1} is decreasing in x_t and x_{t+1} but increasing in g_{t+1} and g_{t+2}.*

Proof. In appendix B.

The mechanics of these comparative dynamics are similar to those pertaining to g_t in the last lemma. Increases in x_t and x_{t-1} raise the arrival rates of the $t+1$th and $t+2$th innovations, which will shorten the expected lifetime of the tth innovation. In equilibrium, the firm invests less. Increases in g_{t+1} and g_{t+2} have opposite effects on the arrival rates of the $t+1$th and $t+2$th innovations and hence opposite effects on investment in the $t-1$th interval.

The steady state is obtained by removing the interval subscripts t on g and x in (14) and (16).[16]

$$\left(\lambda'(g)\, g+\lambda(g)\right)\left(\frac{\overline{h}-\eta+3\delta}{3\delta+\eta-\overline{h}}\right)^2+\left(\lambda'(g)\,(1+g)+\lambda(g)\right)\frac{\varphi(x)\,\lambda(g)\, g}{r+\varphi(x)\,\lambda(g)}=0, \quad (17)$$

$$\frac{\varphi'(x)\,\lambda(g)\, g}{r+\varphi(x)\,\lambda(g)}\left\{\frac{\left(\overline{h}-\eta+3\delta\right)^2}{18\delta}+\frac{\left(3\delta+\eta-\overline{h}\right)^2}{18\delta}\frac{\varphi(x)\,\lambda(g)}{r+\varphi(x)\,\lambda(g)}(1+g)\right\}=1. \quad (18)$$

Lemma 5 *A unique steady state exists.*

Proof. In appendix B.

Proposition 2 *When the intermediate good market is a natural duopoly,*

(a) an increase in $\overline{h}$ raises steady state investment, $dx/d\overline{h}>0$,

[16]The stablitity of the steady state equilibrium cannot in general be guarnateed much in the way as in the previous case of a natural monopoly intermediate good market and the model of Aghion and Howitt (1992).

(b) and for a sufficiently small interest (discount) rate r, the targeted rate of improvement in each interval also increases, $dg/d\overline{h} > 0$,

(c) an increase in δ lowers the targeted rate of improvement chosen in each interval, $dg/d\delta < 0$,

(d) and if $\lambda''(g) \leq 0$ for $g \leq g^*$, steady state investment x rises in response, $dx/d\delta > 0$.

Proof. In appendix C.

Assume for what follows that the conditions in proposition 2 hold. To understand these results, note that the two firms competing in prices in any given interval benefit from an increase in the average demand for quality, as well as from more dispersion in demand. The first effect is obvious. The second effect is due to the fact that the price competition between the two firms is less "intense" when there exists more dispersion in workers' productivities in the sense that under a larger δ, a given price increase by one firm triggers less decline in the market share of the firm for the density of buyers is lower at each h. Indeed, the lower quality firm benefits more from an increase in dispersion than the high quality firm does as the basis for the former to acquire a positive market share is that there exists enough dispersion in the demand for quality to begin with. Conversely, it is the high quality firm that benefits more from an increase in the average demand.

Fixing δ, an increase in $\overline{h}$ raises the average demand but lowers the relative dispersion. There are both positive and negative effects on the firms' payoffs then. The positive effect dominates for the top firm, whereas the negative effect dominates for the second firm. The tth innovator is the top firm in the tth interval and the second firm in the $t+1$th interval. Hence the firm's payoff rises in the first but declines in the second halves of its lifetime. It turns out that the sum of discounted payoffs increases nevertheless, resulting in greater investment in equilibrium. Now recall that the firm's expected payoffs in the tth and $t+1$th intervals are maximized at $g_t = g^*$

and $g_t = \widehat{g} < g^*$ respectively. The increase in the payoff in the tth and the decline in the $t+1$th intervals strengthen the case for choosing a g_t closer to the larger g^*.[17]

Fixing $\overline{h}$, an increase in δ raises the dispersion in the demand for quality and leaves the average demand intact. Hence both the top and second firms benefit unambiguously under a greater δ. As to the effect on g, since the increase in dispersion benefits the second firm more than the top firm, the payoff for the tth innovator in the $t+1$th interval goes up more than it does in the tth interval, strengthening the case for choosing a smaller g. When the payoffs in both intervals increase, investment increases in equilibrium.[18] Now inequality has a positive effect on investment, in contrast to the negative effect when the market for the intermediate good is a natural monopoly. There is then no straightforward relationship between the two. I shall

[17]As stated in the proposition, this apparently intuitive result is only guaranteed for a sufficiently small interest rate. This is due to a general equilibrium consideration. The argument that the increase in $\overline{h}$ raises the payoff in the tth interval but lowers the payoff in the $t+1$th interval and would result in a larger g_t is valid to the extent that x_t and x_{t+1} are held fixed. But in fact the increases in x_t and x_{t+1} caused by the increase in $\overline{h}$ have opposite effects on the relative payoffs in the two intervals for $r > 0$; the payoff in tth interval goes down more than the payoff in the $t+1$th interval does for given increases in x_t and x_{t+1}. Hence, the result holds only when the secondary effect through the changes in x does not overwhelm the primary effect. Note that the qualification that $r > 0$ is only a sufficient condition. In some extensive numerical experiments, I fail to uncover a single instance where the result does not obtain.

[18]Just like the effect of $\overline{h}$ on g, this apparently intuitive result is only guaranteed under some qualifications as stated in the proposition. Increases in δ raise the payoffs in the two intervals to the extent that there is not too large an adverse change in the expected lengths of the two intervals. This may happen for g_{t+1} and g_{t+2} decline in response to the increase in δ, raising the arrival rates of the $t+1$th and $t+2$th innovations. Under the condition that $\lambda'' < 0$ for $g \in (\widehat{g}, g^*)$, the increases in the arrival rates would be small so that the secondary effect never overwhelms the primary effect. Again, the condition $\lambda'' < 0$ is a only sufficient condition and the result probably holds under much weaker conditions. For example, it is possible to verify that it holds under $\lambda(g) = e^{-\rho g}$ and $\lambda(g) = (1+g)^{-\rho}$, both of which violate $\lambda'' < 0$.

defer a more systematic analysis of this point to the next section.

Before doing so, I first note that parts (c) and (d) of the proposition imply that,

Corollary 1 *At a fixed $\overline{h}$, an increase in δ results in more rapid innovations as the increase raises the arrival rate of each innovation, $\varphi(x)\lambda(g)$.*

This result is intuitive. More dispersion results in less intense price competition among the top and the second firms in each interval. The payoffs to successful innovation rise and more investment follows. In the mean time, the fall in the intensity of price competition benefits the second firm in each interval more. It then becomes more important for the innovator to choose a g_t closer to smaller $\widehat{g}$ than to the larger g^*. When it does so, it innovates sooner too.

Finally, it is instructive to note that in the model of Aghion and Howitt (1992), $g = \widehat{g}$ when the innovation is drastic in the sense that the innovator's pricing decision is not constrained by the competition from the previous grade, while $g > \widehat{g}$ under non-drastic innovations whereby the innovator is constrained to charge a limit price due to the competition from the previous grade. And then for given gs, whether an innovation is drastic or otherwise depends on the elasticity of demand facing the intermediate good monopolist. The results discussed above are in fact closely related to the established results. In the present model, g is closer to the larger g^* and further away from the smaller $\widehat{g}$ just when the interval in which the innovator charges the limit price has a greater weight in the firm's total payoff. As in the Aghion and Howitt model, the elasticity of demand plays the crucial role in determining g for it affects the relative importance of the two intervals that affects the firm's choice between $\widehat{g}$ and g^*. In particular, g is closer to $\widehat{g}$ for a less elastic demand curve; the case of greater dispersion. The contribution of the present analysis is in relating the opposing tendencies to the distributions of the demand for quality in a transparent and intuitive manner. The relationship is harder to understand and arguably less intuitive in the Aghion and Howitt model where the elasticity of demand is related to the returns to scale in the final good production.[19]

4. THE DISTRIBUTION OF HUMAN CAPITAL, INVESTMENT AND GROWTH

In this section, I make use of the results established in previous sections to present a systematic analysis of how the distribution of human capital affects investment in technology and growth. Before doing so, I first note that if investment in quality improvements and the assignment of intermediate goods to workers remain constant when δ varies, aggregate output would remain unchanged. This means that the comparison is over a feature of the economy that should not alter investment and growth but would do so in a free market equilibrium.

Recall that as δ varies from 0 to $\frac{1}{3}\left(\overline{h}-\eta\right)$, the market for the intermediate good remains a natural monopoly with each worker equipped with the highest grade intermediate good at each point in time. Aggregate output per unit of time in the tth interval is then

$$Y_t = \overline{h} q_t$$

which implies that it increases from one interval to the next according to

$$Y_t = (1+g)\, Y_{t-1}. \tag{19}$$

[19] In the Aghion and Howitt model, final good output is given by $y = A_t F(x)$ where A_t denotes the quality of tth innovation and x the quantity of the intermediate good used. The (inverse) demand curve facing the intermediate good monopolist for a drastic innovation is therefore $p = A_t F'(x)$. For the innovation to be drastic, the unconstrained price $p = \arg\max \{A_t F'(x)\, x - wx\}$ where w is the unit cost of producing the intermediate good must be below the limit price A_t/A_{t-1}. Whether the condition holds depends on the elasticity of the demand curve, which in turn depends on the returns to scale in $F(x)$. The relationship is a complicated one. Specifically, a fall in the elasticity may cause an innovation more likely to become drastic only for an initially small elasticity whereas the opposite holds for an initially large elasticity; see equation (5.3) of their paper.

The length of each interval is exponentially distributed with hazard $\varphi(x)\lambda(g)$. Aghion and Howitt (1992) shows that the average growth rate in such circumstances is equal to[20]

$$\begin{aligned}\overline{g} &= \varphi(x)\lambda(g)\ln(1+g) \\ &\simeq \varphi(x)\lambda(g)\,g. \end{aligned} \tag{20}$$

Since the rate of improvement is fixed at g^* when the market for the intermediate good remains a natural monopoly, the average growth rate is a simple increasing function of investment. Thus the negative relationship between dispersion and investment established previously carries over to a negative relationship between dispersion and growth.

When δ reaches $\frac{1}{3}\left(\overline{h}-\eta\right)$, the market for the intermediate good turns into a natural duopoly. Appendix A shows that in the tth interval, the tth innovator covers the market from some $h' = \frac{2\overline{h}-\eta}{3}$ to the top end, while the $t-1$th innovator serves the lower end, with the respective market shares of the two firms given by

$$\begin{aligned} s &= \frac{1}{2}+\frac{\overline{h}-\eta}{6\delta}, \\ s' &= \frac{1}{2}-\frac{\overline{h}-\eta}{6\delta}. \end{aligned}$$

This shows that an increase in dispersion raises the second firm's market share at the expense of the top firm's. This reshuffling of market shares is inefficient because all workers should be matched with the highest grade available in each interval. This is so for the net output of a production unit

$$\widehat{y} = hq - \eta q$$

[20]The approximation in (20) is exact in a model where each worker is matched with a continuum of intermediate goods as in $y = h\int_0^1 q(j)\,dj$ and where the quality improvement of each variety of intermediate goods is as described above. In this model, growth is no longer stochastic because the uncertainty in each sector is averaged out in the aggregate. The actual growth rate of output is as given in (20).

is increasing in q for all $h \in \left[\overline{h}-\delta, \overline{h}+\delta\right]$ since $\overline{h}-\delta > \eta$. And it can be verified that the inefficiency is reflected in a lower aggregate output

$$
\begin{aligned}
Y_t &= \frac{1}{2\delta} \int_{\overline{h}-\delta}^{h'} h q_{t-1} dh + \frac{1}{2\delta} \int_{h'}^{\overline{h}+\delta} h q_t dh \\
&= \frac{q_{t-1}}{4\delta} \left\{ \left(\frac{2\overline{h}-\eta}{3} \right)^2 - \left(\overline{h}-\delta\right)^2 \right\} + \frac{q_t}{4\delta} \left\{ \left(\overline{h}+\delta\right)^2 - \left(\frac{2\overline{h}-\eta}{3} \right)^2 \right\} \qquad (21)
\end{aligned}
$$

for fixed q_t and q_{t-1}. We may summarize the result as

Proposition 3 *Fixing the qualities of the intermediate goods, increases in* δ *for* $\delta > \frac{1}{3}\left(\overline{h}-\eta\right)$ *lower aggregate output, representing a negative level effect.*

The introduction of a higher grade does not completely replace the existing grades because the innovator finds it not privately optimal to do so. Its profit is maximized at a price above the level that will drive out the incumbent. The gradual obsolescence in the present model contrasts previous growth models of gradual obsolescence where there are economic reasons for the continued employment of older vintages. In Chari and Hopenhayn (1991), human capital is specific to a vintage of technology and production requires both young and old workers skilled in the same vintage. It is not optimal for all young workers to receive training only in the latest vintage because there are not enough old workers skilled in the same vintage. In Young (1993), old and new vintages are complementary so that it is obviously not optimal for only one vintage to be used. In Caballero and Hammour (1996), it pays for existing vintages not to be scraped immediately upon the arrival of the latest vintage because capital goods are long-lived. In Caballero and Jaffe (1993) and Lai (1998), production exhibits the usual returns to variety so that even though the latest vintage supplies more efficiency unit, it is not optimal to scrape all the existing vintages. In contrast, the gradual obsolescence in the present model is purely wasteful and is due to the fact that the innovator cannot perfectly price discriminate. There would be no gradual obsolescence had there been little dispersion in human capital in the workforce. And since a greater number of firms selling different grades will be accommodated in equilibrium with greater dispersion, the rate of obsolescence will be further and further below the optimum when dispersion increases.

As to the effect on investment, greater dispersion results in more investment, not less as in the case of a natural monopoly. But this does not guarantee that the average growth rate will also be higher because it is a function of the rate of improvement made from one interval to the next as well. Specifically, (21) implies that the growth in aggregate output from one period to the next is also governed by (19) with the associated average growth rate given by (20). While it is true that an increase in δ results in more rapid innovations due to the increase in the arrival rate $\varphi(x)\lambda(g)$, higher growth does not necessarily follow because g is lowered. Indeed, the fall in g causes $\lambda(g)g$ to fall too for $g < g^*$ in equilibrium. Essentially, though more investment follows from greater dispersion, the research projects chosen become more conservative in the meantime so that the resulting average growth rate increasingly deviates below the maximum attainable for given investment levels. As will be seen in the next section, for each investment level, the socially efficient choice of g is at g^* because of the assumed linear preference so that the deviation of the average growth rate below the maximum attainable is an increasingly inefficient deviation.

Whether the effect from more investment or from a smaller rate of improvement in each interval dominates in determining the change in the average growth rate that follows from greater dispersion is generally impossible to ascertain. But there appears to be a better case for the average growth rate to increase as it can be verified that this is so under any combinations of $\varphi(x) = x^{\beta}$ or $\varphi(x) = \log(1+x)$ and $\lambda(g) = e^{-g\rho}$ or $\lambda(g) = (1+g)^{-\rho}$. On the other hand, I fail to uncover a single instance where the average growth rate declines. Still the possibility remains that growth may decline in the midst of increasing investment.

While x and g are continuous functions of δ for $\delta < \frac{1}{3}\left(\overline{h}-\delta\right)$ and for $\delta > \frac{1}{3}\left(\overline{h}-\delta\right)$, I have yet to establish whether there may be any jumps in x and g at $\delta = \frac{1}{3}\left(\overline{h}-\delta\right)$ caused by the change in market structure that results in an entirely different set of equilibrium conditions. It turns out that at $\delta = \frac{1}{3}\left(\overline{h}-\delta\right)$, the first equilibrium conditions for a natural duopoly (17) implies that $g = g^*$ and the second one (18) reduces to the corresponding condition for a natural monopoly in (8). This suffices to rule out any possible discontinuities and allows a complete characterization of investment and growth as a function of δ.

Proposition 4 *As δ increases from 0,*

(a) x first declines, reaches a minimum at $\delta = \frac{1}{3}\left(\overline{h}-\delta\right)$ and increases from thereafter,

(b) g is constant at g^* until δ reaches $\frac{1}{3}\left(\overline{h}-\delta\right)$ and decreases from thereafter,

(c) the average growth rate $\varphi(x)\lambda(g)g$ decreases until δ reaches $\frac{1}{3}\left(\overline{h}-\delta\right)$ and may increase from thereafter,

(d) but investment and the average growth rate will not recover to their respective levels at $\delta = 0$ even when δ reaches the upper bound for the market to remain as a natural duopoly at $\delta = \frac{3}{5}\left(\overline{h}+\delta\right)$.

Proof. Parts (a) to (c) follow from the preceding discussion. To establish (d), note that at $\delta = \frac{3}{5}\left(\overline{h}-\eta\right)$, the left side of the steady state condition for a natural duopoly given in (18) becomes

$$
\begin{aligned}
& \frac{\varphi'(x)\left(\overline{h}-\eta\right)\lambda(g)g}{r+\varphi(x)\lambda(g)}\left\{\frac{98}{135}+\frac{8}{135}\frac{\varphi(x)\lambda(g)}{r+\varphi(x)\lambda(g)}(1+g)\right\} \\
< \quad & \frac{\varphi'(x)\left(\overline{h}-\eta\right)\lambda(g)g}{r+\varphi(x)\lambda(g)}\frac{106}{135} < \frac{\varphi'(x)\left(\overline{h}-\eta\right)\lambda(g^*)g^*}{r+\varphi(x)\lambda(g^*)}.
\end{aligned}
$$

for any $x > 0$. The first inequality follows from the condition that $r > \varphi(x)\lambda(g)g$ which, as will be seen in the next section, must hold for utility to be finite. The last inequality follows because $g < g^*$ in equilibrium for $\delta > \frac{1}{3}\left(\overline{h} - \eta\right)$. The final expression is the left side of (9) that determines investment in a natural monopoly at $\delta = 0$. This shows that investment will not recover to the level at $\delta = 0$ when δ reaches $\frac{3}{5}\left(\overline{h} - \eta\right)$. The same conclusion for the average growth rate $\varphi(x)\lambda(g)g$ follows immediately as $g < g^*$. ■

To summarize, the effects of greater dispersion on investment and growth depends on how much dispersion exists to begin with. Starting with low inequality in the distribution of human capital, greater inequality makes it harder for the monopolist to appropriate its technology investment, lowering investment and growth as a result. Further increases in inequality beyond the point where the market for the intermediate good has turned into a natural duopoly, however, lower the intensity of price competition between the oligopolists. This raises the payoff to innovation and results in more investment in equilibrium. However, the increase in investment is accompanied by both greater static and dynamic inefficiencies. Greater static inefficiencies follow because more workers are inefficiently matched with low quality intermediate goods. And then the targeted rate of improvement deviates further below the optimal rate g^*, causing greater dynamic inefficiencies. As such, the equilibrium cannot be first best for $\delta > \frac{1}{3}\left(\overline{h} - \eta\right)$. In fact, the equilibrium in this model is never first best. To see this, I now turn to a more systematic analysis of efficiency.

5. INEFFICIENT INEQUALITY

As remarked previously, it is optimal to assign each worker the highest grade of the intermediate good available in each interval for $\overline{h} - \delta > \eta$. The maximum attainable flow of aggregate output net of the cost of intermediate good production and research expenditure per unit of time in the tth interval is

$$\widehat{Y}_t = \left(\overline{h} - \eta - x\right) q_t.$$

The assumption that each worker has a linear preference over the final good implies that expected aggregate welfare can be expressed as[21]

$$U = \int_0^{\tau} e^{-r\tau} \sum_{t=0}^{\infty} \prod (t, \tau) \left(\overline{h} - \eta - x\right) q_t d\tau \tag{22}$$

where $\prod (t, \tau)$ is the probability that there will be exactly t innovations up to time τ which has the following expression

$$\prod (t, \tau) = (\lambda (g) \varphi (x) \tau)^t e^{-\lambda(g)\varphi(x)\tau} / t!.$$

Evaluating (22)

$$U = \frac{\left(\overline{h} - \eta - x\right) q_0}{r - \varphi (x) \lambda (g) g} \tag{23}$$

where q_0 is the quality of the intermediate good at time 0. This shows that for utility to be finite, the average growth rate (or the instantaneous expected rate of improvement) must be below the discount rate, i.e. $\varphi (x) \lambda (g) g < r$. The planner chooses the rate of targeted improvement g in each interval and the (normalized) flow rate of investment x to maximize (23).

It is immediately apparent that the optimal rate of improvement is at g^* as conjectured previously. Taking first order condition of (23) yields

$$\frac{\varphi' (x) \left(\overline{h} - \eta - x\right) \lambda (g^*) g^*}{r - \varphi (x) \lambda (g^*) g^*} = 1. \tag{24}$$

The Inada condition on $\phi (x, s)$ ensures that a positive and finite optimal x exists. Note that optimal investment is independent of δ, confirming the claim made in the last section that changes in dispersion should not alter allocations in the present model.

Proposition 5 *There is always suboptimal investment and growth in equilibrium, with equilibrium investment and the average growth rate closest to the optimum at* $\delta = 0$.

[21] The formulation of the planning problem follows closely Aghion and Howitt (1992, section 4).

Proof. It suffices to compare the equilibrium with identical workers, i.e. $\delta = 0$, to the optimum. In doing so, subtract the left side of the equilibrium condition (9) at $\delta = 0$ from the left side of the optimality condition (24)

$$\begin{aligned} & \varphi'(x)\lambda(g^*)g^*\left\{\frac{\overline{h}-\eta-x}{r-\varphi(x)\lambda(g^*)g^*}-\frac{\overline{h}-\eta}{r+\varphi(x)\lambda(g^*)}\right\} \\ = \quad & \varphi'(x)\lambda(g^*)g^*(r-\varphi(x)\lambda(g^*)g^*)\left\{\frac{\varphi(x)\lambda(g^*)\left(\overline{h}-\eta\right)(1+g^*)}{r+\varphi(x)\lambda(g^*)}-x\right\}. \quad (25) \end{aligned}$$

Denote equilibrium x as x^e. Now the above evaluated at x^e must be positive once it is recognized from (7) that the expected revenue of the research firm in the $t-1$th interval is simply

$$q_{t-1}\left(\frac{\varphi(x^e)\lambda(g^*)\left(\overline{h}-\eta\right)g^*}{r+\varphi(x^e)\lambda(g^*)}-x^e\right)$$

which is clearly positive in equilibrium. This ensures the positivity of the expression inside the curly bracket in (25). For utility to be finite, $r-\varphi(x^e)\lambda(g^*)g^* > 0$ too. All together, we have the downward sloping first order condition for optimal x evaluated at x^e positive, from which it follows that optimal x is above x^e. From proposition 4, investment is greatest at $\delta = 0$. For any $\delta > 0$, x^e must be further below the optimum. The same conclusion on the average growth rate follows trivially. ■

This result stands in contrast to the result obtained from the standard quality-ladder growth model in which there is no unambiguous comparison between equilibrium and optimal investment. In the standard model, the different distortions affect how the equilibrium differs from the optimum in opposite directions. In particular, the "business stealing" effect causes excessive investment, whereas imperfect intertemporal and intratemporal appropriability of investment in quality improvements cause suboptimal investment.

To see the reason for the difference in conclusion reached in this paper, take the case of $\delta = 0$. Recall that this is the case where the innovator sells to the whole market of size one. The last condition is crucial because it implies that the usual under-provision caused by the monopolist's inability to perfectly price-discriminate is absent. Furthermore, at $\delta = 0$, the firm limits price according to the identical demand for quality of all buyers, so that the rent earned is precisely equal to the increase in output made possible by the innovation. In this way, there is no imperfect intratemporal appropriability of investment that turns the monopolist's profit below the increase in output in the usual model.[22]

In fact, the assumption of a market fixed in size coupled with limit pricing in Bertrand competition also eliminate the usual "business stealing". For the "business stealing" effect to be operative, there has to be a transfer of rent from the incumbent monopolist to the innovator. There is no such transfer in the present model as the rent earned by the innovator is just equal to, but no more than, the increase in output made possible by the innovation for $\delta = 0$ because of limit pricing in Bertrand competition.[23]

The only distortion that remains is the imperfect intertemporal appropriability of investment as the innovator may only earn a rent equal to its contribution to output increases for a limited duration, before the arrival of the next innovation that will drive the firm out of the market. And this unambiguously lowers investment incentives below the optimum. Since investment is greatest with identical workers. Any heterogeneity only lowers investment further below the optimum.

Although further increases in inequality beyond the point where the market has turned into a natural duopoly raise investment incentives, part (d) of proposition 4 establishes that investment incentives never recover to the level at $\delta = 0$. This means that any increase in investment brought by greater inequality may always be forthcoming through a sufficient reduction in inequality instead. The latter change is a

[22]See Romer (1994) for an excellent discussion of this point.

[23]This result was first shown to hold in the early study of Sah and Stigilitz (1987) for a one-shot patent race.

pareto improvement over the former for it results in the rate of targeted improvement set equal to the optimum instead of further away in the case of an increase in inequality, and it eliminates the static allocative inefficiency that a fraction of workers are inefficiently matched with lower grades intermediate goods.

6. CONCLUDING REMARKS

For the sake of tractability, I have not taken up the analysis for $\delta > \frac{3}{5}\left(\overline{h} - \delta\right)$ at which point the intermediate good market may support more than 2 firms in equilibrium. This restriction clearly does not affect the conclusion that the effect of greater inequality on investment depends on how much inequality exists to begin with. This conclusion may only be reinforced by extending the analysis to an intermediate good market where three or more firms may acquire market shares in equilibrium because there is then reason to believe that greater inequality must eventually lower investment again, just like when $\delta < \frac{1}{3}\left(\overline{h} - \delta\right)$. This is because with the average demand remaining constant but with an increasingly greater number of firms competing for market shares as δ rises, it is inevitable that each firm's payoff may only fall. The welfare conclusion that investment is always suboptimal remains valid when δ is allowed to increase beyond $\frac{3}{5}\left(\overline{h} - \delta\right)$ as along as investment is greatest at $\delta = 0$. At $\delta = 0$, the innovator sells to the whole market and appropriate the entire increase in output as its payoff. It does not appear possible that the payoff can be any higher.[24]

The analysis of inequality and growth cannot be complete without both being endogenously determined. In this paper, I have taken inequality to be fixed for the sake of brevity and because the more interesting effect in the model is from inequality to investment and growth. I plan to take up endogenous inequality within the framework of this paper in future work.

[24]This cannot be concluded unambiguously though without a formal analysis because the firm may stay in the market for a longer period of time when δ becomes larger.

APPENDIX

A. Production market competition

In this section, I present the details of the Bertrand price competition among the intermediate good firms. The top firm in the quality ladder, firm t, will sell from the top end of the market $\overline{h} + \delta$ to some marginal worker $h_t \geq \overline{h} - \delta$. If the marginal worker h_t is strictly above $\overline{h} - \delta$, she must be indifferent between purchasing from firm t at price p_t or from firm $t-1$ at price p_{t-1}.

$$h_t = \max \left[\overline{h} - \delta, \, \frac{p_t - p_{t-1}}{q_t - q_{t-1}} \right]. \tag{26}$$

The firm's profit maximization is hence

$$\max_{p_t} \left\{ \frac{p_t - \eta q_t}{2\delta} \left(\overline{h} + \delta - \max \left[\overline{h} - \delta, \, \frac{p_t - p_{t-1}}{q_t - q_{t-1}} \right] \right) \right\}.$$

Assuming an interior solution,

$$p_t = \frac{1}{2} \left\{ p_{t-1} + \eta q_t + \left(\overline{h} + \delta \right) (q_t - q_{t-1}) \right\}. \tag{27}$$

From (26), the solution for p_t is not an interior solution if the above satisfies

$$p_t < p_{t-1} + \left(\overline{h} - \delta \right) (q_t - q_{t-1}).$$

Substituting in (27) and simplifying

$$(q_t - q_{t-1}) \left(3\delta - \overline{h} \right) < p_{t-1} - \eta q_t.$$

If the condition holds, firm t's best response is to charge a price low enough to attract all buyers. A p_{t-1} strictly above ηq_{t-1} that satisfies the inequality cannot be equilibrium because in this case firm $t-1$ may increase profit from 0 to a positive amount by lowering price. But if the inequality is met at $p_{t-1} = \eta q_{t-1}$, firm $t-1$ will indeed be priced out in equilibrium.

$$(q_t - q_{t-1}) \left(3\delta - \overline{h} + \eta \right) < 0 \Leftrightarrow \delta < \frac{1}{3} \left(\overline{h} - \eta \right). \tag{28}$$

If the condition is satisfied, there is no price above the unit production cost that firm $t-1$ may charge for which firm t's best response is not to charge a price low enough to serve the whole market. The market for the intermediate good is a natural monopoly. The equilibrium price charged by firm t is obtained by setting $\frac{p_t - p_{t-1}}{q_t - q_{t-1}} = \overline{h} - \delta$ at $p_{t-1} = \eta q_{t-1}$.

$$p_t = \left(\overline{h} - \delta\right)(q_t - q_{t-1}) + \eta q_{t-1}.$$

With a market size of one, the firm's profit is equal to

$$\pi_t = \left(\overline{h} - \eta - \delta\right)(q_t - q_{t-1}).$$

Conversely, if the inequality in (28) is not met, there always exists a price above the unit production cost that firm $t-1$ may charge for which firm t's best response is not to serve the whole market. For these values of $\left\{\overline{h}, \delta, \eta\right\}$, at least two firms may serve the market in equilibrium. Consider the second firm in the quality ladder $t-1$. It has a marginal buyer with the lower quality firm $t-2$ defined by

$$h_{t-1} = \max\left[\overline{h} - \delta,\, \frac{p_{t-1} - p_{t-2}}{q_{t-1} - q_{t-2}}\right]. \tag{29}$$

The firm's profit maximization is

$$\max_{p_{t-1}} \left\{ \frac{p_{t-1} - \eta q_{t-1}}{2\delta} \left(\frac{p_t - p_{t-1}}{q_t - q_{t-1}} - \max\left[\overline{h} - \delta,\, \frac{p_{t-1} - p_{t-2}}{q_{t-1} - q_{t-2}}\right] \right) \right\}.$$

Assuming an interior solution

$$p_{t-1} = \frac{1}{2}\left\{ p_t \frac{q_{t-1} - q_{t-2}}{q_t - q_{t-2}} + p_{t-2} \frac{q_t - q_{t-1}}{q_t - q_{t-2}} \right\} + \frac{1}{2}\eta q_{t-1}. \tag{30}$$

From (29), the solution for p_{t-1} is not an interior solution if p_{t-1} as given above satisfies

$$p_{t-1} < p_{t-2} + \left(\overline{h} - \delta\right)(q_{t-1} - q_{t-2}) \tag{31}$$

in which case firm $t-1$ finds it most profitable to sell all the way to the low end of the market. My task now is to establish the sufficient condition for which the above holds. Plugging (30) into (31), it is easily seen that the inequality will hold for a sufficiently large p_{t-2}. Any p_{t-2} strictly above ηq_{t-2} that satisfies (31) cannot be equilibrium because in this case firm $t-2$ may lower price to raise profit from 0 to a positive amount. The lowest price that firm $t-2$ may charge is its unit production cost ηq_{t-2}. Hence if the inequality holds for this value of p_{t-2}, there is no price above the unit production cost firm $t-2$ may charge for which firm $t-1$'s best response is not to price firm $t-2$ out. Whether or not the inequality holds also depends on the value for p_t. In particular, it holds for a sufficiently small p_t. If the equilibrium is such that the third firm is priced out, the highest price that firm $t-1$ may charge is

$$p_{t-1} \leq \eta q_{t-2} + \left(\overline{h} - \delta\right)(q_{t-1} - q_{t-2}) \tag{32}$$

which from (27) implies that the highest price firm t may charge is

$$p_t \leq \frac{1}{2}\eta\,(q_t + q_{t-2}) + \frac{1}{2}\left\{\left(\overline{h} - \delta\right)(q_{t-1} - q_{t-2}) + \left(\overline{h} + \delta\right)(q_t - q_{t-1})\right\} \tag{33}$$

Since the inequality in (31) holds for a sufficiently small p_t and the above is the highest p_t that may be charged in equilibrium when the third firm is priced out, it suffices to ensure that the inequality indeed holds for this value of p_t. Simplifying and rearranging, the inequality reduces to

$$-2\delta\frac{q_{t-1}}{q_t} + 3\left(\overline{h} - \eta - \delta\right)\frac{q_{t-2}}{q_t} < 3\overline{h} - 5\delta - 3\eta.$$

Since $\overline{h} - \delta - \eta > 0$ by assumption, the left side is below $\frac{q_{t-1}}{q_t}\left(3\underline{h} - 5\delta - 3\eta\right)$. Hence the above is satisfied if and only if

$$3\overline{h} - 3\eta - 5\delta > 0.$$

Together with the converse of (28), this is the sufficient condition under which exactly two firms will produce in equilibrium.

To solve for the equilibrium prices and quantities, rewrite the profit maximization of firm $t-1$ as

$$\max_{p_{t-1}} \left\{ \frac{p_{t-1} - \eta q_{t-1}}{2\delta} \left(\frac{p_t - p_{t-1}}{q_t - q_{t-1}} - \left(\overline{h} - \delta\right) \right) \right\}$$

subject to (32). Assuming an interior solution, the first order condition of the above and (27) yield the equilibrium prices

$$p_{t-1} = \frac{1}{3} \left\{ \left(3\delta + \eta - \overline{h}\right) q_t - \left(3\delta - \overline{h} - 2\eta\right) q_{t-1} \right\}, \quad (34)$$

$$p_t = \frac{1}{3} \left\{ \left(\overline{h} + 2\eta + 3\delta\right) q_t - \left(\overline{h} - \eta + 3\delta\right) q_{t-1} \right\}. \quad (35)$$

The solution is an interior solution if (34) indeed satisfies (32)

$$\frac{3\left(\overline{h} - \eta - \delta\right)}{3\delta + \eta - \overline{h}} > \frac{q_t - q_{t-1}}{q_{t-1} - q_{t-2}}. \quad (36)$$

Otherwise p_{t-1} is equal to the corner given by (32) which leads to a best response of p_t given by (33). Which case applies depends on the exact values of (q_t, q_{t-1}, q_{t-2}). Accordingly, the firms' payoffs as functions of the qs are not differentiable at the corner. To avoid dealing with the complexity that results, I shall ignore the possibility that the alternative price functions may apply and assume that (36) is satisfied in equilibrium. Let $q_t = (1 + g_t)\, q_{t-1}$. Then the condition becomes

$$\frac{3\left(\overline{h} - \eta - \delta\right)}{3\delta + \eta - \overline{h}} > \frac{g_t}{g_{t-1}} (1 + g_t).$$

It is shown in the text that the firms' choices of g will lie in the interval $(\widehat{g}, g^*)$ where $\widehat{g}$ and g^* are as defined in (A1c) and (15) respectively. Hence the inequality is guaranteed to hold if

(A3) $\frac{3\left(\overline{h}-\eta-\delta\right)}{3\delta+\eta-\overline{h}} > g^* (1 + g^*) / \widehat{g}$.

The calculations to follow are simpler under the set of price functions in (34) and (35) than the price functions in (32) and (33) while the results of the paper stand irrespective of which set of price functions is used in the calculations.

Substituting (34) and (35) into (26) yields the marginal consumer between the two firms

$$h_t = \frac{2\overline{h} + \eta}{3}.$$

The quantities sold are equal to

$$\begin{aligned} z_t &= \frac{\overline{h} + \delta - h_t}{2\delta} = \frac{\overline{h} - \eta + 3\delta}{6\delta}, \\ z_{t-1} &= \frac{h_t - \overline{h} + \delta}{2\delta} = \frac{3\delta + \eta - \overline{h}}{6\delta}. \end{aligned}$$

The unit net revenues are

$$\begin{aligned} p_t - \eta q_t &= \frac{1}{3}\left(\overline{h} - \eta + 3\delta\right)(q_t - q_{t-1}), \\ p_{t-1} - \eta q_{t-1} &= \frac{1}{3}\left(3\delta + \eta - \overline{h}\right)(q_t - q_{t-1}). \end{aligned}$$

The profit functions can be obtained by taking the product of the quantities sold and the unit net revenues.

B. Proofs of Lemmas 1-5

Proof of Lemma 1.—

By (A1c), the first term of the left side of (14) is initially positive, reaches 0 at $g_t = g^*$ and eventually turns negative while by (A1b), (A1c) and (A2), the second term is also initially positive, reaches 0 at some $\widehat{g} < g^*$ and turns negative thereafter. Hence there is a unique $g_t > 0$ that solves (14) and the condition is sufficient for optimum.

Proof of Lemma 2.—

The first order condition (14) can only be satisfied at a point where the first term is positive because the second term turns negative at a smaller g_t than the first term does. But the first term is the product of $\partial\left(\lambda\left(g_t\right) g_t\right)/\partial g_t$ which is positive only for $g_t < g^*$ and a positive term that does not depend on g_t.

Proof of Lemma 3.—

Call the left sides of (17) and (18) F^1 and F^2 respectively. From lemma 2, the search for the steady state equilibrium can be restricted to $g \in (\widehat{g},\ g^*)$. Then the first term of F^1 is positive and the second term and in particular $\lambda'(g)(1+g)+\lambda(g)<0$. It follows that $F^1_x<0$ in equilibrium, where the subscript denotes partial derivatives. Suppose the term $\frac{\varphi(x)\lambda(g)g}{r+\varphi(x)\lambda(g)}$ stays constant when g is varied. Lemma 1 implies that $F^1_g<0$ at the point where the condition is satisfied. Now since $\lambda'(g)(1+g)+\lambda(g)<0$ for $g>\widehat{g}$, the effect of g on F^1 through the term $\frac{\varphi(x)\lambda(g)g}{r+\varphi(x)\lambda(g)}$ is likewise negative. With F^1_x and F^1_g both negative, (17) defines a negatively sloped relationship between g and x for $g \in (\widehat{g},\ g^*)$. And it is straightforward to verify that as $g \to \widehat{g}$, $x \to \infty$ and as $g \to g^*$, $x \to 0$. As to (18), the proof of lemma 4 and the strict concavity of φ imply that $F^2_x<0$ and $F^2_g>0$ for $g<g^*$. This defines a positively sloped relationship between g and x. It is not difficult to see that as $g \to 0$, $x \to 0$ and as $g \to g^*$, x tends to some finite positive number. Therefore the two curves must intersect once and only once at some $g \in (\widehat{g},\ g^*)$.

Proof of Lemma 4.—

It follows from lemma 1 that left side of (14) is decreasing at the optimal g_t, and from lemma 2 that $\lambda'(g_t)(1+g_t)+\lambda(g_t)<0$. Hence g_t is decreasing in $\frac{\varphi(x_t)\lambda(g_{t+1})g_{t+1}}{r+\varphi(x_{t+1})\lambda(g_{t+2})}$. The expression is clearly increasing in x_t and g_{t+2} and decreasing in x_{t+1}. Lemma 2 implies that the $t+1$th innovator will also choose a $g_{t+1}<g^*$ from which it follows that varying g_{t+1} raises $\lambda(g_{t+1})g_{t+1}$.

Proof of Lemma 5.—

Denote the solution to (14) as $g_t = g(x_{t+1}, x_{t+2}, g_{t+1}, g_{t+2})$ and substitute it for g_t in (16). This defines x_{t-1} as a function of x_{t+1}, x_{t+2}, g_{t+1} and g_{t+2} only. Call the left side of (16) G. Note that $\partial G/\partial x_{t-1}<0$ because of the strict concavity of φ. In evaluating dG/da where $a = x_{t+1},\ x_{t+2},\ g_{t+1}$ or g_{t+2}, note that

$$\frac{dG}{da} = \frac{\partial G}{\partial a} + \frac{\partial G}{\partial g_t}\frac{dg_t}{da} = \frac{\partial G}{\partial a}$$

because by (14), $\partial G/\partial g_t = 0$ at $g_t = g\left(x_{t+1}, x_{t+2}, g_{t+1}, g_{t+1}\right)$. This means that the indirect effect of the variable concerned on G via g_t can be ignored. Assumption (A3) in Appendix A implies that $\frac{3\delta+\overline{h}-\eta}{3\delta-\overline{h}+\eta} > g_{t+1}\frac{1+g_t}{g_t}$, under which G is decreasing in x_t and x_{t+1} and increasing in g_{t+1} and g_{t+2}.

C. Proof of Proposition 2

By straightforward differentiation, $F^1_{\overline{h}} > 0$, $F^2_{\overline{h}} > 0$, $F^1_{\delta} < 0$ and $F^2_{\delta} > 0$. Recall from the proof of lemma 5 that $F^1_x < 0$, $F^1_g < 0$, $F^2_x < 0$ and $F^2_g > 0$. Totally differentiate the system (17) and (18)

$$\begin{aligned}
\frac{dg}{d\delta} &= \frac{F^1_{\delta}\left(F^2_x/F^1_x\right) - F^2_{\delta}}{F^2_g - F^1_g\left(F^2_x/F^1_x\right)} < 0,\\
\frac{dx}{d\delta} &= \frac{F^1_{\delta}\left(F^2_g/F^1_g\right) - F^2_{\delta}}{F^2_x - F^1_x\left(F^2_g/F^1_g\right)} \gtrless 0\,(?)\,,\\
\frac{dg}{d\overline{h}} &= \frac{F^1_{\overline{h}}\left(F^2_x/F^1_x\right) - F^2_{\overline{h}}}{F^2_g - F^1_g\left(F^2_x/F^1_x\right)} \gtrless 0\,(?)\,,\\
\frac{dx}{d\overline{h}} &= \frac{F^1_{\overline{h}}\left(F^2_g/F^1_g\right) - F^2_{\overline{h}}}{F^2_x - F^1_x\left(F^2_g/F^1_g\right)} > 0.
\end{aligned}$$

The signs of $dx/d\delta$ and $dg/d\overline{h}$ cannot be established simply by referring to the signs of the components making up the respective expressions. Part (b) can be proved by appealing to the following arguments. From (17) and at $r = 0$

$$\left(\lambda'\left(g\right)g + \lambda\left(g\right)\right)\left(\frac{\overline{h} - \eta + 3\delta}{3\delta + \eta - \overline{h}}\right)^2 + \left(\lambda'\left(g\right)\left(1+g\right) + \lambda\left(g\right)\right)g = 0.$$

The term $\left(\frac{\overline{h}-\eta+3\delta}{3\delta+\eta-\overline{h}}\right)^2$ is increasing in $\overline{h}$, while $\lambda'\left(g\right)g + \lambda\left(g\right) > 0$ in equilibrium. Therefore at a higher $\overline{h}$, the condition can only be satisfied at a larger g given that

$F_g^1 < 0$. This shows that $dg/d\overline{h} > 0$ at $r = 0$. Since $dg/d\overline{h}$ is a continuous function of r, $dg/d\overline{h} > 0$ holds for r sufficiently close to 0. Part (d) can be established by evaluating $dx/d\delta$ from substituting in the expressions for the individual components that constitutes $dx/d\delta$.

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