A Model of International Airline Alliances

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Abstract

This paper develops a theoretical model to investigate the effect of airline alliances on market outcome for fairly general demand and cost specifications. Two typical alliance types are examined: complementary and parallel alliances. The complementary alliance refers to the case where two firms link up their existing networks so as to feed traffic to each other, while the parallel alliance refers to collaboration between two firms who, prior to their alliance, are competitors on some routes of their networks. We find that a complementary alliance is likely to increase total output whereas a parallel alliance is likely to decrease it. The results of an empirical test from trans-Atlantic alliance routes for the 1990-94 period confirm the theoretical predictions on partners' output and total output.

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A MODEL OF INTERNATIONAL AIRLINE ALLIANCES

1. INTRODUCTION

International strategic alliances have occurred in a broad spectrum of industries including the automobile, commercial aircraft, electronic equipment, steel, and telecommunication industries (*Economist,* September 11, 1993). Among these industries, the airline industry has the largest number of alliances. More than 50 new alliances have been formed every year since 1994, and the number of alliances as of 1997 reached 502 (*Airline Business*, June 1998). This is up from almost none a decade ago. These alliances have been spurred on by regulatory barriers such as the lack of access to domestic markets by foreign carriers, limits on foreign ownership, or simply the fear of being left behind (Gallacher and Odell, 1994).

In order to attract more passengers in an increasingly competitive environment, international airlines have been seeking to extend the range of their networks and access new markets.[1] International alliances allow carriers to expand the reach of their networks and services to many parts of the world where it may not be economical to do so on their own, or where there may be a lack of authority to operate their own flights. Alliances may provide opportunities for the partner airlines to reduce costs by coordinating activities in various fields: joint use of ground facilities such as lounges, gates and check-in counters; codesharing[2] or joint operation; block space sales;[3] joint advertising and promotion; exchange of flight attendants; and so on. As a result, the partners may become more cost-effective and increase their competitiveness.

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[1] In airline markets, there are demand forces such that consumers prefer airlines which serve a large number of points to ones which serve a small number of points, with all other factors (such as prices) being held constant (Tretheway and Oum, 1992).

[2] A codesharing agreement is a marketing arrangement between two airlines whereby one airline's designator code is shown on flights operated by its partner airline. For example, Lufthansa has been codesharing on United Airlines' flight between Frankfurt and 25 U.S. interior cities via two of United's hubs (Chicago O'Hare and Washington Dulles).

[3] If two carriers make a block space sale agreement, each carrier can buy a block of seats in the other carrier's flights and resell them to passengers. For example, Air Canada and Korean Air have signed such an agreement on the Seoul-Vancouver-Toronto route, under which each buys 48 seats from the other's flights on the route.

Alliances can also provide benefits to consumers. Alliance partners can better coordinate flight schedules to minimize travellers' waiting time betw en flights while providing sufficient time for connections. Joint baggage handling eliminates the need to retrieve and re-check baggage a connecting places, and thus reduces the risk associated with interline handling in which no single carrier has the sole responsibility for the baggage. Consumers can enjoy greater choices due to alliances. Consider, for example, a passenger who wants to fly from Indianapolis to Lyon. She could fly Indianapolis-Washington, D.C.-Frankfurt-Lyon on United/Lufthansa partners' flights. She could also fly Indianapolis-Pittsburgh-London-Lyon on British Airways/USAir alliance flights. Alternatively, she could fly Indianapolis-Detroit-Amsterdam-Lyon on KLM/Northwes alliance flights. Without the alliances, she would have to interline on several different carriers with great inconvenience.

Despite these benefits, concer s have been expressed on potential anticompetitive effects of international alliances. Most international airline routes have a few competitors. Thus, an alliance between any two significant competitors on an international route may adversely affec the degree of competition on the route. For example, European Union officials recently expressed concerns about the increase in concentration on some trans-Atlantic routes after major alliances, and seemed bent on pushing ahead w th measures to restrict them (e.g., *Business Week*, 2 March 1998).

In this paper we attempt to examine whether the increase in post-alliance concentration on certain routes should be viewed as a cause for concern. More specifically, the paper develops a theoretical model to investigate the effect of airline alliances on market outcome for fairly general demand and cost specifications. Two typ al alliance types are examined: complementary and parallel alliances. The complementary alliance refers to a situation where two firms link up their existing networks so as to feed traffic to each other, while the parallel alliance refers to collaboration between two firms who, prior to their alliance, are competitors on some routes of their networks. We find that a complementary alliance is likely to increase total output whereas a parallel alliance is likely to decrease it. We further conduct an empirical test using data from trans-Atlantic alliance routes for the 1990-94 period. The results appear to confirm the theoretical predictions on partners' output and total output.

The effects of international airline alliances have previously been investigated elsewhere. The empirical investigation includes Gellman Research Associates (USDOT, 1994), Youssef and Hansen (1994), the U.S. General Accounting Office (USGAO, 1995), Oum, *et al.* (1996), Brueckner and Whalen (1998), and Park and Zhang (1999). In particular, Oum, *et al.* (1996) examined the effect of codesharing agreements on firm conduct and air fares by focusing on trans-Pacific markets. They found that a codesharing agreement between small carriers ("non-leaders") increases the market leader's output and reduces its price. Brueckner and Whalen (1998) examined whether alliance partners charge lower interline fares than non-allied carriers. They found that an alliance between two previous competitors would raise fares by about 5 percent in their gateway markets, but the effect is not statistically significant. Park and Zhang (1999) investigated alliance effects on air fares, passenger volume, and consumer surplus in Nor h Atlantic aviation markets. They found that, on average, output increases and air fares fall on the routes served by allying carriers.

Theoretical work on international airline alliances is relatively rare and more recen . Brueckner (1997) and Park (1997) examined theoretically the effects of alliances on traffic leve and welfare using linear demand and linear marginal cost functions. In particular, Brueckner (1997) pointed out that an alliance would reduce competition in the gateway (or interhub) market which was previously served by the codesharing partners, but cooperative pricing of trips by the partners would increase traffic in the connecting market since portions of a connecting trip are complements. Brueckner showed that in the presence of increasing returns to traffic density (Caves, *et al.*, 1984; Brueckner and Spiller, 1994), the beneficial effect of codesharing outweighs its harmful effect for most parameter values of his theoretical model. The present paper is in the same spirit of Brueckner (1997) and Park (1997), but it investigates the issues for more genera demand and cost specifications and for two typical alliance types, namely, complementary and parallel alliances.

The paper is organized as follows. In the next section, a theoretical model is developed to compare pre- and post-alliance situations for both complementary and parallel alliances. The effects on market outcome of complementary and parallel alliances are examined in Sections 3 and 4, respectively. Section 5 presents test results for our theoretical predictions and Section 6 concludes.

2. THE MODEL

We begin by constructing a pre-alliance situation where none of airlines have yet to make any type of alliance. As depicted in Figure 1, a simple air transport network is considered, consisting of three gateway cities located in different countries: a, b and h. The network consists of two route segments, *ah* and *hb*, where three air carriers are operating. Firm H is assumed to serve both segments using its hub-spoke network with city h being the hub, while the other tw firms, A and B, serve the *ah* and *hb* segments, respectively.[4] Notice that there are both local (i.e., a to h, h to b) passengers and connecting (.e., a to b) passengers (traffic also includes passengers returning from h to a, b to h, and b to a). For local traffic *ah* (*hb*, respectively), travellers can fly with carriers A or H (carriers B or H, respectively). For connecting traffic *ab*, travellers change

[4] Note that two national carriers are assumed to operate on each route of the network. Since international air services between two cities are mainly decided by bilateral agreements between the two countries involved, this assumption seems to be reasonable. Our results can be derived from a more realistic network structure considered, for example, by Nero (1996), although it makes analysis more complex.

aircraf at city h. They can either fly with firm H on segments *ah* and *hb*, or fly with A on segment *ah* and then change to B's flight on segment *hb*.[5]

**** FIGURE 1 ABOUT HERE ****

For tractability of the analysis, we consider segment-based demand specifications. The firm-specific demand functions for each segment may be written as:

$$Q^i = D^i(\rho^1, \rho^2, \rho^3, \rho^4) \quad for\ i = 1,2,3,4$$

where ρ^1 is the "full" price of travelling with firm H on segment (route) *ah*, ρ^2 is the "full" price of travelling with firm H on *hb*, ρ^3 is the "full" price of travelling with firm A on *ah*, and ρ^4 is the "full" price of travelling with firm B on *hb*. Note that the route demand for each carrier depends on the full prices of both routes offered by all the carriers. For example, Q^1 depends not only on ρ^1 and ρ^3, the full prices of travelling on route *ah* offered by carriers H and A, but als on ρ^2 and ρ^4, the full prices of travelling on route *hb* offered by carriers H and B. This is because some passengers travelling on *ah* will connect to *hb* in order to complete travel between cities a and b. Hence, changes in ρ^2 or ρ^4 will shift the demand by these connecting passengers and thereby affect the demand on route *ah*.

In the above demand specification, the full price, ρ^i, is taken to be the sum of ticket price, denoted p^i, and non-ticket cost of travel borne by passengers (De Vany, 1974; Panzar, 1979). The most important cost is the schedule delay cost which arises from the difference between a passenger's desired departure and actual departure time. Research has found that the

5 If connections must be made at connecting airports or hubs, less travel time will usually be required with a single airline than when the trip involves switching airline. This is because a single airline's connecting flights are more likel to reduce waiting time at the connecting airports and have a lower probability of baggage being lost than interlining flights.

schedule delay cost on a route depends largely on a carrier's flight frequency, which in turn depends on its *total traffic* on the route (e.g., Douglas and Miller, 1974). Thus, if Q^i represents total passengers carried by a carrier on route i, then the schedule delay cost may be written as $g^i(Q^i)$. The full price is thus

$$\rho^i = p^i + g^i(Q^i).$$

The schedule delay cost for the connecting service is the sum of the schedule delay cost of each local route, $g^i(Q^i) + g^j(Q^j)$.

Solving the demand functions for ρ^i yields the following inverse demand functions:

$$\rho^i = d^i(Q^1, Q^2, Q^3, Q^4) \quad for\ i = 1,2,3,4.$$

A carrier's cost function on route i may be expressed as $C^i(Q^i)$, implying its (round-trip) cost of carrying Q^i passengers on the route. Note that $\dot{Q}$ represents *total passengers* carried by the airline on the route. Given these demand and cost specifications, the carriers' profit functions can be expressed as:

$$\begin{aligned} \Pi^H \equiv\ & Q^1\left[d^1(Q^1,Q^2,Q^3,Q^4) - g^1(Q^1)\right] - C^1(Q^1) \\ & + Q^2\left[d^2(Q^1,Q^2,Q^3,Q^4) - g^2(Q^2)\right] - C^2(Q^2) \end{aligned} \qquad (1)$$

$$\begin{aligned} \Pi^A &\equiv Q^3\left[d^3(Q^1,Q^2,Q^3,Q^4) - g^3(Q^3)\right] - C^3(Q^3) \\ \Pi^B &\equiv Q^4\left[d^4(Q^1,Q^2,Q^3,Q^4) - g^4(Q^4)\right] - C^4(Q^4). \end{aligned} \qquad (2)$$

We assume that, owing to the presence of connecting passengers, a higher traffic volume on the *hb* route will have a positive effect on the demand and thus the carriers' marginal profits on the *ah* route, and vice versa. This implies that there are network complementarities between the two local routes *ah* and *hb* for carrier H or for carriers A and B (Oum, *et al.*, 1996).

Specifically,

$$\Pi^H_{12} > 0, \quad \Pi^A_{34} > 0, \quad \Pi^B_{43} > 0 \tag{3}$$

where subscripts denote partial derivatives with respect to Q. Oum, *et al.* (1995) contain an explicit derivation of network complementarities in a hub-spoke network. The main economic forces behind network complementarities are the increasing returns to traffic density (i.e., the cos per passenger on a given falls as the number of passengers travelling on the route rises) and the decreasing schedule delay cost of an airline in its traffic on a given route (i.e., $g^{/}(Q^{i}) < 0$).

Next we assume that outputs of rival carriers on each route are strategic substitutes in the terms of Bulow, Geanakoplos and Klemperer (1985), i.e.,

$$\Pi^H_{13} < 0, \quad \Pi^A_{31} < 0, \quad \Pi^H_{24} < 0, \quad \Pi^B_{42} < 0. \tag{4}$$

This assumption is fairly standard in oligopolistic competition with quantity as the strategy variable (e.g., Dixit, 1986). Furthermore, owing to the effects of connecting traffic, the outputs of rival carriers on different route segments will also be strategic substitutes, i.e.,

$$\Pi^H_{14} < 0, \quad \Pi^B_{41} < 0, \quad \Pi^H_{23} < 0, \quad \Pi^A_{32} < 0. \tag{5}$$

A "complementary" alliance between firms A and B is modelled as follows. Both firms, while continuing to provide local services, now jointly provide connecting services for passengers travelling between cities a and b. In order to compete with firm H's connecting services, the partners enhance the quality of their connecting services by adjusting arrival and departure flights so as to minimize waiting time between flights while providing sufficient time for connections. They can further re-locate departure gates for connecting flights close to arrival gates, coordinate baggage transfer, and cooperate other joint activities at the connecting airport. The partners w also coordinate their output decisions so as to maximize their joint profit.

Next, consider another post-alliance situation where firms H and A make a "parallel" alliance in the sense that they are competitors on the *ah* segment of the network before the alliance. After the alliance, each partner continues to provide local services in the *ah* segment but chooses quantities to maximize the joint profit. As an example, Air Canada (a hub partner) and Korean Air (a non-hub partner) have implemented this type of parallel alliance on the Seou - Vancouver-Toronto route since 1993.

3. EFFECTS OF COMPLEMENTARY ALLIANCE

3.1 Output and Profit

We first analyze the effects of the complementary alliance. We consider an equilibriu that arises when carrier H and the aligned partners play a Cournot game on each segment of the network. To facilitate the comparison with the pre-alliance situation, we formulate the carriers' problem as follows:

$$\begin{aligned} &\max_{Q^1, Q^2} \ \Pi^H \\ &\max_{Q^3} \ \Pi^A + \theta \Pi^B \\ &\max_{Q^4} \ \Pi^B + \theta \Pi^A. \end{aligned} \tag{6}$$

Clearly, $\theta = 0$ and 1 correspond to the pre-alliance situation and the complementary alliance, respectively. We thus need to compare the case of $\theta = 1$ with the case of $\theta = 0$. Unfortunately, it is extremely hard to compare output and profit directly, even in some special cases. To overcome this difficulty we use the differential techniques. Specifically, we treat θ as a continuous variable in the range $0 \le \theta \le 1$, and assume that carriers' output, $Q^i(\theta)$, is differentiable in θ in the entire range. With these assumptions, the overall effect of a complementary alliance can be calculated as the integral of small changes $d\theta$:

$$\Delta Q^i \equiv Q^i(1) - Q^i(0) = \int_0^1 \left[dQ^i(\theta)/d\theta\right] d\theta .$$

It turns out to be easy to sign the infinitesimal effect, $dQ^i(\theta)/d\theta$. Consequently, the overall effect, ΔQ^i, can be determined as well if the sign of the infinitesimal effect remains unchanged in the range $0 \le \theta \le 1$, a condition one can check.[6]

The first-order conditions for the profit-maximization problem (6) are:

$$\begin{aligned} \Pi_1^H &= 0 \\ \Pi_2^H &= 0 \\ \Pi_3^A + \theta\Pi_3^B &= 0 \\ \Pi_4^B + \theta\Pi_4^A &= 0. \end{aligned} \tag{7}$$

Assume that the second-order conditions are also satisfied and that the equilibrium is stable.[7]

Proposition C-1. Under the complementary alliance, firm H produces less output on both routes *ah* and *hb,* while the alliance partners produce greater output on both routes, than under the pre-alliance situation.

Proof. See appendix.

The intuitive explanations for Proposition C-1 are as follows. Owing to the presence of connecting passengers, an increase in the output of carrier A on the *ah* route feeds more traffic

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[6] Farrel and Shapiro (1990) and Oum, *et al.* (1995) used similar techniques in their analysis of horizontal merger effects and airline hubbing effects, respectively.

[7] The stability condition is important. If an equilibrium is not stable, then a slight deviation by one player will cause deviations of other players farther away from the equilibrium and thereby make the equilibrium untenable. In this situation the equilibrium may be unreachable and hence unobservable in practice. The stability of Cournot-Nash equilibrium has been studied by, among others, Seade (1980), Dixit (1986), Slade (1994), and Zhang and Zhang (1996).

to carrier B on the *hb* route and enhances B's marginal profit. This is the network complementarity discussed in (3). Without the alliance, the (positive) effect of carrier A's output on carrier B's profit is not considered by A because A only acts to maximize its own profit. Similarly the effect of B's output on A's profit is ignored by carrier B. With the alliance, by contrast, the carriers would behave cooperatively so that externalities from the route output decision would be internalized. In other words, since profits are jointly maximized, the alliance partners will increase their respective quantities so as to take advantage of the network complementarity. When both alliance partners become more aggressive in supplying output, carrier H would reduce its output on both routes since rivals' outputs in the same market are strategic substitutes.

Although firm H reduces its output on routes *ah* and *hb*, it does not necessarily imply a reduction in its profit. This is because profit is affected not only by output in these markets, but also by corresponding air fares. Now, we investigate whether each firm's profit increases or decreases due to the complementary alliance.

Proposition C-2. Under the complementary alliance, firm H earns less profit, but the alliance partners earn more profit, as compared to the pre-alliance situation.

Proof. Substituting the equilibrium quantities (Q^1,Q^2,Q^3,Q^4) into (6), and differentiating with respect to θ, we have:

$$\frac{d\Pi^H}{d\theta} = \sum_{k=1}^{2} \frac{\partial \Pi^H}{\partial Q^k}\frac{dQ^k}{d\theta} + \sum_{k=3}^{4} \frac{\partial \Pi^H}{\partial Q^k}\frac{dQ^k}{d\theta} \tag{8}$$

$$\frac{d\Pi^i}{d\theta} = \sum_{k=1}^{2} \frac{\partial \Pi^i}{\partial Q^k}\frac{dQ^k}{d\theta} + \sum_{k=3}^{4} \frac{\partial \Pi^i}{\partial Q^k}\frac{dQ^k}{d\theta}, \qquad i = A,B. \tag{9}$$

By the first-order conditions, the network complementarity, and the results of Proposition C-1, it can be easily verified tha $d\Pi^H/d\theta < 0$, $d\Pi^A/d\theta > 0$, $d\Pi^B/d\theta > 0$. *Q.E.D.*

3.2 Total Output, Full Price and Welfare

According to Proposition C-1, it is not clear whether *total* output on each route increases or decreases due to the complementary alliance since firm H decreases output in each marke, while the aligned partners increase their output. In this section we examine the effects of the alliance on total output, full price and welfare.

Proposition C-3. The complementary alliance results in (i) increased total outpu t an d ( ) decreased full price, on at least one of the *ah* and *hb* routes. Thus, consumers in at least one of these markets are better off due to the complementary alliance. Furthermore, in the special case where the demand is symmetric between routes *ah* and *hb* and carriers A and B have the same cost function, the complementary alliance results in incr sed total output and decreased full price on both routes.

Proof. See appendix.

We have shown that, after a complementary alliance, the partners will increase output on each route segment and the full price will decline in at least one market. The effect of a complementary alliance on total welfare of the network is ambiguous, however. To show this, we assume a partial equilibrium framework in which consumer demand for air travel in eac h market is derived from a utility function which can be approximated by the for

$$U(Q^1, Q^2, Q^3, Q^4) + Z$$

where Z is expenditure on a competitively supplied *numeraire* good, and $\partial U/\partial Q^i = \rho^i$. Recalling that ρ^i is the full price of travelling on route i, i.e., $\rho^i = p^i + g^i(\cdot)$, the consumer surplus in this framework can be written as:

$$CS = U(Q^1, Q^2, Q^3, Q^4) - \sum_{i=1}^{4} \rho^i Q^i \tag{10}$$

Total surplus can then be written as:

$$W = CS + \left(\Pi^H + \Pi^A + \Pi^B\right) \tag{11}$$

where W may be interpreted as "world welfare" since the markets under consideration involve different countries.

Substitution of (1) and (2) into (11) yields the following expression for W:

$$W = U(Q^1, Q^2, Q^3, Q^4) - \sum_{i=1}^{4} \left[g^i(Q^i) \cdot Q^i + C^i(Q^i)\right]. \tag{12}$$

Differentiating (12) with respect to θ and using $\partial U / \partial Q^i = \rho^i = p^i + g^i$, we obtain

$$\frac{dW}{d\theta} = \sum_{i=1}^{4} [p^i - g^{i\prime} Q^i - C^{i\prime}] \frac{dQ^i}{d\theta}. \tag{13}$$

The signs of the terms in the brackets depend on the degree of mark-up of the carriers. From the first-order conditions, these signs are ambiguous. Hence, the overall effect of the complementar alliance on world welfare is not clear.

4. EFFECTS OF PARALLEL ALLIANCE

This section examines the effect of the parallel alliances. Recall that the two partners continue to individually provide local services after their alliance. Based on the assumption tha carriers H and A choose to be alliance partners, we further assume that the connecting traffic handled by carriers A and B before the alliance is negligible. Hence,

$$\Pi^A = \Pi^A(Q^1, Q^3), \quad \Pi^B = \Pi^B(Q^2, Q^4). \tag{14}$$

Now, we show that unlike the complementary alliance, parallel alliance partners are likely to decrease their total output on the *ah* route after their alliance.

Proposition P-1. If the non-hub partner (i.e., carrier A) produces the same amount of output after the parallel alliance, then the hub partner (i.e., carrier H) produces less output on both routes, and the non-partner (i.e., c rrier B) produces more output on the *hb* route than under the pre-alliance situation.

Proof. See appendix.

The intuition behind Proportion P-1 is that by forming a parallel alliance and maximizing the joint profit, the hub partner chooses Q^1 taking account of the negative externalities of the hub partner's output on the non-hub partner's profit. This leads to decreases in the hub partner's output on route *ah*. Consequently, the hub partner decreases its *bh* traffic as well, owing to the network complementarity. Similarly, we can show:

Proposition P-2. If the hub partner (i.e., firm H) produces the same amount of output after the parallel alliance, then t e non-hub partner (i.e., firm A) decreases its output, and the non-partner (i.e., firm B) produces the same amount of output, as compared to the pre-alliance situation.

The next question naturally arises: what if the outputs of both alliance partners are chosen endogenously? In this case, it is not possible for both partners to produce greater outputs on route *ah*. The result is stated in Proposition P-3:

Proposition P-3. $dQ^1/d\theta$ and $dQ^3/d\theta$ cannot both be positive.

Proof. Now the optimization problem may be set as follows:

$$\max_{Q^1,Q^2} \ \Pi^H + \theta\Pi^A$$

$$\max_{Q^3} \ \Pi^A + \theta\Pi^H$$

$$\max_{Q^4} \ \Pi^B$$

Differentiating the first-order conditions with respect to θ, we have

$$\Psi\frac{dQ}{d\theta} = -\begin{bmatrix}\Pi_1^A \\ 0 \\ \Pi_3^H \\ 0\end{bmatrix} \tag{15}$$

where

$$\Psi = \begin{bmatrix}\Pi_{11}^H + \theta\Pi_{11}^A & \Pi_{12}^H & \Pi_{13}^H & 0 \\ \Pi_{21}^H & \Pi_{22}^H & 0 & \Pi_{24}^H \\ \Pi_{31}^A + \theta\Pi_{31}^H & 0 & \Pi_{33}^A + \theta\Pi_{33}^H & 0 \\ 0 & \Pi_{42}^B & 0 & \Pi_{44}^B\end{bmatrix}.$$

From the fourth equation in (15), it can be easily verified that $dQ^2/d\theta$ and $dQ^4/d\theta$ have opposite signs. $dQ^1/d\theta$ and $dQ^3/d\theta$, however, are interdependent with each other. By the third equation in (15),

$$\frac{dQ^3}{d\theta} = \frac{-1}{\Pi_{33}^A + \theta\Pi_{33}^H}\left[\Pi_3^H + (\Pi_{31}^A + \theta\Pi_{31}^H)\frac{dQ^1}{d\theta}\right].$$

Due to the assumptions of strategic substitutes,

$$\Pi_3^H < 0, \quad \Pi_{31}^A < 0, \quad \Pi_{31}^H < 0.$$

Also, by second-order condition,

$$\Pi_{33}^A + \theta\,\Pi_{33}^H < 0.$$

Hence $dQ^1/d\theta$ and $dQ^3/d\theta$ cannot both be positive. *Q.E.D.*

We note that, although both $dQ^1/d\theta$ and $dQ^3/d\theta$ cannot be positive at the same time, it is possible that both $dQ^1/d\theta$ and $dQ^3/d\theta$ are negative.

To sum up the effects of the parallel alliance on each firm's output, the partners' total output is likely to decrease, while the non-partner output may increase (by Proposition P-1), remain unchanged (by Proposition P-2), or decrease (by Proposition P-3). Thus, consumer surplus on the *ah* route is likely to decrease due to this type of alliance.

5. EMPIRICAL TEST

Our model predicts that after the complementary alliance, the partners increase output while the non-partner decreases output in each of the market egment (Proposition C-1). Overall, the total output is likely to increase as a result (Proposition C-3). On the other hand, from the analysis of the parallel alliance, the partners are likely to decrease output on the market segmen where they form the alliance (Propositions P-1 and P-2). Changes in non-partners' outputs are uncertain. Overall, total output on the market segment is likely to decrease (Propositions P-3).

We tested these predictions by using panel data of trans-Atlantic alliance routes for the 1990-94 period. To identify trans-Atlantic alliance routes, we first selected 17 US and 12 European gateway cities. Among the routes from the combination of these cities, four major alliances between US and European carriers took place on seventeen routes during the 1990-94 period: British Airways/USAir, Delta/Sabena/Swissair, Northwest/KLM, and United/Lufthansa (see Table 1). The alliance routes were identified by using mainly the *Official Airline Guides (OAG): Worldwide Edition.* Since t e OAG indicates the flights of airlines involving alliances as the "★" symbol, those routes with that symbol were regarded a s alliance routes. The February and July issues of the OAG were used to check whether a particular alliance continued on the alliance route. We also supplemented the *Airline Business* (1994), *International Civil Aviation Organ zation (ICAO) Journal* (1990-94), *USDOT* (1994), and *USGAO* (1995) to verify our identification of alliance routes. For each of the seventeen alliance routes, we collected alliance partners' traffic, non-partners' traffic, and total traffic data from the International Civil Aviation Organization (ICAO) publication, *Traffic By Flight Stage*. To control for the firm size of non-partners, we restricted our analysis to the largest non-partner airlines on the route. The total numbers of observations available for the alliance partners (and thus for total traffic) and the largest non-partner carriers are 151 and 97, respectively. The mean value for the alliance partner's passenger volume during the period is 108,200 people, while the mean value for the total traffic is 247,770 people. The number of carriers on each route was also obtained from the ICAO publication.

**** TABLE 1 ABOUT HERE ****

The alliance type for each of the seventeen routes was determined based on the OAG and the numerous publications mentioned above. If alliance partners fed their traffic to each other beyond an alliance route, that alliance was regarded as a comp ementary alliance. Complementar alliances took place on twelve out of the seventeen alliance routes. If two competing airlines on an route formed an alliance and collaborated only on that route, that alliance was regarded as a parallel alliance. Parallel alliances occurred on three out of seventeen alliance routes. However, alliances on the other two routes had both characteristics of complementary and those of parallel alliances since the alliance partners were competitors on those routes prior to their alliances and they cooperated not only on those routes but also beyond those routes. We treated those alliances as both complementary and parallel alliances. Therefore, our database consists of fourteen complementary-alliance routes and five parallel-alliance routes, as shown in Table 1. We then demarcated pre-alliance and post-alliance periods for each alliance-route. Due to the nature of the yearly traffic data we used, we relied on a subjective cut-off date (i.e., June 30) for determining the post-alliance period. If an alliance took place prior to June 30, we regarded tha year as the starting year of the post-alliance period. However, if the alliance took place after June 30, we regarded the following year as the starting point of the post-alliance period.

For the test on the partners' traffic, we ran the following regression:

$$Q_P = \beta_1\, CA + \beta_2\, PA + \beta_3\, NUM + \sum \beta_t\, YR_t + \sum \beta_j\, RTE_j + \epsilon_P \qquad (16)$$

where Q_P is alliance partners' traffic, CA is the dummy variable coded as one for the post-alliance period of complementary alliance and zero otherwise, PA is the dummy variable coded as one for the post-alliance period of parallel alliance and zero otherwise, NUM is the number of airlines on a given route, YR_t is an year-specific dummy variable, RTE_j is a route-specific dummy variable, and ϵ_p is a random-error term. For the test of non-partners' traffic and total traffic, the non-partners' traffic and total traffic were used for dependent variables in (16) instead of Q_P. Route Atlanta-Amsterdam and year 1990 is used as a base route and year in the regression.

**** TABLE 2 ABOUT HERE ****

Table 2 shows the test results for our predictions from the theoretical model. The test results generally confirm the theoretical predictions. First, as shown in the first column of Table 2, the test results on alliance partners' outputs areconsistent with the corresponding propositions. As excepted, all the coefficients of CA are estimated as positive, regardless of specifications. More importantly, the coefficients of CA are estimated as highly significant under the specifications (1) and (2). These results confirm that each of complementary alliance partners increases its traffic after the alliance. For parallel alliance partners' outputs, all coefficients are estimated as negative, implying that demand shift effects on the partners' outputs are weak. The coefficients of PA under the specifications (3) and (4) are estimated as negative and significant.

Second, the last column of Table 2 shows that test results on total output are high y consistent with the corresponding predictions. The coefficients of CA and PA are estimated as positive and negative (both significant at least at 5% level), respectively. Following the complementary alliance, total traffic increases by an average of 11-17 percent of the average tota traffic In contrast, total traffic decreases by an average of 11-15 percent of the average total traffic due to the parallel alliance. Third, the second column of Table 2 indicates that the test results on non-partners' outputs are partly consistent with the corresponding propositions. In general, the signs of the coefficients are consistent with the propositions, but statistically insignificant.

6. CONCLUDING REMARKS

This study analyzes the effects on market outcome and welfare of two typical types of alliances: complementary and parallel alliances. We find that a complementary alliance is likely to increase total output and decrease full price. Thus, consumer surplus is likely to increase as a result. On the other hand, a parallel alliance is likely to decrease total output and increase fu price on the market segment where the alliance is formed. Consequently, consumer surplus is likely to decrease on at least that segment. The results of an empirical test generally confirm the theoretical predictions on alliance partners' outputs and total output. The results indicate that the partners' traffic increases due to the complementary alliance, while the partners' traffic decreases due to the parallel alliance. Total traffic increases by an average of 11-17 percent in the cases of complementary alliances, while total traffic decreases by an average of 11-15 percent in the cases of parallel alliances.

These findings have some important policy implications. Government agents should be very careful in allowing would-be parallel alliance partners to have antitrust immunity. Since the partners are significant competitors in the same markets, competition may be reduced if they are able to integrate their operations with the protection of antitrust immunity. However, under certain conditions, allowing more complementary alliances may create a more competitive environment and improve welfare.

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FIGURE 1. A simple air transport network

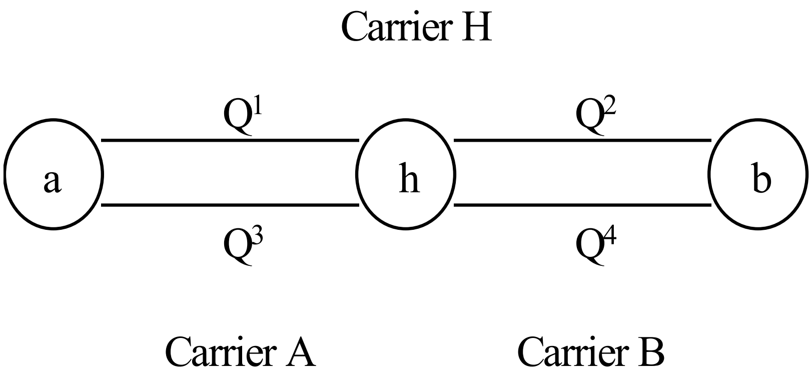

TABLE 1. Sample alliance routes

Route	Alliance partners	Alliance type[a]
Atlanta-Amsterdam	Northwest/KLM	C (94)
Atlanta-Brussels	Delta/Sabena	P (93)
Atlanta-Frankfurt	United/Lufthansa	C (94)
Boston-Amsterdam	Northwest/KLM	C (93)
Boston-London	British Airways/USAir	C (93)
Baltimore-London	British Airways/USAir	C (94)
Chicago-Frankfurt	United/Lufthansa	C + P (94)
Detroit-Amsterdam	Northwest/KLM	C (93)
Los Angeles-London	British Airways/USAir	C (94)
Minneapolis/St.Paul-Amsterdam	Northwest/KLM	C (93)
New York-Amsterdam	Northwest/KLM	C (94)
New York-Brussels	Delta/Sabena	P (94)
New York-London	British Airways/USAir	C (94)
New York-Zurich	Delta/Swissair	P (93)
Philadelphia-London	British Airways/USAir	C (93)
Washington D.C.-Amsterdam	Northwest/KLM	C (94)
Washington D.C.-Frankfurt	United/Lufthansa	C + P (94)

[a]The number in parenthesis is the starting year of the post-alliance period.

TABLE 2. Results on partners' traffic, non-partners' traffic, and total traffic

	Partners' traffic (N=151)				Non-partners' traffic (N=97)				Total traffic (N=151)			
Variables	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
CA	20,174***	21,397***	5,300	7,496	1,189	1,965	-1,262	3,400	42,180***	43,480***	29,670**	27,205**
	(5.84)[a]	(6.39)	(1.09)	(1.49)	(0.33)	(0.57)	(-0.20)	(0.54)	(5.11)	(5.11)	(2.40)	(2.12)
PA	-2,273	-6,800	-12,639***	-13,895**	3,172	-536	-560	-1,836	-28,033**	-32,843**	-37,046***	-35,636**
	(-0.41)	(-1.25)	(-2.31)	(-2.53)	(0.62)	(-0.10)	(-0.01)	(-0.34)	(-2.14)	(-2.44)	(-2.68)	(-2.55)
NU		-6,793***		-3,454		-5,579***		-5,386**		-7,216		3,876
YR91			-7,480**	-4,425			1,503	7,065			-33,490***	-36,919***
			(-1.98)	(-1.06)			(0.36)	(1.56)			(-3.52)	(-3.46)
YR92			10,318***	11,168***			15,380***	16,478***			5,340	4,386
			(2.77)	(2.98)			(3.83)	(4.22)			(0.57)	(0.46)
YR93			10,044**	10,229**			17,284***	18,563***			1,395	1,186
			(2.51)	(2.57)			(4.19)	(4.62)			(0.14)	(0.12)
YR94			22,822***	21,860***			13,071*	10,328			9,521	10,601
			(3.94)	(3.78)			(1.77)	(1.43)			(0.65)	(0.72)
ATL-BRU	-17,550**	-12,778	-19,451**	-16,741**					-21,081	-16,012	-23,475	-26,516
	(-1.90)	(-1.42)	(-2.34)	(-1.99)					(-0.95)	(-0.72)	(-1.12)	(-1.24)
ATL-FRA	-2,086	-2.086	-2,086	-2,086	43,418***	43,418***	43,418***	43,418***	41,332**	41,332**	41,332***	41,332***
	(-0.31)	(-0.32)	(-0.34)	(-0.35)	(7.44)	(7.74)	(8.53)	(8.83)	(2.53)	(2.54)	(2.69)	(2.69)
BOS-AMS	13,646**	6,609	16,621***	12,728*					-51,756***	-59,232***	-49,254***	-44,885***
	(1.99)	(0.95)	(2.70)	(1.94)					(-3.15)	(-3.46)	(-3.17)	(-2.68)
BOS-LON	67,487***	79,469***	70,462***	76,239***	41,651***	51,537***	42,141***	50,903***	214,110***	226,840***	216,610***	210,130***
	(9.82)	(10.60)	(11.44)	(10.77)	(7.08)	(7.76)	(8.04)	(8.38)	(13.03)	(12.24)	(13.92)	(11.66)
BWI-LON	-39,379***	-43,754***	-36,772***	-39,242***					-99,122***	-103,770***	-98,018***	-95,246***
	(-4.17)	(-4.76)	(-4.34)	(-4.58)					(-4.38)	(-4.56)	(-4.57)	(-4.37)
CHI-FRA	29,034***	36,732***	31,107***	34,812***	-4,780	1,540	-4,034	1,607	78,597***	86,776***	80,400***	76,242***
	(4.19)	(5.22)	(5.04)	(5.32)	(-0.81)	(0.25)	(-0.77)	(0.29)	(4.74)	(4.98)	(5.15)	(4.57)
DET-AMS	2,129	-5,153	2,511	-1,418					-67,674***	-75,411***	-70,715***	-66,306***
	(0.25)	(-0.61)	(0.33)	(-0.18)					(-3.34)	(-3.61)	(-3.63)	(-3.24)
LAX-LON	137,530***	160,630***	137,530***	149,270***	10,037*	28,301***	6,413	24,059***	375,050***	399,590***	375,050***	361,870***
	(20.12)	(16.97)	(22.62)	(15.85)	(1.85)	(3.42)	(1.34)	(2.94)	(22.95)	(17.08)	(24.41)	(15.09)
MSP-AMS	25,398***	21,023**	28,005***	25,535***					-50,217**	-55,866**	-49,114**	-46,342***
	(2.69)	(2.29)	(3.30)	(2.95)					(-2.22)	(-2.41)	(-2.29)	(-2.12)
NYC-AMS	127,220***	144,890***	127,220***	136,200***	8,238	21,069***	8,238	20,625***	171,010***	189,770***	171,010***	160,930***
	(18.61)	(17.27)	(20.92)	(16.64)	(1.41)	(2.92)	(1.62)	(3.02)	(10.46)	(9.15)	(11.13)	(7.72)
NYC-BRU	36,464***	58,672***	35,562***	46,959***	-21,447***	-3,256	-21,191***	-3,307	97,238***	120,830***	96,535***	83,748***
	(5.21)	(6.25)	(5.71)	(5.02)	(-3.57)	(-0.38)	(-4.03)	(-0.39)	(5.81)	(5.21)	(6.14)	(3.52)
NYC-LON	392,920***	421,450***	392,920***	407,430***	102,600***	124,850***	98,980***	120,470***	111,530***	114,560***	111,530***	109,900***
	(57.49)	(39.49)	(64.62)	(37.82)	(15.93)	(12.49)	(17.47)	(12.17)	(68.24)	(43.43)	(72.59)	(40.05)
NYC-ZRH	38,204***	47,052***	39,375***	43,771***	-18,488***	-11,271*	-17,486***	-10,657*	56,545***	65,945***	57,648***	52,715***
	(5.25)	(6.29)	(6.07)	(6.26)	(-2.93)	(-1.72)	(-3.17)	(-1.80)	(3.25)	(3.56)	(3.52)	(2.96)
PHL-LON	24,382***	25,496***	27,357***	27,608***					-16,017	-14,834	-13,515	-13,797
	(3.55)	(3.85)	(4.44)	(4.51)					(-0.98)	(-0.91)	(-0.87)	(-0.88)
WAS-AMS	-17,899*	-18,266**	-22,729***	-22,374***					-50,064**	-50,454**	-55,216***	-55,614***
	(-1.97)	(-2.08)	(-2.77)	(-2.74)					(-2.30)	(-2.33)	(-2.66)	(-2.68)
WAS-FRA	14,669**	23,725***	16,742***	21,137***	-11,566*	4,855	-15,992**	-9,442	81,836***	91,458***	83,639***	78,706***
	(2.44)	(3.72)	(3.11)	(3.53)	(-1.67)	(-0.69)	(-2.56)	(-1.47)	(5.69)	(5.80)	(6.15)	(5.16)
Constant	47,131***	60,471***	42,965***	48,807***	60,763***	71,765***	51,806***	60,606***	103,730***	117,900***	109,680***	103,120***
R^2	0.98	0.98	0.98	0.99	0.88	0.89	0.92	0.92	0.99	0.99	0.99	0.99

[a] Numbers in parentheses are t-statistics.
*** significant at the 1% level, ** significant at the 5% level, * significant at the 10% level.

APPENDIX

Proof of Proposition C-1.

Differentiating (7) with respect to θ yields

$$\Phi \frac{dQ}{d\theta} + \frac{d\Pi}{d\theta} = 0 \qquad \text{(A1)}$$

where

$$\Phi = \begin{bmatrix} \Pi_{11}^{H} & \Pi_{12}^{H} & \Pi_{13}^{H} & \Pi_{14}^{H} \\ \Pi_{21}^{H} & \Pi_{22}^{H} & \Pi_{23}^{H} & \Pi_{24}^{H} \\ \Pi_{31}^{A} & \Pi_{32}^{A} & \Pi_{33}^{A} & \Pi_{34}^{A} \\ \Pi_{41}^{B} & \Pi_{42}^{B} & \Pi_{43}^{B} & \Pi_{44}^{B} \end{bmatrix}, \qquad Q = \begin{bmatrix} Q^{1} \\ Q^{2} \\ Q^{3} \\ Q^{4} \end{bmatrix}, \qquad \frac{d\Pi}{d\theta} = \begin{bmatrix} 0 \\ 0 \\ \Pi_{3}^{B} \\ \Pi_{4}^{A} \end{bmatrix}.$$

Note that Φ can be manipulated to become

$$\begin{aligned} \Phi &= diag[\Pi_{11}^{H}\ \Pi_{22}^{H}\ \Pi_{33}^{A}\ \Pi_{44}^{B}] \begin{bmatrix} 1 & -R_{12}^{H} & -R_{13}^{H} & -R_{14}^{H} \\ -R_{21}^{H} & 1 & -R_{23}^{H} & -R_{24}^{H} \\ -R_{31}^{A} & -R_{32}^{A} & 1 & -R_{34}^{A} \\ -R_{41}^{B} & -R_{42}^{B} & -R_{43}^{B} & 1 \end{bmatrix} \\ &\equiv diag[\Pi_{11}^{H}\ \Pi_{22}^{H}\ \Pi_{33}^{A}\ \Pi_{44}^{B}][I - R] \end{aligned}$$

where

$$R_{ij}^{K} \equiv -\left(\Pi_{ii}^{K}\right)^{-1}\Pi_{ij}^{K}$$

is the derivative of carrier K's reaction function. Solving (A1) for $dQ/d\theta$, we obtain

$$\frac{dQ}{d\theta} = -\Phi^{-1}\frac{d\Pi}{d\theta} = -[I-R]^{-1} diag[\Pi_{11}^{H}\ \Pi_{22}^{H}\ \Pi_{33}^{A}\ \Pi_{44}^{B}]^{-1} \begin{bmatrix} 0 \\ 0 \\ \Pi_3^B \\ \Pi_4^A \end{bmatrix}. \qquad (A2)$$

By the second-order condition, the diagonal matrix is negative. Furthermore, by network complementarit due to connecting traffic, Π_3^B and Π_4^A are both positive. Now, we determine the sign of $[I - R]^{-1}$. The

stability of the Cournot-Nash equilibrium implies that the magnitude of the eigenvalues of the matrix R must be less than unity (Zhang and Zhang, 1996). Hence, by the Neumann lemma,[8] $[I-R]^{-1}$ exists and

$$[I-R]^{-1} = I + R + R^2 + \cdots + R^n + \cdots. \qquad (A3)$$

By (3), (4), (7) and the second-order condition, it can be shown that the matrix R has the following signs:

$$R = \begin{bmatrix} 0 & + & - & - \\ + & 0 & - & - \\ - & - & 0 & + \\ - & - & + & 0 \end{bmatrix}.$$

Hence, by (A3), $[I-R]^{-1}$ must have the following signs:

$$[I-R]^{-1} = \begin{bmatrix} + & + & - & - \\ + & + & - & - \\ - & - & + & + \\ - & - & + & + \end{bmatrix}.$$

---

[8] Neumann lemma is that if R is a real square matrix and the magnitude of eigenvalues of R is less than one, then $(I-R)^{-1}$ exists and $(I-R)^{-1} = \sum_{i=0}^{\infty} R^i$. See, for example, Ortega and Rheinboldt (1970, p.45).

Then, according to (A2), we have

$$\frac{dQ}{d\theta} = \begin{bmatrix} - \\ - \\ + \\ + \end{bmatrix}.$$

Q.E.D.

Proof of Proposition C-3.

Let the vector of changes in total output on routes *ah* and *hb* be denoted b

$$\frac{d\tilde{Q}}{d\theta} \equiv \frac{d}{d\theta}\begin{bmatrix} Q^1+Q^3 \\ Q^2+Q^4 \end{bmatrix} = \frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix}.$$

Partition the Hessian matrix Φ into

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}$$

where each of ϕ_{ij} is a 2×2 matrix. Then, (A1) may be written as

$$\phi_{11}\frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \phi_{12}\frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix} = 0,$$

$$\phi_{21}\frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \phi_{22}\frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix} = -\frac{d\Pi^{A,B}}{d\theta}.$$

Solving the first equation and substituting, we obtain

$$\frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} = R^H_{A,B}\frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix}$$

where matrix $R^H_{A,B} \equiv -\phi_{11}^{-1}\phi_{12}$ can be interpreted as the derivative of the reaction function of carrier H.

By the condition that the equilibrium is stable, the norm of this matrix must be less than unity (Zhang and Zhang, 1996), i.e., $\|R^H_{A,B}\| < 1$. It follows that either $\left|\frac{dQ^1}{d\theta}\right| < \left|\frac{dQ^3}{d\theta}\right|$ *or* $\left|\frac{dQ^2}{d\theta}\right| < \left|\frac{dQ^4}{d\theta}\right|$.

Thus, by the result of Proposition C-1,

$$\frac{d\tilde{Q}}{d\theta} = \frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix}$$

must have at least one positive element. This in turn implies that the "full" price of passengers after the complementary alliance must decline in at least one market between AH and HB.

In the special case where the demand is symmetric between routes *ah* and *hb* and carriers A and B have the same cost function, we will have $dQ^1 = dQ^2$ and $dQ^3 = dQ^4$. Then, as shown above, the stabilit condition will imply that

$$\frac{d\tilde{Q}}{d\theta} = \frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \end{bmatrix} + \frac{d}{d\theta}\begin{bmatrix} Q^3 \\ Q^4 \end{bmatrix} > 0.$$

Q.E.D.

Proof of Proposition P-1.

Given that the non-hub partner, carrier A, does not change its output in the parallel alliance, the optimization problem faced by carriers H and B can be set as follows:

$$\max_{Q^1,Q^2} \Pi^H + \theta\Pi^A$$

$$\max_{Q^4} \Pi^B$$

where $\theta = 0$ or 1, representing the situation before and after the alliance respectively. The first-order conditions for the carriers may be written as

$$\Pi_1^H + \theta\Pi_1^A = 0$$

$$\Pi_2^H + \theta\Pi_2^A = 0 \qquad \text{(A4)}$$

$$\Pi_4^B = 0\,.$$

Using (14) and differentiating (A4) with respect to θ yields

$$\Psi \frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \\ Q^4 \end{bmatrix} = -\begin{bmatrix} \Pi_1^A \\ 0 \\ 0 \end{bmatrix}$$

where, since $\Pi_2^A = 0$,

$$\Psi = \begin{bmatrix} \Pi_{11}^H + \theta\Pi_{11}^A & \Pi_{12}^H & \\ \Pi_{21}^H & \Pi_{22}^H & \Pi_{24}^H \\ & \Pi_{42}^B & \Pi_{44}^B \end{bmatrix},$$

Rearrange,

$$\Psi = diag[\psi_{11}\,\psi_{22}\,\psi_{44}]\begin{bmatrix} I & -R_{12} & \\ -R_{21} & I & -R_{24} \\ & -R_{42} & I \end{bmatrix}$$

$$\equiv diag[\psi_{11}\,\psi_{22}\,\psi_{44}]\,[I - R]$$

where ψ_{ij} refers to the element of matrix Ψ and $R_{ij} = -\psi_{ii}^{-1}\psi_{ij}$. By second-order condition, $\psi_{ii} < 0$, and

by (3), (4) and (7), the matrix R has the following signs

$$R = \begin{bmatrix} 0 & + & 0 \\ + & 0 & - \\ 0 & - & 0 \end{bmatrix}. \tag{A5}$$

Assume that the equilibrium is stable, we have

$$\Psi^{-1} = [I - R]^{-1} diag[\psi_{11}\,\psi_{22}\,\psi_{44}]^{-1}$$
$$= \left(\sum_{k=0}^{\infty} R^k \right) diag[\psi_{11}^{-1}\,\psi_{22}^{-1}\,\psi_{44}^{-1}].$$

According to (A5), it can be easily verified that

$$\Psi^{-1} = \begin{bmatrix} - & - & + \\ - & - & + \\ + & + & - \end{bmatrix}.$$

Since $\Pi_1^A < 0$,

$$\frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \\ Q^4 \end{bmatrix} = -\Psi^{-1}\begin{bmatrix} \Pi_1^A \\ 0 \\ 0 \end{bmatrix}$$

has the sign of

$$\frac{d}{d\theta}\begin{bmatrix} Q^1 \\ Q^2 \\ Q^4 \end{bmatrix} = \begin{bmatrix} - \\ - \\ + \end{bmatrix}.$$

Q.E.D.