Knowledge Spillovers, Infant Industry, and Import Quota

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Abstract: We model an economy with an infant industry consisting of several oligopolists that potentially can operate under conditions of learning-by-doing and knowledge spillovers. In the absence of protection, the infant industry cannot compete with foreign firms, and its output is zero for ever. We consider a welfare improving policy (as compared with free trade) which consists of both (i) prohibiting importation of the good in question, and (ii) giving subsidies to domestic firms in order to internalize knowledge spillovers and under-production. We derive a continuum of subsidy rules that guide oligopolists to achieve the socially optimal output path, given zero imports. In our model firms maximize over an infinite horizon. They may choose pre-commitment strategies, or feedback strategies.We obtain a rather surprising result about the optimal subsidy-cum-tax rule: when the stock of knowledge is low, firms may be required to pay a tax (i.e., the subsidy per unit of output is negative) on their outputs, even though the outputs yield positive externalities. This tax, however, encourages firms to produce more, because according to the subsidy-cum-tax rule, they will lower the tax or possibly receive a subsidy when the stock of knowledge they generate becomes sufficiently great.

We next consider a better policy that would achieve the same path of knowledge and domestic output as above, but that would generate higher welfare. This better policy does not involve direct taxes or subsidies on the outputs of domestic firms. Instead, it consists of setting a quota on imports, and adjusting the quota over time, with the adjustment being conditioned on the domestic stock of knowledge. We show that the quota can be large when the stock of knowledge is small, but tightened as this stock grows. The intuition behind this is that the government should put domestic firms under a tough competition in their infancy with a promise to make their life easier as their knowledge grows. This has a major advantage of of increasing the quantity available for consumption when the domestic industry is in its infancy and lowering the imports when the domestic production reaches the maturity level.

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1 Introduction

When an activity generates knowledge spillovers that benefit other economic agents, the usual policy recommendation is to subsidize that activity. Infant industries are important instances of knowledge spillovers, in conjunction with learning-by-doing. While each firm accumulates knowledge through experience, i.e., learning-by-doing, it also confers benefits to other firms in the industry through knowledge transmission (e.g., because learning-by-watching takes place, or because workers in the industry learn from each other through conversation). In the context of international trade, it is quite often the case that domestic infant industries are protected by tariffs or quotas on imports of foreign-produced substitutes.

Economists typically point out that in the case of an infant industry facing foreign competition, there are two main sources of market failures[1]. The first source of market failure arises from imperfections in the capital market. In their early learning stage, the unit production cost of domestic firms is typically higher than the world-market price of the same product. In the absence of tariffs on foreign-produced perfect substitutes, or of domestic production subsidies, domestic firms do not have, during their infancy, sufficient revenues to meet input costs. In a world with a perfect capital market, these losses can be financed by borrowing from financial institutions as long as the expected future profits are sufficient to pay back the loans, with interests. When the capital market is imperfect, due to informational asymmetry creating moral hazard or adverse selection problems, the market outcomes might involve credit rationing, and might not be efficient. Under these conditions, there might be a case for government loan subsidies, or production subsidies; but many economists are not convinced[2] that governments can improve upon the allegedly imperfect capital market. The second source of market failure lies in the fact that firms that generate knowledge spillovers do not get paid for this service, and hence do not have sufficient incentive to provide the service at the socially optimal level. The ideal remedies for these market market failures would be to treat them at their sources: to improve the efficiency of the capital market, and to subsidize firms for their production externalities.

A third source of market failure associated with many infant industries is

[1]See, for example, Kemp (1962), Clemhout and Wan (1970). This literature focusses on the case of perfectly competitive firms, abstracting from the inefficiency that would arise if firms have market power. In this paper, we deal with this additional complication.

[2]See Dixit (1987).

that domestic firms may not take price or subsidy rates as beyond their control. When domestic firms are oligopolists, and foreign supplies are banned, or restricted by quotas, there is another source of inefficiency: firms restrict outputs to raise price above the competitive level. In a static world, the traditional remedy for underproduction is a production subsidy, which depend on three factors: production cost, number of oligopolists, and demand. However, in a world where knowledge is accumulated over time, so that production cost is gradually and endogenously changing, it is not obvious how production subsidies should be designed. At first sight, it might seem that the government should announce at the outset a time path of the subsidy rate: for example, the rate is 20% for the first year, 15% for the second year, 10% for the third year, and so on. The government would hope that firms take such a time path as given when they plan their production; in other words, it would hope that firms accept its role as the open-loop Stackelberg leader. Unfortunately, as shown by Kydland and Prescott (1977), such a policy would be, in general, time inconsistent: it is likely that after the first year, the government will find it optimal to renege from its previously announced subsidy path[3].

One way of overcoming the time-inconsistency problem is to look for a time-invariant rule which prescribes the subsidy rate at any point of time on the basis of the currently observed values of the states of the system. Such rule would be optimal if it can be shown that the firms reacting to the rule actually replicate what a hypothetical central planner with full control over outputs would produce. In this paper, we seek optimal time-consistent policies for the case of an infant industry consisting of firms that behave as Cournot oligopolists.

In the real world, infant industries are often given protection against foreign competition, by means of tariffs or quotas, as well as production subsidies[4]. An extreme form of protection would be to ban imports and to subsidize domestic production. In this paper, we examine optimal policies under two alternative scenarios. In the first scenario, we *take as given* that imports are banned, and ask the following questions. (i) can one find a time path of production subsidies induced by a subsidy rule, that encourages

[3]The time-inconsistency property explained in Kydland and Prescott (1977) has been given fuller treatment in several books and articles. See, for example, Karp and Livernois (1992), Xie (1998), Dockner et al. (2000).

[4]In South Korea, the car industry was heavily protected by imports restrictions. The aircraft producers in Brazil and Cabada have received a great deal of subsidies.

domestic oligopolists to produce and generate knowledge at the rate that would replicate what would be achieved by a central planner who has direct control over outputs? (ii) would such a rule involve a subsidy rate that rises, or falls, over time? (iii) should the subsidy rate for each point of time t be dependent of the knowledge stock at t?

In the second scenario, direct subsidies are not allowed and imports are not banned, but controlled by quotas. We show that there exists a quota rule that generates a time path of quota, and an associated path of domestic output (and hence knowledge path) which is *identical* to the one obtained under the first scenario. *No production subsidies* are required. Social welfare under the second scenario is greater than under the first scenario, because domestic outputs and production costs are the same under both scenarios, but consumers enjoy a greater level of consumption, at a lower price. In both scenarios, we assume that the capital market is perfect, and that there is no uncertainty nor informational asymmetry. This simplifying assumption allows us to focus on policy measures to remedy externality due to knowledge spillovers, and to correct for market power due to oligopoly.

We obtain a rather surprising result about optimal subsidy rules: when the stock of knowledge is low, firms may be required to pay a tax on their outputs (i.e., the subsidy per unit of output is negative), even though the outputs yield positive externalities. This tax, however, encourages firms to produce more as compared with what they would do in the absence of the policy rule, because according to the rule, they will later on receive a subsidy or at least reduce the tax rate imposed, when the stock of knowledge they generate becomes sufficiently great.

In the case of quotas on imports from a foreign mature firm, we show that there is a continuum of special quota rules that ensure domestic firms will follow the optimal output path found in scenario 1 above. It follows that each of the two instruments (the subsidy and the import quota rules) can separarately achieve the same path of knowledge accumulation for the infant industry, but the quota rule is superior, because welfare is greater under scenario 2. Another surprising result is that the optimal import quota can be a decreasing function of the stock of knowledge: as the domestic industry gradually matures, the quota is tightened. The intuition behind this result is as follows. The optimal action of the government is to put domestic firms under a tough competition in their infancy with a promise to "make life easier" for them as their knowledge grows. This is exactly the opposite of the policy followed by many governments who strongly protect infant industries in their infancy and reduce the protection as the domestic industry grows.

2 The Basic Model

Assume there are n domestic firms. Let $q_i(t)$ denote firm i's output at time t. (We will simplify notation and write q_i for $q_i(t)$ when there is no confusion.) We assume that there is only one state variable, $K(t)$. It represents the stock of knowledge in the industry. All firms in the industry have access to the same stock of knowledge. The industry output adds to the stock K; this may be interpreted as the result of both learning-by-doing and free transfer of knowledge among workers. Knowledge depreciates at the rate $\delta > 0$:

$$\dot{K} = \sum q_i - \delta K \tag{1}$$

For eacf firm i, the total cost of producing output level q_i depends on K and on q_i. We assume that, for a given level of knowledge $K \geq 0$, total cost is increasing and convex in q_i. An increase in K lowers the cost of producing any output level, as long as K is below some level $\bar{K} > 0$. The industry is said to be mature when the level $\bar{K}$ is reached. Afterwards, any further increase in K will have no effect on cost.

$$C(K, q_i) \geq 0, \quad C(K, 0) = 0, \quad C_K \leq 0, \quad C_q > 0, \quad C_{qq} > 0 \tag{2}$$

$$C_K = 0 \text{ if } K \geq \overline{K} \tag{3}$$

An example: The following cost function satisfies the properties (2) and (3) above:

$$C(K, q) = \max[0, \overline{K} - K]\gamma q + \frac{1}{2}\mu q^2, \quad \gamma > 0, \quad \mu > 0, \quad \overline{K} > 0. \tag{4}$$

This cost function has the property that the average cost curve for each firm has a vertical intercept that shifts down as knowledge is gradually accumulated, provided that $K < \bar{K}$. Once $K = \bar{K}$ there is no further shift.

Let Q denote the output of the domestic industry

$$Q = \sum_{i=1}^{N} q_i$$

and let the demand function be $P(Q)$. We assume that $P(0) > C_q(K_0, 0)$.

2.1 The social optimum under autarky

We now study the optimal time path of output under autarky, under the assumption that the government has direct control over the output of each domestic firm. This serves as a useful benchmark for evaluating the welfare implications of policies under decentralization. The utility of consuming quatity Q is

$$U(Q) = \int_0^Q P(Z)dZ$$

Since the N firms are identical, we write $Nq = Q$. The *flow of welfare* under autarky at time t is

$$W^A(t) = U(Q(t)) - NC(K(t), q(t)) \tag{5}$$

The optimization problem facing the central planner is a simple optimal control problem: choose the time path of output of the representative firm, so as to maximize *total welfare*, defined as the integral of discounted *flow of welfare:*

$$V(K_0) \equiv \max_q \int_0^\infty \{U(Q) - NC(K,q)\} \exp(-rt)dt \tag{6}$$

subject to the constraints $Q \geq 0$,

$$\dot{K} = Q - \delta K \tag{7}$$

and $K(0) = K_0$.

The Hamiltonian is

$$H = U(Q) - NC(K, Q/N) + \lambda[Q - \delta K]$$

and the necessary conditions are

$$P(Q) = C_q - \lambda \tag{8}$$

$$\dot{\lambda} = (r + \delta)\lambda + NC_K \tag{9}$$

Condition (8) says that, at the optimal output level at time t, the price is equated to marginal cost net of the marginal future benefit generated by output, λ. Equation (9) describes how this marginal benefit evolves over time. These equations give

$$[-P' + \frac{1}{N}Cqq]\dot{Q} + C_{qK}\dot{K} = (r+\delta)[C_q - P] + NC_K \tag{10}$$

Replacing Q by $\dot{K} + \delta K$ and $\dot{Q} = \ddot{K} + \delta\dot{K}$ in (10), we obtain a second order differential equation in K . Let K_∞^{so} be the steady state for that second order differential equation. Then K_∞^{so} must satisfy the condition

$$P(\delta K_\infty^{so}) - C_q(K_\infty^{so}, \delta K_\infty^{so}/N) = \frac{1}{(r+\delta)} NC_K(K_\infty^{so}, \delta K_\infty^{so}/N) < 0 \tag{11}$$

This condition says that at the steady state, price must be below marginal cost by the amount that reflects the contribution of output to knowledge capital.

The corresponding steady-state output is

$$Q_\infty^{so} = \delta K_\infty^{so} \tag{12}$$

The system of differential equations (7) and (10) typically displays saddle-point stability, as illustrated by the linear-quadratic example below. The stable branch of the saddle is denoted by $\widehat{Q}(K)$. It represents the optimal control rule in feedback form: for any given K, the optimal output level is given by

$$\widehat{Q} = \widehat{Q}(K) \tag{13}$$

In what follows, we will use the symbol $\widehat{Q}_\infty$ to denote $\widehat{Q}(K_\infty^{so})$.

A linear-quadratic example

Take the cost function specification (4) and assume linear demand, $P = a - bQ$.We assume that γ is sufficiently small so that

$$b\delta + \frac{\delta\mu}{N} - \frac{\gamma(r+2\delta)}{r+\delta} > 0 \tag{14}$$

and

$$a > \gamma\overline{K}, \qquad (b\delta + \frac{\delta\mu}{N} - \frac{\gamma\delta}{r+\delta})\bar{K} > a. \tag{15}$$

(**Remark:** Assumption (14) and (15) ensure that it is optimal to converge to a positive, finite $K < \bar{K}$.)

Using (11), we obtain a unique steady-state level of knowledge:

$$K_\infty^{so} = \frac{a - \gamma\overline{K}}{b\delta + \frac{\delta\mu}{N} - \left[\frac{r+2\delta}{r+\delta}\right]\gamma} \tag{16}$$

Equation (10) becomes

$$\ddot{K} - r\dot{K} + JK = \frac{\left(\gamma\bar{K} - a\right)(r+\delta)}{\left[b + \frac{\mu}{N}\right]} \tag{17}$$

where

$$J \equiv \frac{\gamma(r+2\delta)}{b + \frac{\mu}{N}} - \delta(r+\delta) < 0 \tag{18}$$

This differentential equations has two real roots. We take the negative root, denoted by β, which ensures convergence to the steady state

$$\beta = \frac{r}{2} - \frac{1}{2}\sqrt{r^2 - 4J} < 0 \tag{19}$$

Since K_0 is smaller than K_∞^{so}, the optimal time path of K is monotone increasing, with $K(t)$ approaching K_∞^{so} asymptotically:

$$K(t) = K_\infty + (K_0 - K_\infty^{so})\exp(\beta t) \tag{20}$$

from which, using the fact that $Q = \dot{K} + \delta K$, we obtain the optimal path of industry output

$$Q(t) = -\beta K_\infty^{so} + (\delta + \beta)K(t) \tag{21}$$

where $\delta + \beta > 0$, as can be verified from

$$2(\delta + \beta) = (2\delta + r) - \sqrt{(2\delta + r)^2 - 4\left[\frac{\gamma(r+2\delta)}{b + \frac{\mu}{N}}\right]}$$

From (21) we obtain the optimal control rule in feedback form, where the optimal output level at any time is expressed as an increasing function of the stock of knowledge capital at that time, independently of the calendar time:

$$\hat{Q}(K) = -\beta K_\infty^{so} + (\delta + \beta)K. \tag{22}$$

2.2 Optimal production subsidy rule

We now show that the social optimal path can be achieved by a suitably designed production-subsidy rule that is time-independent and is therefore time-consistent. This rule determines the subsidy rate per unit of output at any time as a function of the currently observed level of K. From such a rule, each firm can determine from the rule how much subsidy it gets, given the output of its rivals, as a function of its current output and the observed value of the state variable K.

Consider a rate of subsidy per unit of output $s(K)$, so that a firm's profit is

$$\pi_i(q_i, Q_{-i}, K) = P(q_i + Q_{-i})q_i + s(K)q_i - C(K, q_i) \tag{23}$$

where

$$Q_{-i} \equiv \sum_{j \neq i} q_j$$

To determine the optimal subsidy rule, i.e., the functional form for $s(K)$, the government must know if firms are playing an open-loop Cournot game among themselves, or a closed-loop Cournot game. These concepts are explained in more detail below.

2.2.1 The case of open-loop Cournot game

In the open-loop Cournot case, each firm i takes the time paths of outputs of its rivals as given, and works out its optimal time path $q_i(.)$ to which it commits itself for the entire horizon. The subsidy rule $s(K)$ is known to all firms. Firm i's problem is

$$\max_{q_i} \int_0^\infty e^{-rt} \pi_i(q_i, Q_{-i}, K) dt \tag{24}$$

where π_i is given by (23), and where $Q_{-i}(t)$ is taken as given. The maximization is subject to

$$\dot{K} = q_i + Q_{-i} - \delta K$$

with $K(0) = K_0$ (given).

Given the subsidy rule $s(K)$ which all firms take as outside their control, an open-loop Nash equilibrium (OLNE)[5] for the Cournot oligopolists is defined as a profile of stategies $(q_1^*(.), q_2^*(.), ..., q_N^*(.))$ such that, for each firm i, given that $Q_{-i}(t) = \sum_{j\neq i} q_j^*(t)$, the strategy $q_i^*(.)$ is the solution of problem (24). In what follows, we focus on symmetric equilibrium. To find a symmetric equilibrium, we first consider the necessary conditions for representative firm i.

The Hamiltonian for problen (24) is

$$H_i = \pi_i + \lambda_i[q_i + Q_{-i} - \delta K]$$

This gives the necessary conditions

$$q_i P' + P = C_q - s(K) - \lambda_i \tag{25}$$

and

$$\dot{\lambda}_i = (r+\delta)\lambda_i - s'(K)q_i + C_K \tag{26}$$

Condition (25) says that, at the equilibrium output level at time t, firm i equates its marginal revenue to marginal cost net of subsidy and net of the private marginal future benefit generated by its output, λ_i. Equation (26) describes how this marginal benefit evolves over time. From these equations we get a differential equation

$$(r+\delta)\left[C_q - s(K) - P - q_i P'\right] - q_i s' + C_K =$$

$$\left[C_{qq} - N q_i P'' - (N+1)P'\right]\dot{q}_i + (C_{qK} - s')\dot{K} \tag{27}$$

Note that at the steady state, equation (27) yields the following condition

$$q_i P' + P = C_{q_i} - s(K) - \frac{1}{r+\delta}\left[q_i s'(K) - C_K\right] \tag{28}$$

along with

$$N q_i = \delta K \tag{29}$$

[5]For a concise treament of open-loop and closed-loop equilibria in differential games, see Clemhout and Wan (199..). See also Fudenberg and Tirole (19., chapters....). Dockner et al. (2000) provide a comprehensive development of differential games.

That is, each firm equates its marginal revenue to its marginal cost, net of subsidy, and net of the present value of the stream of future benefit generated by the output.

Substituting $q_i = [\dot{K} + \delta K]/N$ and similarly for $\dot{q}_i$, into (27), we obtain a second order differential equation in K. If that equation has a solution that leads to a positive steady state K_{∞}^{ol} then that solution is a symmetric open-loop Nash equilibrium with a positive production at the steady Q_{∞}^{ol}. In what follows, we assume such a solution pair $(K(t), Q(t))$ exists, and the time paths of K and Q are monotone. (Such a pair does exist in the linear-quadratic case.)

Since $Q(t)$ and $K(t)$ are monotone, we can represent the OLNE in the feedback form $Q(K)$. Substituting $N\dot{q}_i = Q'(K)\dot{K} = Q'(K)[Q(K) - \delta K]$ into (27) we obtain the result that oligopoly production path, represented in the feedback form, satisfies the following differential equation

$$(r+\delta)\left[C_q - s(K) - P - \frac{Q(K)}{N}P'\right] - \frac{Q(K)}{N}s' + C_K =$$

$$[C_{qq} - Q(K)P'' - (N+1)P'] \frac{Q'(K)[Q(K) - \delta K]}{N} + (C_{qK} - s')(Q(K) - \delta K) \quad (30)$$

with the boundary condition

$$Q(K_{\infty}^{ol}) = Q_{\infty}^{ol} \quad (31)$$

where K_{∞}^{ol} and Q_{∞}^{ol} are determined by (28) and (29).

To ensure that the OLNE coincides with the socially optimal time path of output that we found earlier, the social planner must choose an appropriate subsidy rule $s(K)$, such that the optimal control in feedback form $\hat{Q}(K)$, given in (13), or, for the linear-quadratic case, (22), satisfies equation (30) and (31).

For an OLNE to coincide with the social optimum, the subsidy rule must satisfy the following differential equation

$$(r+\delta)\left[C_q - s(K) - P - \frac{\hat{Q}(K)}{N}P'\right] - \frac{\hat{Q}(K)}{N}s' + C_K =$$

$$\left[C_{qq} - \hat{Q}(K)P'' - N + 1)P'\right] \frac{\hat{Q}'(K)\left[\hat{Q}(K) - \delta K\right]}{N} + (C_{qK} - s')(\hat{Q}(K) - \delta K) \quad (32)$$

where $\hat{Q}(K)$ is the feedback representation of the socially optimal path. An optimal subsidy rule is thus a solution of the following first-order linear differential equation

$$s(K) + A(K)s'(K) = B(K) \tag{33}$$

where

$$A(K) \equiv \frac{\hat{Q}(K)\left[\frac{1}{N} - 1\right] + \delta K}{(r+\delta)}$$

and

$$B(K) \equiv \left[-P - \frac{\hat{Q}(K)}{N}P' + C_q\right] + \frac{C_K}{(r+\delta)} - \frac{C_{qK}}{(r+\delta)}\left[\hat{Q}(K) - \delta K\right] +$$

$$\frac{1}{N(r+\delta)}\left[\hat{Q}(K)P'' + (N+1)P' - C_{qq}\right]\hat{Q}'(K)\left[\hat{Q}(K) - \delta K\right] \tag{34}$$

We now argue that any subsidy rule solution of (33) induces the oligopolists to follow the socially optimal production path : there is thus a continuum of optimality inducing subsidy rules[6]. Our argument consists of showing that, for any subsidy rule which is a solution of (33), (i) firstly, the socially optimal production $\hat{Q}(K)$ satisfies (30), and (ii) secondly, that K_∞^{so} and Q_∞^{so} solve (28) and (29). Part (i) is true for any solution of (33) by construction of (33). To show (ii), note first that by (12), K_∞^{so} and Q_∞^{so} satisfy (29) for all subsidy rules. Furthermore, for any subsidy rule which is a solution of (33), equation (32) is satisfied for all K including K_∞^{so}. At K_∞^{so}, we have

$$(r+\delta)\left[C_q - s(K_\infty^{so}) - P - \frac{\hat{Q}(K_\infty^{so})}{N}P'\right] - \frac{\hat{Q}(K_\infty^{so})}{N}s' + C_K = 0$$

which is exactly (28). Thus for any subsidy rule which solves (33), K_∞^{so} and Q_∞^{so} solve (28) and (29).

To sum up, for any subsidy rule which satisfies (33), the socially optimal production path is the equilibrium production path for the oligopoly.

[6]Provided that the Hamiltonian associated with the problem of each firm remains concave with respect to the stock of knowledge and that firms'profits remain non negative.

LEMMA 1: If $s_L(K)$ is a subsidy rule that leads the oligopoly to achieve the socially optimal path, then so does the modified subsidy rule $s_M(K)$ defined by

$$s_M(K) = s_L(K) + \phi(K) \tag{35}$$

where $\phi(K)$ is any **concave** function that satisfies the equation

$$(r+\delta)\phi(K) = \phi'(K)\left\{\left[\hat{Q}(K) - \delta K\right] - \frac{1}{N}\hat{Q}(K)\right\} \tag{36}$$

REMARK 3: We insist on concavity in order to ensure that the **sufficiency conditions** for the optimization problem of each oligopolist are satisfied.

Proof of Lemma 1:

Under the subsidy rule $s_L(K)$, the following conditions are satisfied for each oligopolist along the path $\hat{Q}(K(t))$:

$$q_i P' + P - C_q + s_L(K) + \lambda_i = 0 \tag{37}$$

$$\dot{\lambda}_i = (r+\delta)\lambda_i - s_L'(K)q_i + C_K \tag{38}$$

$$\dot{K} = Nq_i - \delta K \tag{39}$$

Now if we replace $s_L(K)$ by $s_M(K)$, then, from (37), q_i will be unchanged provided the shadow price $\lambda_i(t)$ is replaced by

$$\lambda_i^{\#}(t) = \lambda_i(t) - \phi(K(t)) \tag{40}$$

Since equation (40) must hold for all t, it is necessary that

$$\dot{\lambda}_i^{\#}(t) = \dot{\lambda}_i(t) - \phi'(K(t))\dot{K}(t) = (r+\delta)\lambda_i - s_L'(K)q_i + C_K - \phi'(K)(Nq_i - \delta K) \tag{41}$$

But if $\lambda_i^{\#}(t)$ is to be the new shadow price, it must hold that

$$\dot{\lambda}_i^{\#}(t) = (r+\delta)\lambda_i^{\#} - s_L'(K)q_i - \phi'(K)q_i + C_K \tag{42}$$

The two equations (41) and (42) are consistent with each other if and only if

$$-(r+\delta)\phi(K) - \phi'(K)q_i = -\phi'(K)(Nq_i - \delta K)$$

This condition is equivalent to (36). ■

An illustration: The linear-quadratic case.

In this case, the following subsidy rule (linear and *increasing* in K) is a solution to the differential equation (33)

$$s^*(K) = \eta^* + \alpha^* K$$

where

$$\alpha^* = \frac{\gamma\left[(\beta+\delta)\left(1-(1/N)\right)+\frac{b(r+2\delta)}{Nb+\mu}\right]}{r+\delta+(\delta/N)-\beta[1-(1/N)]} > 0 \tag{43}$$

where β was given by (19), and

$$\eta^* = \frac{\left(a-\gamma\bar{K}\right)\left[\delta(r+\delta)(b/N)+\delta\gamma(1-N^{-1})-\alpha^*(r+\delta+(\delta/N))\right]}{\delta(r+\delta)\left[b+\frac{\mu}{N}\right]-\gamma(r+2\delta)} \tag{44}$$

(The denominator (44) is positive due to (14).) These results make sense. In the special case where $\gamma = 0$ (i.e., there is no stock externality) then $\alpha^* = 0$ and $\eta^* > 0$. It can be shown that $s(K_\infty)$ is negative, though it is possible that $S(K) < 0$ for K sufficiently close to zero.

Moreover it can be shown that for any constant C (within a given interval consistent with participation of firms) the following subsidy will also induce the oligopoly to follow the socially production path

$$s^*(K) = CA(K)^{\left(-\frac{1}{A'(K)}\right)} + \eta^* + \alpha^* K$$

since neither (30) nor (30) will be affected by this subsidy modification.

In the case of a monopoly we have

$$s^*(K) = CK^{-\frac{(r+\delta)}{\delta}} + \eta^* + \alpha^* K$$

Many implications can be drawn from the multiciplicity of optimal subsidy rules. First, contrary to most subsidy programs for infant industries, an efficiency inducing subsidy rule need not be an increasing function of the stock of knowledge. Second, though we have determined a family of efficiency inducing subsidy rules we can determine the characteristics of some specific subsidy rules that satisfy particular criteria. For example, we can determine the subsidy rule satisfying an auto-financing condition therefore avoiding one of the most criticism addressed to subsidies in general: their costs. There exists an auto-financed optimum subsidy rule s_s corresponding to the case where the constant C solves:

$$\int_0^\infty s(K)\,\hat{Q}(K)e^{-rt}dt = 0$$

That is

$$s_s(K) = CK^{-\frac{(r+\delta)}{\delta}} + s_l^*(K) \text{ with } C = -\frac{\int_0^\infty s_l^*(K)\,\hat{Q}(K)e^{-rt}dt}{\int_0^\infty K^{-\frac{(r+\delta)}{\delta}}\hat{Q}(K)e^{-rt}dt}$$

The subsidy rule s_s seems at first very appealing , it first induce the oligopolists follow the effecient production path and second the present value of the total cost of the program is zero. The subsidy rule s_s has however a major drawback, the value of the parameter C depends on the initial condition $K(0) = K_0$. If the game is stopped at any time t and the stock of knowledge is at $K^*(t)$, the level at which it should be if all firms had followed the optimal production path, then the auto-financed subsidy program corresponds to the initial level of C picked at date 0. However, if firms are aware of the objective of an auto-financed program, and the dependance of the subsidy rule specified on K_0, they might voluntarily deviate from the optimal path just to force the regulator to modify the initial subsidy rule: the auto-financed program may be time inconsistent.

An alternative objective could be seeked without giving rise to time-inconsistency would be to determine the subsidy rule that converges to zero in the long-run: $s(K_\infty) = 0$. The corresponding subsidy rule would then be

$$s(K) = CK^{-\frac{(r+\delta)}{\delta}} + s_l^*(K) \text{ where } C = -\frac{s_l^*(K_\infty)}{K_\infty^{-\frac{(r+\delta)}{\delta}}}$$

that is

$$s(K) = s_l^*(K) - s_l^*(K_\infty)\left(\frac{K}{K_\infty}\right)^{-\frac{(r+\delta)}{\delta}}$$

Note that since $\left(\frac{K}{K_\infty}\right)^{-\frac{(r+\delta)}{\delta}} > 1$ for $K < K_\infty$ and since $\alpha^* > 0$ we have

$$s_l^*(K_\infty) > s_l^*(K)$$

if $s_l^*(K_\infty) > 0$ (which is always the case if N is sufficiently large) we have

$$s(K) = s_l^*(K) - s_l^*(K_\infty)\left(\frac{K}{K_\infty}\right)^{-\frac{(r+\delta)}{\delta}} < 0$$

the optimum subsidy rule that yields no intervention in the long-run would then be a tax, however $s'(K) > 0:$ as the stock of knowledge increases the tax is reduced. Moreover since the choice of C and thus of the subsidy rule does not depend on K_0 this subsidy rule is time consistent.

2.2.2 The case of closed-loop Cournot game

Let us turn to the case where firms use closed-loop strategies (or Markov-perfect strategies). Each firm i believes that its rivals follow a strategy which determines their output at each time t as a function of the observed level of knowledge $K(t) : q_j(t) = \chi_j(K(t))$, where $j \neq i$. Firm i then seeks a rule $\chi_i(.)$ that maximizes its discounted stream of profits

$$\max_{q_i} \int_0^{\infty} e^{-rt} \pi_i(q_i, Q_{-i}, K) dt \tag{45}$$

Given the subsidy rule $s(K)$ which all firms take as outside their control, a Markov perfect Nash equilibrium (MPNE) for the closed-loop Cournot oligopolists is defined as a profile of stategies $(\chi_1^*(.), \chi_2^*(.), ..., \chi_N^*(.))$ such that, for each firm i, given that $Q_{-i}(t) = \sum_{j \neq i} \chi_j^*(K(t))$, the strategy $\chi_i^*(.)$ is the feedback form of the solution of problem (45). In what follows, we consider the symmetric case: firm i thinks $\chi_j(.) = \chi_h(.) \equiv \chi(.)$ for all $j, h \neq i$.

Let $M = N - 1$. Firm i's profit is

$$\pi_i = q_i P(q_i + M\chi) + q_i s(K) - C(K, q_i)$$

and the firm seeks to maximize

$$\int_0^{\infty} \pi_i(t) e^{-rt} dt$$

subject to

$$\dot{K} = q_i + M\chi(K) - \delta K$$

The Hamiltonian is

$$H_i = \pi_i + \lambda_i \left[q_i + M\chi(K) - \delta K \right]$$

and the necessary conditions are

$$q_i P' + P + s(K) - C_q + \lambda_i = 0$$

$$\dot{\lambda}_i = (r + \delta - M\chi')\lambda_i - q_i s'(K) + C_K - q_i P' M \chi'$$

These conditions yield an equation that characterize a symmetric MPNE where $q_i = \chi(K)$

$$-(\chi P' + P + s(K) - C_q)(r + \delta - M\chi') - \chi s' - \chi P' M \chi' =$$

$$\{C_{qK} + C_{qq}\chi'(K) - \chi P'' N \chi' + (N+1)P' N \chi'\}\,[N\chi - \delta K] \quad (46)$$

This is a first order differential equation in χ. Give the subsidy rule $s(.)$, a MPNE that admits a steady state is a solution to the differential equation (46) with the 'boundary condition'

$$N\chi(K_\infty^{\#}) - \delta K_\infty^{\#} = 0$$

where $K_\infty^{\#}$ is unspecified.

(For the linear quadratic case, with a subsidy rule of the form $s(K) = \eta + \alpha K$, a MPNE can be found with linear strategies $\chi(K) = g + hK$. See the Appendix.)

To guide firms to achieve the socially optimum path as a MPNE, the subsidy rule must satisfy a first order differential equation analogous to, but not the same as, the linear differential equation (33). For the linear quadratic case, the following subsidy rule leads the oligopolists to follow the optimal production path

$$s(K) = \eta^{**} + \alpha^{**} K$$

It is possible, but tedious, to compare α^{**} and η^{**} with the values α^{*} and η^{*} of the optimum subsidy rule under OLNE.

3 Optimal Quota Rule

In Section 2 we have abstracted from international trade, by considering a closed economy. (Alternatively, that section may correspond to the case where the government bans the imports.) We found, under those conditions, the optimal output path for the infant industry, and the corresponding path of knowledge capital. The optimum was found to be decentralizable, even when firms are oligopolists. The steady-state stock of knowledge was denoted by K_∞^{so} and the steady-state price was $P_\infty = P(Q_\infty^{so}) = P(\delta K_\infty^{so})$. From (11) we know that this price is lower than marginal cost, though possibly higher than average cost, of the representative firm. The gap reflects the fact that output generates externalities (or at least helps maintain the steady-state knowledge level). The *flow of welfare* per period at the steady state is

$$W_\infty = U(Q_\infty) - NC\left[K_\infty^{so}, \frac{1}{N}Q_\infty^{so}\right] \tag{47}$$

and *total welfare*, if $K_0 = K_\infty^{so}$, is

$$V(K_\infty^{so}) = \int_0^\infty e^{-rt} W_\infty dt = \frac{1}{r} W_\infty$$

Now we consider an alternative scenario. Suppose that the good in question can be imported at a price $\bar{P}$. Assume that $\bar{P} < P(Q_\infty^{so})$ where Q_∞^{so} is the optimal steady state output found under autarky. In the absence of the domestic industry, free trade would give the per period welfare flow

$$W_F = U(\bar{Q}) - \bar{Q}\bar{P} \tag{48}$$

where $U'(\bar{Q}) = \bar{P}$. Since $C(K, q)$ is convex in q (i.e., the marginal cost curve is upward-sloping), it is possible that $W_\infty > W_F$: the steady state autarkic welfare flow can exceed the welfare flow under free trade with no domestic production. Under these circumstances, if K_0 is not too far from K_∞, a policy of autarky (banning imports) with production subsidies can be superior to the regime of free trade without production subsidies (assuming that at the free trade price $\bar{P}$, and without subsidies, no domestic firm, starting at knowledge level K_0, can earn a stream of profits with a positive present value). In what follows, we assume that this is the case. Then we can conclude that a policy of banning imports for ever, plus production subsidies, is strictly better than free trade without production subsidies. With K_0 close to K_∞^{so} the former policy yields the total welfare $V(K_0) \simeq \frac{1}{r}W_\infty > \frac{1}{r}W_F$.

This however does not mean that there does not exist a superior policy that yields higher total welfare than both $\frac{1}{r}W_\infty$ and $V(K_0)$. In what follows, we show that a policy of import restriction under a suitably designed quota rule, *without* production subsidies, is one such superior policy, which in addition ensures that the infant industry will generate exactly the same path of knowledge capital as under autarky, but social welfare is higher.

The quota rule that we propose works as follows. The government announces that if the current level of knowledge capital of the infant industry is $K(t)$, then current permitted level of imports is $Q^f(K(t))$, where $Q^f(.)$ denotes the function relating K to imports. We call $Q^f(.)$ a *quota rule*. We will show that there exists a quota rule that allows domestic firms to produce (and hence learn) as much as they would under the scenario of section two, and yet yields higher welfare.

Consider a rate of import $Q^f(K)$, so that, without production subsidies, the representative domestic firm's profit is

$$\pi_i(q_i, K) = P(q_i + Q_{-i} + Q^f)q_i - C(K, q_i) \tag{49}$$

where Q_{-i} is the total output of the domestic firms other than firm i. We will focus on symmetric equilibrium, so that q is the output of the representative domestic firm. The flow of welfare is under the quota rule is

$$W^R(t) = U(Z(t)) - \bar{P}Q^f(K(t)) - NC(K(t), q(t)) \tag{50}$$

where $Z = Nq + Q^f$. Note that as long as $Q^f(K(t)) > 0$, it is the case that $W^R(t) > W^A(t)$ in (5) when the quantities $K(t)$ and $q(t)$ are the same both under autarky and under trade with the quota rule. This is because $U(Z) - U(Nq) > \bar{P}Q^f$.

To determine the quota rule that ensures the achievement of the output path obtained in the optimal solution in autarky found in section 2, the government must know if firms are playing an open-loop game among themselves, or a closed-loop game.

3.1 The open-loop case

In the open-loop case, each firm i takes the time paths of outputs of its rivals as given, and works out its optimal time path $q_i(.)$ to which it commits itself for the entire horizon. The quota rule $Q^f(K)$ is known to all firms. Foreign production does not contribute to the accumulation of knowledge. Firm i's Hamiltonian is

$$H_i = P(q_i + Q_{-i} + Q^f)q_i - C(K, q_i) + \lambda_i[q_i + Q_{-i} - \delta K]$$

This gives the necessary conditions

$$q_i P' + P - C_q + \lambda_i = 0 \tag{51}$$

and

$$\dot{\lambda}_i = (r + \delta)\lambda_i - P' Q^{f\prime}(K) q_i + C_K \tag{52}$$

The analysis in section 2 is applicable here, with minor modifications. Thus, we get

$$(r+\delta)\left[C_q - P - q_i P'\right] - P' Q^{f\prime}(K) q_i + C_K =$$
$$\left[C_{qq} - N q_i P'' - (N+1)P'\right]\dot{q}_i + \left(C_{qK} - \left(P' + q_i P''\right) Q^{f\prime}(K)\right)\dot{K} \qquad (53)$$

Substituting $q_i = [\dot{K} + \delta K]/N$ and similarly for $\dot{q}_i$, into (53), we obtain a second order differential equation in K. If that equation has a negative root and a positive steady state value K_∞^{dom} then we obtain a symmetric open-loop Nash equilibrium with a positive production at the steady Q_∞^{dom}.

Substituting $N\dot{q}_i = Q'(K)\dot{K} = Q'(K)\left[Q(K) - \delta K\right]$ into (53) we obtain the oligopoly production path in the feedback form as solution to the following differential equation

$$-(r+\delta)\left(P'\frac{Q}{N} + P - C_q\right) + C_K - \left(C_{qq} - (1+N)P' + QP''\right)\frac{Q'\left[Q - \delta K\right]}{N} +$$
$$\left((P' + P''\frac{Q}{N})(Q - \delta K) - P'\frac{Q}{N}\right) Q^{f\prime} - C_{qK}(Q - \delta K) = 0 \qquad (54)$$

with the following boundary condition

$$Q\left(K_\infty^{dom}\right) = Q_\infty^{dom} \qquad (55)$$

To ensure that the OLNE yields the same path of domestic output as under the social optimum in section 2, we must choose an appropriate quota rule such that the optimal control in feedback form $\hat{Q}(K)$, given in (13), or, for the linear-quadratic case, (22), is solution of the differential equation (53) with (55).

Substituting (13) into (54), it follows that the import rule is the solution of the following first-order differential equation

$$-(r+\delta)\left(P'\frac{\hat{Q}}{N} + P - C_q\right) + C_K - \left(C_{qq} - (1+N)P' + \hat{Q}P''\right)\frac{\hat{Q}'\left[\hat{Q} - \delta K\right]}{N} +$$
$$\left((P' + P''\frac{\hat{Q}}{N})(\hat{Q} - \delta K) - P'\frac{\hat{Q}}{N}\right) Q^{f\prime} - C_{qK}(\hat{Q} - \delta K) = 0 \qquad (56)$$

As in the case of optimum subsidy rules, there is thus a continuum of optimality inducing quota rules. For any given optimal quota rule (solution of

(56)) the optimal production will be the only solution that satisfies (54) with the boundary condition (55). As a matter of fact, the optimal production is solution to (54) by construction of the optimal quota rules (56) and K_{∞}^{so} and Q_{∞}^{so} obviously satisfy (55) since $Q_{\infty}^{so} = \delta K_{\infty}^{so}$ and $\hat{Q}\left(K_{\infty}^{so}\right) = \delta K_{\infty}^{so}$.

An illustration: The linear-quadratic case.

In this case, the differential equation (56) can be written as

$$Q^{f} + A\left(K\right)Q^{f\prime} = \frac{B\left(K\right)}{(r+\delta)b} \tag{57}$$

with

$$A\left(K\right) = \frac{\left(-\beta(K-K_{\infty}) + \frac{\hat{Q}(K)}{N}\right)}{(r+\delta)}$$

and

$$B\left(K\right) = (r+\delta)\left(-\left(b+\mu\right)\frac{\hat{Q}\left(K\right)}{N} + a - b\hat{Q}\left(K\right) + \gamma K\right)$$

$$+\gamma\frac{\hat{Q}\left(K\right)}{N} + \left(\mu + (1+N)b\right)\right)\frac{\left(\delta+\beta\right)\beta(K-K_{\infty})}{N} - \gamma\beta(K-K_{\infty})$$

where $\hat{Q}\left(K\right)$ is given by (22).

Since $A\left(K\right)$ and $B\left(K\right)$ are both linear there exist a linear quota rule that is solution to the differential equation above

$$Q_{l}^{f}\left(K\right) = \eta^{f} + \alpha^{f}K$$

Furthermore it can be shown that for any constant C (within a given intervalle allowing participation of firms) the following import rule will also induce the oligopoly to follow the production path characterized in section 3:

$$Q_{C}^{f}(K) = CA(K)^{\left(-\frac{1}{A'(K)}\right)} + \eta^{f} + \alpha^{f}K$$

In the case of a monopoly we have

$$Q_{C}^{f}(K) = CK^{-\frac{(r+\delta)}{\delta}} + \eta^{f} + \alpha^{f}K$$

There exists optimum import rule Q_s^f for which there are will be no imports in the long run, that is corresponding to the case where the constant C solves:

$$Q^f(K_\infty) = 0$$

Furthermore,

$$\text{for all } C > \bar{C} \text{ where } \bar{C} = \alpha^f \frac{\delta}{r+\delta} \bar{K}^{\frac{r+\delta}{\delta}} \text{ we have } Q_C^{f\prime}(K) < 0$$

A numerical example:

Assume $a = 2$, $b = 1$, $r = 0.1$, $\delta = \gamma = \mu = N = 1$ then Q^f is solution to

$$Q^f + 0.91KQ^{f\prime} = 2 - 0.14K_\infty^{so} - 0.96K$$

The optimal linear quota rule is

$$Q_l^f(K) = 2 - 0.14K_\infty^{so} - \frac{0.96}{1.91}K$$

and is thus a decreasing function of the stock of knowledge Furthermore it can be checked that for $\bar{K} = 1.9$, $Q_l^f(K)$ remains positive for $K \leq K_\infty^{so}$

Furthermore the following family of quotas yield the same domestic production as the linear quota rule above

$$Q_C^f(K) = CK^{-\frac{1}{0.91}} + 2 - 0.14K_\infty^{so} - \frac{0.96}{1.91}K$$

The optimal quota rule can be a decreasing function of the stock of knowledge: the quota is tightened as knowledge grows. This is a rather surprising result. The usual policy in the case of infant industry is to block foreign entry during the early infancy of the industry and gradually open the market as the industry gains maturity. In our model the optimal quota rule could recommend just the opposite. The intuition behind this is as follows. In their infancy domestic firms are put under tough competition, at the same time, they are however encouraged to increase production to increase the stock of knowledge, under the promise that imports will be decreased as knowledge grows. Notice moreover that, under the optimal quota rule, at the steady state the quota can be positive, which is similar to the negative production subsidy (a tax) at the steady state in the previous section. One main advantage of having a quota rule that is a decreasing function of the sock of knowledge is that the the consumer surplus increase due to imports is more "efficiently" spread accross time: imports will be higher when the domestic production is still in its infancy (when the domestic market needs most imports) and lower when the domestic industry is mature and has reached a higher production level (when the domestic market needs least imports).

The idea to ban any imports of goods produced by infant industries is thus not necessarily a good one. One can achieve the same knowledge growth path that could be acheived under an optimal subsidy program, without using any subsidies but simply by using import programs conditioned on the level of the stock of knowledge. Note that import rule and the production subsidies (when positive) have opposite distribution effects. The former adversely affect firms and positively affect consumers whereas the later does exactly the inverse. We thus have two policies with two opposite distribution effects leading to the same knowledge growth path and having the same effects on the domestic production. Furthermore it can easily be shown that the change of welfare due to the import rule wrt the closed economy optimum is positive.

3.2 Closed-loop equilibrium

Let us turn to the case where firms use closed-loop strategies (or Markov-perfect strategies). As in the closed-loop equilibrium for the subsidy case, each firm i believes that its rivals follow a strategy which determines their output at each time t as a function of the observed level of knowledge $K(t)$: $q_j(t) = \psi_j(K(t))$, where $j \neq i$. Firm i then seeks a rule $\psi_i(.)$ that maximizes its discounted stream of profits. In what follows, we consider the symmetric case: firm i thinks $\psi_j(.) = \psi_h(.) \equiv \psi(.)$ for all $j, h \neq i$.

Let $M = N - 1$. Firm i's profit is

$$\pi_i = q_i P(q_i + M\psi + Q^f) - C(K, q_i)$$

and the firm seeks to maximize

$$\int_0^\infty \pi_i(t)e^{-rt}dt$$

subject to

$$\dot{K} = q_i + M\psi(K) - \delta K$$

The Hamiltonian is

$$H_i = \pi_i + \lambda_i \left[q_i + M\psi(K) - \delta K\right]$$

and the necessary conditions are

$$q_i P' + P - C_q + \lambda_i = 0$$

$$\dot{\lambda}_i = (r + \delta - M\psi')\lambda_i + C_K - q_i P'(Q^{f\prime} + M\psi')$$

These conditions yield an equation that characterize a symmetric MPNE where $q_i = \psi(K)$

$$-(\psi P' + P - C_q)(r + \delta - M\psi') - \psi P'(Q^{f\prime} + M\psi') + C_K =$$

$$\{C_{qK} + C'_{qq}\psi(K) - \psi P''(N\psi' + Q^{f\prime}) + \left((N+1)\psi' + Q^{f\prime}\right) P'\} [N\psi - \delta K] \quad (58)$$

This is a first order differential equation in ψ. Given the import rule $Q^f(.)$, a MPNE that admits a steady state is a solution to the differential equation (58) with the 'boundary condition'

$$N\psi(K_\infty) - \delta K_\infty = 0$$

where K_∞ is given by (16).

To guide firms to achieve the socially optimum path as a MPNE, the import rule must satisfy a first order differential equation analogous to, but not the same as, the linear differential equation (56). This differential equation is obtained after substituting $\psi(K)$ by $\frac{\hat{Q}(K)}{N}$ into (58)

$$-(\frac{\hat{Q}(K)}{N}P' + P - C_q)(r + \delta - M\frac{\hat{Q}'(K)}{N}) - \frac{\hat{Q}(K)}{N}P'(Q^{f\prime} + M\frac{\hat{Q}'(K)}{N}) + C_K =$$

$$\{C_{qK} + C'_{qq}\frac{\hat{Q}(K)}{N} - \frac{\hat{Q}(K)}{N}P''(N\frac{\hat{Q}'(K)}{N} + Q^{f\prime})\} \left[N\frac{\hat{Q}(K)}{N} - \delta K\right]$$

$$+ \left((N+1)\frac{\hat{Q}'(K)}{N} + Q^{f\prime}\right)\left[N\frac{\hat{Q}(K)}{N} - \delta K\right] P' \quad (59)$$

The results obtained in the open-loop case can be generalized to the closed case. First, it is possible to design an import rule that will guide the domestic firms follow the socially optimum production path. Second, the import rule could be a decreasing function of the stock knowledge, for initial stocks of knowledge around the steady state.

4 Concluding remarks

We determined a socially optimal production path in the case of an oligopolistic infant industry. We then determined the optimal subsidy rule that guides domestic firms follow this production path while imports are not allowed. We then considere the possibility of using imports to influence domestic production. We determine an import rule that also implements the same domestic production path under followed under the optimal subsidy rule.

We thus have two different instruments with opposite distributional effects yieding the same domestic production path. Two surprising situations can occur. In the case of subsidies, the subsidy rule can be a tax. And in the case of import rules, we provide numerical examples where the import rule is a decreasing function of the stock of knowledge. These results hold in both the open-loop and closed-loop case.

It is quite common that governments allow some imports in a market where the domestic industry is still in its infancy. But it is usually for other reasons that these imports are allowed: free trade agreements, avoid reciprocity in other markets etc. In this paper we show that a country could open its frontiers even in the case of an infant industry and where the domestic industry if left alone would under produce. The government could use imports to hasten its own domestic production and accumulation of the stock knowledge.

APPENDIX

Markov-Perfect Nash Equilibrium: The linear-quadratic case

Given a subsidy rule of the form $s(K) = \eta + \alpha K$, firm i assumes that all other firms follow a Markov strategy $q_j = \chi(K) = X + YK$, where X and Y will be determined below. Firm i's profit is

$$\pi_i = (a + \eta + \alpha K)q_i - b\{M(X + YK) + q_i\}q_i - \gamma\{\bar{K} - K\}q_i - \frac{1}{2}\mu q_i^2 \quad (60)$$

Let $V_i(K)$ denote firm i's value function. Then the Bellman equation is

$$rV_i = \max\left[\pi_i + V'(K)\left(M(X + YK) + q_i - \delta K\right)\right] \quad (61)$$

Let $V_i = \frac{1}{2}AK^2 + BK + C$, where A, B, and C are to be determined. Maximizing the right-hand side of (61) with respect to q_i yields

$$(a + \eta + (\alpha + \gamma)K) - bM(X + YK) - (2b + \mu)q_i + AK + B = 0 \quad (62)$$

Substitute $q_i = X + YK$ into (62) to get

$$Y = \frac{A + \alpha + \gamma}{b(N+1) + \mu} \equiv Y(A)$$

and

$$X = \frac{a + \eta + B}{b(N+1) + \mu} \equiv X(B)$$

Substitute these into (61) to get

$$\frac{r}{2}AK^2 + BK + C = \left[a + \eta - \gamma\bar{K} + NB + (\alpha + \gamma + NA)K\right](X(B) + Y(A)K)$$

$$- [bN + (\mu/2)] (X(B) + Y(A)K)^2 - (AK + B)\delta K \qquad (63)$$

From (63), we can solve for A, B and C.Collecting all terms having K^2 as a common factor, we have a quadratic equation in A

$$A^2 \left[2bN^2 + (2N-1)\mu\right] + A\left[2(\alpha+\gamma)(bN^2 + b + n\mu - (r + 2\delta)(b(N+1) + \mu)^2\right]$$

$$+2(\alpha + \gamma)^2 \left[b + \frac{\mu}{2}\right] = 0$$

from which we obtain two roots for A. Choose the one that ensures convergence.

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