THE SONNENSCHEIN-DEBREU-MANTEL PROPOSITION AND THE THEORY OF INTERNATIONAL TRADE

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Abstract

Much of the comparative statics of trade theory rests on the unrealistic assumption that in each trading country all agents are alike or behave collectively as though they are alike. In the present paper we show that two well-known comparative statical propositions are highly sensitive to the relaxation of the assumption and suggest that many other results are equally fragile.

1. Introduction

Much of the comparative statics of trade theory rests on the following

> *Conventional Assumption* (*CA*): In each trading country *either* all agents are alike *or* (perhaps because any disturbance is accompanied by lumpsum compensatory payments) they behave collectively as though they are alike.

The *CA* is of dubious realism. That it continues to be accepted (on grounds of "analytical convenience") betrays a widespread failure to understand the possibly disastrous consequences of abandoning it. The latter flow from the Sonnenschein-Debreu-Mantel (*SDM*) "anything goes" proposition.[1]

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In the present paper we describe the consequences of abandoning the *CA* in terms of two well-known comparative-static propositions from conventional trade theory. Many other propositions would have served as well. A potential application of the *SDM* proposition emerges whenever a disturbance in one country feeds back from trading partners in which agents are sufficiently inhomogeneous and sufficiently numerous. The disturbance may be an act of government policy; it may have its origin in nature; or it may take the form of a change in preferences or technology.[2]

2 Example 1: The protection afforded by an optimal tariff

From conventional treatments of the optimal tariff it emerges that the tariff (*a*) protects industries which are import-competing under free trade and (*b*) leaves the initial direction of trade undisturbed. However those treatments depend on the *CA*. It will be shown that, without the *CA*, an optimal tariff might (*a′*) discourage production of an initially imported commodity and (*b′*) reverse the import/export status of some commodities.[3]

It suffices to consider a world economy with just two commodities and two countries, the home or policy-making country and the passive or foreign country. In each country, the technology is convex and there is a complete absence of market-distorting taxes and externalities; hence perfect competition prevails. In the home country, but not in the foreign country, all households are identical. In Figure 1, E_i and M_i denote the imports and exports of commodity *i* by the home country; *HH* is the conventionally-shaped offer curve of the home country; *FF* is the offer curve of the foreign country, with the "butterfly" form which cannot be ruled out when not all households are identical;[4] and u^1u^1 and u^2u^2 form a pair of non-intersecting home-

country trade indifference curves. Notice that at any given terms of trade *each* country makes a unique offer.

There is a unique free-trade equilibrium at *P*, where the home country exports the first commodity and the terms of trade are indicated by the slope of *OP*.

The imposition of an optimal tariff by the home country shifts the world equilibrium (still unique) to *Q*. At that point, the home country exports the *second* commodity; in other words, the imposition of the tariff has reversed the pattern of trade. The rate of duty on the home country's *new* import is given by the ratio *OA/OB*. As Figure 1 makes clear, both the world and domestic prices of the second commodity (imported by the home country under free trade) are higher under the tariff. This is in sharp contrast to the conventional pre-Lerner conclusion that, in response to a tariff on imports, the world price of the initially imported commodity will decline and the domestic price increase. It also contrasts with the outcome, the possibility of which was discovered by Lerner (1936), in which both the world and domestic prices of the initially imported commodity decline.

Two features of the above demonstration may be emphasized. First, the possibility of perverse protection and trade reversal is not conditional on a multiplicity of equilibria. In Figure 1 both the free-trade and the restricted-trade equilibrium are unique.[5] Second, it has been assumed that the optimal tariff is imposed on the imported commodity. However, from the Bickerdike-Lerner Symmetry Theorem,[6] nothing depends on that assumption.

While it is possible that an optimal tariff will reverse the initial pattern of world trade, that outcome is not inevitable. Whether trade reverses depends on the precise form of the foreign offer curve. It also depends on the elasticity of substitution along the home country's trade indifference curves. Thus suppose that the elasticity of

substitution is infinite so that the indifference curves are positively-sloped and parallel straight lines, like u^1u^1 and u^2u^2 of Figure 2. Suppose moreover that u^1u^1 and u^2u^2 have a common slope slightly less than that of TOT', which forms a tangent to the foreign offer curve at the origin. Then the free-trade equilibrium is represented by point *P*, the optimal-tariff equilibrium by *Q*, and the tariff is trade-reversing. As Figure 1 makes clear this outcome does not require that the elasticity of substitution be infinite. However it is also clear that, with a sufficiently small constant elasticity, the optimal-tariff equilibrium must lie in the first quadrant, so that world trade retains its free-trade pattern. Figure 3 provides an illustration.

3. Example 2: Mill and Edgeworth on Impoverishing Growth

Consider the ancient question posed by J.S. Mill and Edgeworth: What happens to the terms of trade and wellbeing of a country when there is a technical improvement in the export industry of that country? Suppose that the improvement is factor neutral and that in the home country a representative agent's preferences are homothetic; and, for concreteness, suppose that, in an initial world equilibrium, the home country exports the first commodity. Then the improvement will change the home offer curve from *HH* to *H′H′* in Figure 4. Superimposing the "butterfly" offer curve of the foreign country, we see that the improvement may not only turn the terms of trade against the home country and impoverish that country, possibilities well understood by Mill and Edgeworth; it may also reverse the pattern of world trade. Figure 5 illustrates the additional possibility.

4. A Concluding Remark

It is well known that the Arrow-Debreu-McKenzie models of general equilibrium yield quite meagre comparative statics. The best-known comparative statical propositions are associated with the special models of Heckscher and Ohlin, Lerner and Samuelson. In the present paper we have suggested that some of those propositions are vulnerable to a small and realistic adjustment of the Heckscher-Ohlin assumptions, *viz.* the accommodation of heterogeneous households in one or more of the disturbed country's trading partners. In a companion paper [Kemp and Shimomura (1999)], we have shown that other comparative statical propositions are vulnerable to an alternative realistic relaxation of the Heckscher-Ohlin assumptions, *viz.* the recognition that the reallocation of factors within a country subject to a disturbance is costly. Taken together, the two adjustments of Heckscher-Ohlin assumptions rule out most of the familiar comparative static propositions of trade theory, and that without appealing to increasing returns to scale, the multiplicity of equilibrium or the non-competitiveness of markets. However there are exceptions, propositions which are valid for general Arrow-Debreu-McKenzie economies and hence are robust to the changes in assumption considered in this paper. One thinks especially of the normative gains-from-trade and Kemp-Wan propositions.

FOOTNOTES

1 See Sonnenschein (1973), Debreu (1974) and Mantel (1974, 1976). For a recent exposition of the *SDM* proposition, see Mas-Colell *et al.* (1995, Sec. 17.*E*).

2 Elsewhere we have been critical not of the *CA* but of the construction on the basis of the *CA* of non-cooperative game theoretical models of trade; see Kemp and Shimomura (1995).

3 It is well known from the pre-war work of Abba Lerner that a positive tariff on imports might turn domestic prices against the imported commodity; see Lerner (1936) and Metzler (1949). However it was not suggested by either Lerner or Metzler that it might be *optimal* to impose a tariff that does not protect the import-competing industry.

On the other hand, Jan Graaff (1949, 1957), has argued on the conventional "representative agent" assumption that the optimal tariff might be zero, thus denying (*a*) and (*b*). Recently, however, his argument has been discussed critically by Kemp and Shimomura (2000).

4 Johnson (1959) showed that a country's offer curve may have the "butterfly" form if the "representative agent" assumption of conventional Heckscher-Ohlin trade theory is abandoned.

From the later work of Sonnenschein (1973), Debreu (1974) and Mantel (1974, 1976), we now know that any continuous (vector) function which is defined on commodity prices bounded away from zero, which is homogeneous of degree zero and which satisfies Walras' law can be interpreted as an excess demand function for some Arrow-Debreu economy with at least as many households as commodities.

From our point of view, Johnson's achievement consisted in specifying a special Arrow-Debreu economy of which the excess demand functions yield a "butterfly" offer curve. In Johnson's specification, there are two classes of households the typical members of which differ both in their preferences and in their factor endowments (one is endowed with labour only, the other is endowed with capital only).

5 Of course Figure 1 could be redrawn to admit multiple free-trade equilibria. However the possibility of perverse protection and trade reversal would not disappear.

6 See Bickerdike (1907) and Lerner (1936), also Kemp (1969, p.298).

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