Factor Price Equalization (FPE) Implies Product Quality Equalization (PQE)

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This paper examines the issue of the effect of international trade on quality choice by firms in a factor-endowment framework. Factor price equalization implies product quality equalization. In the North-South context it means that the South catches up with the North in product quality due to the competitive pressure from international trade, not because of technology adoption. Free trade may induce the North, the quality-leader in autarky, to improve its product quality - and yet the South would leapfrog and match it in the free trade equilibrium.

Introduction

How does international trade affect product quality? There is a large volume of analytical literature, dating back to the 70s. One would then think that some bench-mark answer already exists in a "text book" form, well-understood to the point of being obvious. Somewhat strangely, it is not the case.

The literature on product quality and trade can be grouped into two categories: one looks at the problem in a static setting and the other in a dynamic setting. Within the first category, there are three strands. An early paper such as Rodiguez (1979) interprets quality as the amount of some uniform service that is packaged into one unit of a product and the consumer derives utility from the total amount of services (quality times quantity) consumed.

Later papers such as Falvey and Kierzkowski (1987) and Flam and Helpman (1987) consider general-equilibrium models of trade in which an economy has a homogenous-good sector and a vertically differentiated sector. There are high-quality and low-quality brands in a continuum of goods, while the quality of any particular brand is fixed. In a two-factor (labor-capital) framework, Falvey and Kierzkowski (1987) assume that a higher quality brand is associated with a more capital-intensive technology. This leads to the prediction that the relatively capital- (labor-) abundant country will export higher- (lower-) quality brands. The Flam-Helpman model assumes one-factor economies: with the North (South) defined as having a higher (lower) endowment of this factor (total amount of effective labor). In addition to the factor endowment difference, their model assumes technology differences - with the North having a comparative advantage in producing high-quality brands. The prediction about the effect of trade is similar to that of the Falvey-Kierzkowski model: the North (South) will specialize and export higher- (lower-) quality brands.[1]

These results may be hastily interpreted as a baseline prediction of how trade affects product quality. But they are not, because the quality levels of *individual* brands are exogenous. Thus, by assumption, trade *cannot* lead to a change in the quality of any particular brand; it only affects a country's pattern of specialization.

The third strand in static models refers to game-theoretic models of quality, quantity and price choice (starting from Das and Donnenfeld (1989) to Zhou et al. (2002), among others), which are partial-equilibrium and thus not designed to answer the effect of opening up trade.

Finally, quality choice in terms of "rising product quality" in a dynamic and international setting is contained in the seminal work by Grossman and Helpman (1993). Quality improvement is synonymous with product innovation. Using two factors, unskilled labor and skill-labor (human capital), and the assumption that the R&D activity is human-capital intensive, their model predicts that international trade enables a relatively skilled-labor abundant country to "capture leadership positions in a large number of high-technology industries as compared to its relative output of traditional goods."

While this prediction is useful, we note that all quality improvements do not stem from new blue prints or patents. Quality variation by firms producing similar products may very well result from different levels and compositions of resource use *within a given technology.* There are products/services whose technologies are quite standard and yet there are international differences in quality, such as wrist watches, cloths, electric appliances and shoes - even a basic, functional car. For example, casual observation tells that in case of electrical appliances, (at least until recently) because of superior raw material and workmanship, those from developed countries were sturdier and more reliable, i.e. quality-wise better than the brands from developing countries. In business-management literature, there is a concept called "total quality control" (e.g, Deming, 1986), which emphasizes quality enhancements in terms of removing defects, improvising product components etc. rather than through any fundamental technological advancement.

Thus, it is our view that the effect of international trade on product quality, at some basic, static level, is not something that is adequately understood as yet. The current paper addresses this issue in a 2 × 2 (skilled and unskilled labor) factor-endowment framework, *assuming a given technology of producing quality, while not assuming any difference in technology or any brand quality being given.* More specifically, in our model there are horizontally differentiated brands and the quality level of each brand can vary. In other words, it is a hybrid model with horizontal and vertical product differentiation. Given the 2 × 2 structure, as the title of the paper suggests, factor-price-equalization (FPE) is quite plausible - and one of our central findings is that FPE implies PQE (product-quality equalization)! A particular version of this framework is contained in Das (2003), where the focus is on the effect of international trade on the relative wages. In this paper we present a much more general and complete analysis that this important issue of international trade and product quality deserves.

In section I we begin with a model of monopolistic competition and trade in the tradition of Krugman (1979) and Dixit and Norman (1980). The Dixit-Stiglitz utility function is generalized to allow for utility gain from quality. Quality choice is a decision variable facing a firm. There is a further generalization: besides scale economies in producing quantity, we allow for scale diseconomies of producing quality. Assuming that quality production is more skill intensive than quantity production, in autarky the firms in the relatively skilled-labor abundant North produce higher quality compared to their counterparts in the South. As free trade is allowed, the relative wage rises in the North and falls in the South, making quality production more costly in the North and less in the South; hence it falls in the North and rises in the South and in the FPE equilibrium they are equalized.

However, since in monopolistic competition each firm is small relative to the entire market, it cannot capture the directly pro-competitive effect of opening up freer trade. In section II, we consider oligopoly competition. Opening up trade increases the number of rivals and hence intensifies strategic competition affecting the first-order rules of profit maximization. This opens the possibility that firms in both North and South increase quality in response to international trade (because of greater competition). If we combine this with PQE, it is then possible that while in autarky North produces a higher quality than South, as (free) trade opens, PQE occurs at a quality level, higher than that in autarky in the North. We interpret this phenomenon as *quality leapfrogging* by the South.

In a nutshell, our findings say that (a) free trade has an element of narrowing quality differences of products that are traded and (b) quality leapfrogging by the South (the developing countries) is not necessarily an outcome of international technology transfer.

I. Monopolistic Competition

There are two countries: North (N) and South (S). We first characterize autarky and then consider free trade between these countries.

In each country, there are two sectors, M (manufactures) and Y (numeraire). The market structure in sector M is monopolistically competitive, while perfect competition prevails in sector Y. There are two primary factors of production, skill labor and unskill labor, and North is relatively more skilled-labor endowed than the South. Both factors are used in each sector. Sector M produces quantity and quality (imagine that there are two divisions, design and production lines), while the output of sector Y is homogeneous and its quality is pre-determined. In sector M, there are increasing returns in producing quantity, while quality is produced under constant or decreasing returns to scale. To begin with, we assume constant-returns in producing quality. The numeraire good is produced under constant-retrurns technology.

Furthermore, quality production in sector M is most skill-labor intensive, followed by quantity production in that sector. The technology of producing good Y is the least skill-labor intensive.

Sector M produces a differentiated product. Preferences are given by a generalized Dixit-Stiglitz utility function, similar to the one used in Das (2003):

$$\left(\int_0^n q_i^{\frac{1}{\epsilon}} c_i^{\frac{\vartheta-1}{\vartheta}} di\right)^{\frac{\gamma\vartheta}{\vartheta-1}} c_y^{1-\gamma}, 0 < \gamma < 1 < \vartheta < \epsilon$$

where $c_i \equiv$ quantity consumed of the ith variety of good M, $q_i \equiv$ its quality and $c_y \equiv$ the consumption of good Y. The restrictions on parameters γ and ϑ are well-known. The rationale behind $\epsilon > \vartheta$ will be noted later. Mark that this utility function is homothetic with respect to the quantities consumed, while both quantity and quality are subject to diminishing marginal utility. A representative consumer maximizes utility subject to the budget constraint: $\int_0^n p_i c_i di + c_y \leqslant I$, where p_i is the price of variety i and I is the income.

Utility maximization leads the following demand functions:

$$(1)\quad c_i = \frac{\gamma q_i^{\tau} I}{p_i^{\vartheta} Z}, \text{ where } Z \equiv \int_0^n q_i^{\tau} p_i^{1-\vartheta} di \text{ and } \tau \equiv \frac{\vartheta}{\epsilon} < 1$$

$$(2)\quad c_y = (1-\gamma) I$$

$$(3)\quad \Rightarrow \frac{\partial c_i}{\partial p_i} = -\frac{c_i}{p_i}\frac{\vartheta Z - (\vartheta-1) q_i^{\tau} p_i^{1-\vartheta}}{Z} < 0; \quad \frac{\partial c_i}{\partial q_i} = \frac{\tau c_i}{q_i}\frac{Z - q_i^{\tau} p_i^{1-\vartheta}}{Z} > 0.$$

Turning to the production side, let w_s and w_u denote wage payment per unit of skilled labor and unskilled labor respectively. We express technologies by cost functions. In the numeraire sector, let $c^y(w_s, w_u)$ denote the unit and marginal cost function. In sector M, in producing quality, $c^q(w_s, w_u)$ defines the unit and marginal cost functions. The total cost of producing quantity x_i of variety i is given by $C^x(w_s, w_u, x_i) = c^x(w_s, w_u) f(x_i)$. The following assumptions are imposed on the $f(.)$ function:

(a) $f' > 0$;

(b) Define the scale function $\Phi(x) \equiv \frac{f(x)}{xf'(x)} = \frac{AC}{MC}$ such that $\Phi(x) > 1$ for all $x > 0$;

(c) $f'' \geq 0$, which implies that the scale economies decrease with output, i.e., $\Phi'(x) < 0$;

(d) $\lim_{x \to 0} \Phi(x) \to \infty$; $\lim_{x \to \infty} \Phi(x) \to 1$.[2]

Perfect competition and free entry and exit in sector Y imply the zero-profit condition:

$$c^y(w_s,\, w_u) = 1. \tag{4}$$

In sector M, the profit of firm i producing variety i has the expression: $\pi_i = p_i x_i - c^x(w_s, w_u) f(x_i) - c^q(w_s, w_u) q_i$. As standard, in viewing the demand function (1), firm i treats I and Z as parameters. Hence the profit-maximizing rule with respect to quantity or price is given by the MR = MC rule:

$$p_i \left(1 - \frac{1}{\vartheta}\right) = c^x(w_s, w_u) f'(x_i). \tag{5}$$

Totally differentiating π_i with respect to q_i (and again treating I and Z are parameters), the quality-setting rule, in view of (1) again, is given by

$$[p_i - c^x(w_s, w_u) f'(x_i)] \frac{\tau x_i}{q_i} - c^q(w_s, w_u) = 0. \tag{6}$$

If we substitute (5) into (6), we have

$$\frac{\tau x_i f'(x_i)}{\vartheta - 1} \frac{c^x(w_s, w_u)}{c^q(w_s, w_u)} = q_i, \tag{7}$$

which relates quantity, quality and factor prices. Given our assumption of $f'' \geq 0$, quality is positively related to quantity in equilibrium. As quality production is more skilled labor intensive than quantity production in sector M, the ratio c^x/c^q decreases with the relative wage, $\omega \equiv w_s/w_u$; intuitively, an increase in the relative wage implies a decrease in the quality produced.

Next, the zero-profit condition in sector M is expressed as:

$$p_i x_i - c^x(w_s, w_u) f(x_i) - c^q(w_s, w_u) q_i = 0. \tag{8}$$

If we substitute the price- and quality-setting rules into this condition, we obtain, after some manipulation,

$$\Phi(x_i) = \frac{\vartheta - \tau}{\vartheta - 1}. \tag{9}$$

This is where the assumption of $\epsilon > \vartheta$ plays a critical role in the model: as it implies $\tau < 1$, the ratio $(\vartheta - \tau)/(\vartheta - 1)$ exceeds unity, such that given the assumptions (c) and (d) on the $\Phi(x)$ function, there exists a solution to eq. (9), which is the long-run level of firm-output. Moreover, assumption (d) implies the solution is unique. Let it be denoted by $\overline{x}$.

Given $x_i = \overline{x}$, it follows that $p_i = p$ and $q_i = q$.[3] We now specify the rest of the relationships in a closed economy:

$$n\left[c_u^q(\omega)\, q(\omega) + c_u^x(\omega)\, f(\overline{x})\right] + c_u^y(\omega)\, Y = \overline{L}_u \tag{10}$$

$$n\left[c_s^q(\omega)\, q(\omega) + c_s^x(\omega)\, f(\overline{x})\right] + c_s^y(\omega)\, Y = \overline{L}_s \tag{11}$$

$$\frac{\gamma\left(w_u \overline{L}_u + w_s \overline{L}_s\right)}{np} = x. \tag{12}$$

The variable n denotes the number of firms too. The partials of $c^j(.)$, $j = q, x, y$ denote the respective input coefficient, e.g., $c_u^q(\omega) \equiv \partial c^q(w_s, w_u)/\partial w_u$ is the unskilled labor coefficient in quality production; Y is the output of the numeraire good and $\overline{L}_u$ ($\overline{L}_s$) is the inelastic endowment of unskilled (skilled) labor in the economy. Eqs. (10) and (11) spell the respective full employment conditions. The last equation is the market clearing condition of any particular variety of good M.

It is easy to see from the full-employment equations that the relative version of the Rybczinski theorem holds: a higher relative endowment of skilled labor implies a higher relative output of good M (n/Y), at any given p, since both quality and quantity producing activities in sector M are more skilled labor intensive relative to the numeraire sector. Further, given the factor endowments, as shown in Appendix A, the n/Y ratio increases with p, i.e. the relative 'supply curve' is upward sloping. Hence the autarky equilibrium is unique.

Denoting countries by respective subscripts and 'autarky' by a, it readily follows that $p_{Na} < p_{Sa}$, $\omega_{Na} < \omega_{Sa}$ and in particular $q_{Na} > q_{Sa}$, where ω denotes the relative wage. All these rankings are intuitive.

Free Trade

Now suppose that the two countries move to a regime of free trade. Since there is no strategic interaction among firms, their price/quantity and quality decision rules remain unchanged. Assuming that both countries incompletely specialize, the equilibrium firm-outputs in *both* countries are given by (9). The full employment equations remain the same. The only change occurs in the market-clearing equation (12). It is now

$$\frac{\gamma q_r^{\tau} I_R}{p_r^{\vartheta}(n_N q_N^{\tau} p_N^{1-\vartheta} + n_S q_S^{\tau} p_S^{1-\vartheta})} = x_r, \tag{13}$$

where $r =$ North, South and I_R is the world income. We now prove FPE and PQE.

Since $x_r = \overline{x}$, (14) implies

$$\frac{q_N^{\tau}}{p_N^{\vartheta}} = \frac{q_S^{\tau}}{p_S^{\vartheta}}. \tag{14}$$

Suppose $p_N < p_S$. Then (14) implies $q_N < q_S$. In view of (7), this implies $(c^x/c^q)_N < (c^x/c^q)_S$. As this ratio is a decreasing function of ω, $\omega_N > \omega_S$. But, by the Stolper-Samuelson relation, $p_N < p_S \Rightarrow \omega_N < \omega_S$. Hence there is a contradiction. Similar contradiction arises if $p_N > p_S$. It then follows from $p_N = p_S$, implying $\omega_N = \omega_S$ *and* $q_N = q_S$. That is, product quality equalization accompanies factor price equalization. This is a central result of our paper. Intuitively, the quality level chosen by a firm depends on the wage ratio. This implies that, if technologies are same between the trading countries and wages are equalized, product quality is equalized too. Let us denote $p_r = p_R,\ w_{sr} = w_{sR},\ w_{ur} = w_{uR},\ \omega_r = \omega_R$ and $q_r = q_R$.

How does the free-trade equilibrium compare to the respective autarky equilibria? Factor endowment differences imply that the relative wage rises in the North and falls in the South. Therefore, quality production falls in the North and improves in the South. Factor endowment differences also imply that the North exports good M to the South (proved in Appendix A).[4]

Proposition 1 *If the two countries incompletely specialize in the trading equilibrium, FPE holds, implying PQE. The North (South) is a net exporter (importer)* *of good M, and, compared to autarky, the relative wage rises (falls) and product quality falls (rises) in the North (South).*

The well-known Dixit-Norman technique of analyzing 'integrated equilibrium' yields the pattern of specialization. This is outlined in Appendix A

Decreasing Returns to Scale in Quality Production

Increasing returns to scale in producing quantity in sector M capture the assembly-line nature of mass production. But producing quality is quite different. Many partial equilibrium models of quality choice assume increasing costs – for natural reasons.[5] We now show that our central results hold if there are decreasing returns to scale (increasing marginal costs) in quality production.

Define $C^q(w_s, w_u, q) = c^q(w_s, w_u)\, g(q)$, with the properties that the scale elasticity, $g(q)/(qg'(q)) \equiv \Psi(q)$, is less than one, and $\Psi'(q) \leq 0$, i.e. diseconomies continue to increase with quality production. The last assumption implies that $g'' \geq 0$.

A firm's objective function has the expression: $\pi_i = p_i x_i - c^x(w_s, w_u)\, f(x_i) - c^q(w_s, w_u)\, g(q_i)$. Its pricing rule is the same as (5), but its quality-setting rule, after substituting (5) into it, becomes

$$(15) \qquad \frac{\tau x_i f'(x_i)}{\vartheta - 1} \cdot \frac{c^x(w_s, w_u)}{c^q(w_s, w_u)} = q_i g'(q_i).$$

We have the zero-profit condition, $p_i x_i - c^x(w_s, w_u)\, f(x_i) - c^q(w_s, w_u)\, g(q_i) = 0$. Substituting the price and quality-setting rules in it and eliminating p_i and $c^q(.)$, we obtain the following relationship between quantity and quality produced:

$$(16) \qquad \vartheta - \tau\Psi(q_i) = (\vartheta - 1)\Phi(x_i).$$

This equation reveals that, unlike when quality production is subject to constant costs, the long-run output of a firm is not independent of its quality choice. At given factor prices, quality production and quantity production are jointly determined by eqs. (15) and (16). These equations respectively spell a positive and a negative locus in x_i and q_i, implying unique solutions. As a simple comparative statics, we note that $x'(\omega) > 0$ and $q'(\omega) < 0$, because quality production is more skill-intensive than quantity production.

Dividing (5) by (4) yields

$$\frac{f'(x(\omega))c^x(w_s, w_u)}{c^y(w_s, w_u)} = \frac{\vartheta p}{\vartheta - 1} \tag{17}$$

Given $f'' \geq 0$, $x'(\omega) > 0$ and that quantity production in sector M is more skill-intensive than the numeraire sector, the l.h.s. of the above equation is monotonically increasing in ω. Thus $\omega = \omega(p)$ with $\omega'(p) > 0$, i.e. the Stolper-Samuelson implication holds.

The monotonic relationships between ω and x, ω and q, and ω and p imply that, as long as the countries incompletely specialize in the trading equilibrium, FPE and PQE hold.[6]

II. Oligopoly

A monopolistically competitive market structure cannot accommodate the *directly* pro-competitive effect of freer trade, as each firm being arbitrarily small compared to the whole market faces the same price and quality elasticity of quantity demanded in both autarky and free trade. We now assume that the market structure in sector M is an oligopoly – which allows us to incorporate the directly pro-competitive effect of trade and examine its implication. Firms in either country will have to face more competition in the presence of international trade than in its absence, and this leads to a presumption that free trade leads to quality enhancement in both countries. If PQE holds, then it implies a greater quality jump and at the same time a catching-up by the Southern firms, which we have termed as quality leapfrogging by the South.

However, the formal analysis gets quite complex under oligopoly. To keep tractability we impose two assumptions. First, let the increasing-returns technology in producing quantity in sector M be specified by fixed and variable costs, i.e., let the total cost function of producing quantity be of the form: $c^x(w_s, w_u)(x_i + \alpha)$, where $\alpha > 0$. Second, let the scale elasticity in producing quality, Ψ, be constant, i.e., let $g(q)$ be of the form, $g(q) = q^{1+\eta}/(1+\eta)$, $\eta \geq 0$, where $\eta = 0$ or > 0 signifies constant or decreasing returns. We begin with autarky.

Autarky

In sector Y, the zero–profit condition is same as (4), reproduced below for completeness.

$$c^y(w_s,\, w_u) = 1. \tag{18}$$

Assume that, in sector M, firms compete in prices and quality. In view of (3) then, any particular firm's price and quality setting rules are

$$\left[1 - \frac{c^x\,(w_s, w_u)}{p_i}\right]\left[\vartheta - \frac{(\vartheta - 1)\, q_i^{\tau} p_i^{1-\vartheta}}{Z}\right] = 1 \tag{19}$$

$$\left[p_i - c^x(w_s, w_u)\right] x_i \cdot \frac{\tau(Z - q_i^{\tau} p_i^{1-\vartheta})}{Z} = c^q(w_s, w_u) q_i g'(q_i). \tag{20}$$

Note that these conditions take into account the effect of a change in price and quality on Z. Under symmetry, these two equations reduce to:

$$\text{(21)} \qquad \left[1 - \frac{c^x(w_s, w_u)}{p}\right]\left[\vartheta - \frac{\vartheta - 1}{n}\right] = 1$$

$$\text{(22)} \qquad [p - c^x(w_s, w_u)]\, x\tau \left(1 - \frac{1}{n}\right) = c^q(w_s, w_u) q g'(q).$$

The next four equations respective spell, under symmetry, the zero-profit condition, the full-employment conditions and the market-clearing condition for any variety.

$$\text{(23)} \qquad [p - c^x(w_s, w_u)]\, x - \alpha c^x(w_s, w_u) - c^q(w_s, w_u) g(q) = 0$$

$$\text{(24)} \qquad n\left[c_u^q(\omega) g(q) + c_u^x(\omega)(x + \alpha)\right] + c_u^y(\omega) Y = \overline{L}_u$$

$$\text{(25)} \qquad n\left[c_s^q(\omega) g(q) + c_s^x(\omega)(x + \alpha)\right] + c_s^y(\omega) Y = \overline{L}_s$$

$$\text{(26)} \qquad \frac{\gamma(w_s \overline{L}_s + w_u \overline{L}_u)}{pn} = x.$$

Eq. (18) together with (21)-(26) are seven equations having seven variables, w_s, w_u, p, x, q, n and Y.

Unless further structure is imposed, it does not seem possible to compare the autarky equilibria across North and South. But this is immaterial to the issue of whether FPE and PQE hold in free trade. Therefore, we first analyze FPE and PQE at the present level of generality, and, then consider special cases illustrating the trade-effects on product quality in each country.

Free Trade

As countries open up free trade, firms in each country face competition from local *and* foreign firms. Eqs. (19) and (20) still characterize price and quality setting rules of any particular firm, except that the term Z include the price and quality terms of domestic and foreign varieties, i.e.,

$$\text{(27)} \qquad \left[1 - \frac{c^x(w_{sr}, w_{ur})}{p_r}\right]\left[\vartheta - \frac{(\vartheta - 1)\, q_r^{\tau} p_r^{1-\vartheta}}{Z_R}\right] = 1$$

$$\text{(28)} \qquad [p_r - c^x(w_{sr}, w_{ur})]\, x_r \cdot \frac{\tau(Z_R - q_r^{\tau} p_r^{1-\vartheta})}{Z_R} = c^q(w_{sr}, w_{ur}) q_r g'(q_r)$$

where r refers to North or South, and $Z_R = \sum_r n_r q_r^{\tau} p_r^{1-\vartheta}$. The zero-profit and the full employment conditions remain the same:

$$\text{(29)} \qquad [p_r - c^x(w_{sr}, w_{ur})]\, x_r - \alpha c^x(w_{sr}, w_{ur}) - c^q(w_{sr}, w_{ur}) g(q_r) = 0$$

$$\text{(30)} \qquad n_r\left[c_u^q(\omega_r) g(q_r) + c_u^x(\omega_r)(x_r + \alpha)\right] + c_u^y(\omega_r) Y_r = \overline{L}_{ur}$$

$$\text{(31)} \qquad n_r\left[c_s^q(\omega_r) g(q_r) + c_s^x(\omega_r)(x_r + \alpha)\right] + c_s^y(\omega_r) Y_r = \overline{L}_{sr}.$$

Finally, we have the market-clearing condition for any particular variety, expressed as

$$\text{(32)} \qquad \frac{\gamma I_R q_r^{\tau}}{p_r^{\vartheta} Z_R} = x_r,$$

where I_R is the world income.

Unlike under monopolistic competition, it is however not a straightfoward exercise to prove that FPE and PQE hold when both countries incompetely specialize; this is because the long-run firm-output is dependent on the relative wage or the product price ratio. A formal – and a rather long – proof is given in Appendix B, and thus,

Proposition 2 *In free trade equilibrium, as long as each country incompletely specializes, FPE and PQE hold under oligopoly.*[7]

The economic explanation behind product quality equalization under oligopoly is a bit more involved, compared to the monopolistic-competition case. Here, quality production depends on relative wage as well as the number of rivals. Note that free trade provides each firm located in either country an access to the entire global market. Therefore, each firm competes with the same number of firms. This, together with FPE, implies PQE.

Given that the world economy moves from autarky to the integrated equilibrium, insofar as the qualitative effects on firm-output, product price, product quality and relative wages are concerned, *free trade is equivalent to augmenting a country's factor endowments to the respective world factor endowments.* The question before us is: how does product quality in each country adjust when there is a regime change from autarky to free trade?

It is however difficult to answer this question at the current level of generality. In what follows, we consider two special cases. In both, the numeraire good is produced by unskilled labor only so that the unskilled wage is fixed in terms of this good**.** For notational simplicity, let the unskilled labor coefficient be one such that the unskilled wage is equal to unity. Since w_u is given, let $w_s = w$ denote the relative wage. The special cases differ in terms of assumed factor intensity difference between quantity and quality production in sector M.

II..0.1 Same Factor Intensities in Producing Quantity and Quality in Sector M

Suppose that quantity and quality production in sector M are equally skill intensive, i.e., $c^x(w)/c^q(w)$ is constant. This implies that a firm's quality choice depends only on product market competition, not on the relative wage. For notational simplicity choose the unit of q such that $c^x(w)/c^q(w)$ is equal to unity.

Accordingly, define $c^x(w) = c^q(w) \equiv c(w)$.

In autarky, eqs. (21)-(23) and (25)-(26) reduce to

$$p_a = \frac{n_a + (\vartheta - 1)(n_a - 1)}{(\vartheta - 1)(n_a - 1)} \cdot c(w_a) \tag{33}$$

$$\frac{[p_a - c(w_a)]\, x_a \tau (n_a - 1)}{n_a} = \frac{g(q_a) c(w_a)}{\Psi} \tag{34}$$

$$[p_a - c(w_a)]\, x_a - [\alpha + g(q_a)] c(w_a) = 0 \tag{35}$$

$$n_a [g(q_a) + x_a + \alpha] = \frac{\overline{L}_s}{c'(w_a)} \tag{36}$$

$$\frac{\gamma(w_a \overline{L}_s + \overline{L}_u)}{n_a p_a} = x_a. \tag{37}$$

These equations solve p_a, q_a, n_a, w_a and x_a.[8] The first three equations imply

$$x_a = \frac{\alpha(\vartheta - 1)(n_a - 1)}{n_a - \tau\Psi(n_a - 1)} \equiv x(\underset{+}{n_a}); \quad g(q_a) = \frac{\alpha\tau\Psi(n_a - 1)}{n_a - \tau\Psi(n_a - 1)} \equiv q(\underset{+}{n_a}). \tag{38}$$

Check that both quantity and quality are monotonically increasing functions of n, showing the pro-competitive effect of an increase in n. Notice that unlike under monopolistic competition, the long-run output does not depend only on the fixed-cost parameter, α, and the preference parameter ϑ.

Using the expressions in (38), eqs. (36) and (37) are respectively expressed as

$$\frac{\alpha n_a [n_a + (\vartheta - 1)(n_a - 1)]}{n_a - \tau\Psi(n_a - 1)} = \frac{\overline{L}_s}{c'(w_a)} \tag{39}$$

$$\frac{\alpha n_a [n_a + (\vartheta - 1)(n_a - 1)]}{n_a - \tau\Psi(n_a - 1)} = \frac{\gamma\left(w_a \overline{L}_s + \overline{L}_u\right)}{c(w_a)}. \tag{40}$$

Dividing these equations gives

$$\gamma \left(w_a + \frac{\overline{L}_u}{\overline{L}_s} \right) \frac{c'(w_a)}{c(w_a)} = 1, \tag{41}$$

which determines w_a. As expected, an increase in $\overline{L}_u/\overline{L}_s$ implies an increase in w_a. Once w_a is known, either of (39) and (40) determines n_a and thereby output and product quality. In the (w, n) space measuring w along the vertical axis, eq. (39) spells a positive locus, while (41) implies a horizontal line. These are respectively shown as FF_1 and MM_1 in Figure 1.

Figure 1 around here

Note that while relative wage is affected by relative endowment only, the number of firms, output and product quality are affected by both relative endowment and absolute endowments. We consider two comparative statics, which enable us to compare North and South under autarky and compare free trade to autarky.

1. Suppose the absolute endowment of skilled labor remains unchanged but its relative endowment increases. Then MM_1 curve shifts down, showing that the relative wage as well as the number of firms fall. The former effect is direct, and, as w falls, firms in sector M adopt a more skill-intensive technique. The total endowment of skilled labor being given, sector M is able to sustain a smaller number of firms. In turn, less competition implies a lower output and a lower product quality chosen by a firm.

2. Suppose the relative endowments are unchanged, but the absolute endowments increase. Then FF_1 shifts to the right, while the MM_1 curve does not shift. As a result, w_a is unchanged, while n_a is higher (i.e. a greater market size

sustains more firms). More firms mean more competition and hence a greater output and a higher product quality by a firm.

Now define North as the country in which both the relative and absolute endowments of skilled labor are higher. It then follows that under autarky, $w_{aN} < w_{aS}$, while $q_{aN} \lesseqgtr q_{aS}$. However, $q_{aN} > q_{aS}$ if the relative endowment difference is small enough or if the sector M is sufficiently highly skill intensive.[9] We assume that either of these conditions holds.

We are now ready to determine the effects of international trade. As shown earlier, FPE and PQE hold in free trade equilibrium. Furthermore, because North is relatively and absolutely more endowed with skilled labor, in moving from autarky to free trade it essentially 'faces' a decrease in the relative endowment and an increase in the absolute endowment of skilled labor. By similar argument, South faces an increase in the relative and absolute endowment of skilled labor. By using Figure 1 and applying it to the North we find that Northern relative wage increases and so does the quality (since the number of firms in the world economy in free trade exceeds that under autarky). In the South, the relative wage declines but Figure 1 cannot unambiguously indicate the direction of change in product quality. However, given that $q_{aN} > q_{aS}$ and product quality improves in the North, PQE implies that product quality must improve in the South as well.[10] Indeed, South catches up or erases its 'quality deficit' even in the face of North further improving its product quality. In summary, as the world economy moves from autarky to free trade, the relative wage increases in the North and falls in the South, while product quality improves in both countries, along with quality leapfrogging by the South.

It is worth-emphasizing that oligopoly competition underlies quality leapfrogging. The special case under consideration is able to entirely focus on this by suppressing the effect of relative wage changes on product quality. Insofar as the effect on relative wages is concerned, its decline in the South depends critically on the assumption of no factor intensity difference between quantity and quality production in sector M. If we had assumed that quality production is more skill intensive, then relative wage would be related to the quality level and as a consequence leapfrogging can occur together with an increase in the relative wage in both countries. Such a case is considered next.

II..0.2 Quality in Sector M dependent on Skilled Labor Only

This is the case where, in the two extreme ends, the numeraire good is produced by unskilled labor only and the quality production in sector M is undertaken by skilled labor only (with respective input coefficients normalized to one), while quantity production in sector M requires both factors having a Cobb-Douglas technology. Let $c^x(w) = w^\mu$ ($\mu < 1$) and w denotes the relative wage.

The following equations characterize the autarky equilibrium:

$$\frac{n_a + (\vartheta - 1)(n_a - 1)}{(\vartheta - 1)(n_a - 1)} \cdot c^x(w_a) = p_a \tag{42}$$

$$[p_a - c^x(w_a)]\, x_a \cdot \frac{\tau(n_a - 1)}{n_a} = \frac{w_a g(q_a)}{\Psi} \tag{43}$$

$$[p_a - c^x(w_a)]\, x_a - \alpha c^x(w_a) - w_a g(q_a) = 0 \tag{44}$$

$$n_a[g(q_a) + c^x_s(w_a)(x_a + \alpha)] = \overline{L}_s \tag{45}$$

$$x_a = \frac{\gamma(w_a \overline{L}_s + \overline{L}_u)}{n_a p_a}. \tag{46}$$

The first two equations, the price and quality setting rules, follow from (21) and (22). The next three are respectively the zero-profit condition, the full-employment condition of skilled labor and the market-clearing condition. Eqs.

(42) - (44) imply

(47)

$$x_a = \frac{\alpha\,(\vartheta-1)\,(n_a-1)}{n_a-\tau\Psi\,(n_a-1)} \equiv x(\underset{+}{n_a}); \quad g(q_a) = \frac{\alpha\tau\Psi\,(n_a-1)}{[n_a-\tau\Psi\,(n_a-1)]b(w_a)} \equiv h(\underset{+}{n_a},\underset{-}{w_a}),$$

where $b(w) = w/c^x(w)$. Compared to the earlier special case, note that quality choice is influenced by the relative wage rate also. Substituting (47) into (45) and (46),

$$\text{(48)} \qquad \frac{\alpha n_a\{\tau\Psi\,(n_a-1)+\mu[(\vartheta-\tau\Psi)\,(n_a-1)+1]\}}{n_a-\tau\Psi\,(n_a-1)} = b(w_a)\overline{L}_s$$

$$\text{(49)} \qquad \frac{\alpha n_a[n_a+(\vartheta-1)\,(n_a-1)]}{n_a-\tau\Psi\,(n_a-1)} = \frac{\gamma\left(w_a\overline{L}_s+\overline{L}_u\right)}{c^x\,(w_a)}.$$

These are two equations in two variables, n_a and w_a, solving the autarky equilibrium. However, product quality being our focus, it will be convenient to deal with an equation system having q explicitly as a variable. Towards this end, we divide the last two equations and obtain

$$\text{(50)} \qquad \frac{n_a+(\vartheta-1)\,(n_a-1)}{\tau\Psi\,(n_a-1)+\mu[(\vartheta-\tau\Psi)\,(n_a-1)+1]} = \gamma\left(\frac{w_a\overline{L}_s+\overline{L}_u}{w_a\overline{L}_s}\right),$$

which will be used to evaluate the effect of a relative endowment change. Next we implicitly invert the function $h(\underset{+}{n},\underset{-}{w})$ and obtain $n = n(\underset{+}{g},\underset{+}{w})$, and view (48) and (50) determining w_a and g_a. From the definition of the $g(\cdot)$ function, g and q are one-to-one related. Hence a solution of g is equivalent to a solution of q.

Appendix B proves that these equations respectively spell a negative relation and a positive relation between w_a and q_a, shown respectively by FF_2 and MM_2 curves in Figure 2.

Figure 2 around here

Note that not just the relative endowment but absolute endowments matter toward the equilibrium relative wage (and quality choice) – because the price-cost mark-up is not constant. We consider the same two comparative statics as in the previous special case.

First, suppose that the absolute endowment of skilled labor remains unchanged but its relative endowment increases. Then MM_2 shifts to the right. As a result, w_a falls and q_a rises. These results are intuitive.

Next, suppose the relative endowments are unchanged, while the absolute endowments increase. Then FF_2 shifts to the right, with the implication that both w_a and q_a increase.

Again, defining North as the country in which both the relative and absolute endowments of skilled labor are higher, it then follows that under autarky, $q_{aN} > q_{aS}$, while $w_{aN} \lesseqgtr w_{aS}$.

As before, a movement from autarky to free trade (along with FPE and PQE) means that the North essentially faces a decrease in the relative endowment and an increase in the absolute endowment of skilled labor, while South faces an increase in the relative and absolute endowment of skilled labor. In Figure 2, FF_2 curve shifts to the right for both countries, while the MM_2 curve shifts to the left for the North and to the right for the South. Thus we obain the following comparison between autarky and free trade: As the world economy moves from autarky to free trade, in the North the relative wage increases, while product quality may fall or improve, while in the South relative wage may increase or decrease but product quality improves.

Given that $q_{aN} > q_{aS}$, quality leapfrogging *will* occur if the North increases it product quality. Observing the shifts of FF_2 and MM_2 curves it follows that this will happen if the relative endowment differences are not large (implying that the leftward shift of the MM_2 curve is relatively small). Also, in this case, relative wage will rise in the South too, because the effects due to relative endowment differences are small. Hence, we have following proposition.

Proposition 3 *Quality leapfrogging occurs if (a) the relative endowment difference across countries is small enough or (b) if the skill intensity difference between quality and quantity production is small enough and the skill intensity in sector M sufficiently exceeds the skill intensity of the numeraire sector.*

II..0.3 Relation to the Existing Literature on Trade and Quality Leapfrogging

There is a literature on quality leapfrogging in the presence of international trade, meaning a simple catching-up in quality, without the quality leader upgrading its quality further. This is motivated by mutual trade liberalization or country-specific incentives offered to lagging industries in the South.[11] For example, Motta et al. (1997) develop a partial-equilibrium oligopoly model of vertical product differentiation *a la* Shaked and Sutton (1982), in which one country's firm has initial leadership. Their conclusion is that such leadership is likely to persist in the presence of free international trade; leapfrogging is possible only if the country sizes are very similar. Herguera and Lutz (1998) consider a similar model and show that leapfrogging can occur when the lagging country offers special incentives to its industry.[12]

Compared to this literature, the distinguishing features of our analysis are that we consider a more general oligopoly with entry and exit and that too in a general-equilibrium framework in which the costs of producing quality and quantity change in response to international trade - with the implication that the initial leader (the North) may further improve its product quality. Quality leapfrogging occurs, *not* in terms of new technology adoption, but in response to competition *under given technologies of producing quality and quantity.*

III. Concluding Remarks

The literature on product quality and trade policy is vast. Yet, how free trade may induce a change in product quality does not seem to have been addressed adequately. While the partial-equilibrium models are not designed to address this issue, the existing general-equilibrium models typically classify product brands according to their quality in an exogenous fashion and analyze which countries would have comparative advantage which brands – rather than how the quality levels of the brands themselves respond to a change in the trade regime. This paper has developed a baseline, factor-endowment framework, in which the technologies of quantity and quality production are given and available to all (both) trading countries. The model is a hybrid one having both horizontal and vertical product differentiation. Quality choice by firms responds to relative wage changes and changes in the degree of competition. There are two central results. First, factor price equalization (FPE) is shown to imply product quality equalization (PQE). Second, oligopoly competition may imply that product quality improves due to trade in both North and South and yet South leapfrogs and erases the quality deficit in the FPE-PQE equilibrium.

In the oligopoly model in particular, we have assumed that firms treat price and quality as their strategies. Other combinations are possible of course. However, the micro structure of demand based on the generalized Dixti-Stiglitz utility function makes it extremely hard to analytically deal with other pairs of strategic variables among price, quantity and quality, e.g. quantity and quality. But oligopoly firm behavior would yield quality as a function of relative wage and the degree of competition (the number of firms) and therefore the basic insights of our analysis are likely to go through.

The key notion of our model is that of production of quality, not necessarily as a technology innovation but as an outcome of factor combinations. In a dynamic, endogenous-growth framework, it will be interesting to differentiate between basic innovation resulting in the form of a jump in *potential* quality and *actual* quality upgrading based on new technology as well as factor use or organization.

Also, while, in order to emphasize the implications of factor endowment differences, our model maintains the assumption of identical technologies of both quantity and quality production, the analytical difference between these two concepts is applicable to the issue of technology transfer between North and South – insofar as it leads to a difference between potential quality achievable and actual quality achieved.

APPENDICES

A The Monopolistic Competition Model

A.1 Relative Supply Curve: Constant-Returns in Quality Production

Given $x = \overline{x}$, the zero-profit condition in sector Y, namely, (4) and the price-setting rule in sector M, (5), yield the Stolper-Samuelson expressions:

$$\text{(A.1)} \qquad \widehat{w_s} = \frac{\theta_u^y \widehat{p}}{|\theta|}; \quad \widehat{w_u} = -\frac{\theta_s^y \widehat{p}}{|\theta|}; \quad \widehat{\omega} = \frac{\widehat{p}}{|\theta|}, \quad |\theta| = \theta_s^x \theta_u^y - \theta_u^x \theta_s^y > 0$$

where $\theta_s^x = (w_s c_s^x)/c^x$, $\theta_u^x = (\omega_u c_u^x)/c^x, \theta_s^y = (w_s c_s^y)/c^y$ and $\theta_u^y = (w_u c_u^y)/c^y$. Again using $x = \overline{x}$, from the quality-setting rule (7), we obtain $\widehat{q} + \widehat{c^q} - \widehat{c^s} = 0$, implying

$$\text{(A.2)} \qquad \widehat{q} + (\theta_s^q - \theta_s^x)\widehat{\omega} = 0$$

If σ^q, σ^x and σ^y denote the elasticity of factor sustitution respectively in producing quality in sector M, output in sector M and output in sector Y, working through the standard Jones' algebra leads to

$$\text{(A.3)} \qquad \widehat{c_s^q} = -\sigma^q \theta_u^q \widehat{\omega}; \; \widehat{c_u^q} = \sigma^q \theta_s^q \widehat{\omega}$$

(A.4) $$\widehat{c_s^x} = -\sigma^x \theta_u^x \widehat{\omega};\ \widehat{c_u^x} = \sigma^x \theta_s^x \widehat{\omega};\ \widehat{c_s^a} = -\sigma^y \theta_u^y \widehat{\omega};\ \widehat{c_u^a} = \sigma^y \theta_s^y \widehat{\omega}.$$

Next, totally differentiate the full-emplyment equations (10) and (11) at given $\overline{L}_s$ and $\overline{L}_u$. Define μ_u^q (μ_s^q) as the proportion of unskilled (skilled) labor employed in quality production to the total size of unskilled (skilled) labor employment in sector M. Also define λ_j^m as the share of factor j's employment in sector M and similarly λ_j^y as the share of factor j's employment in sector Y, where $j = s,\ u$. Then we have

$$\text{(A.5)} \qquad \lambda_u^m \left[\widehat{n} + \mu_u^q \left(\widehat{c_u^q} + \widehat{q}\right) + (1 - \mu_u^q)\, \widehat{c_u^x}\right] + \lambda_u^y \left(\widehat{c_u^y} + \widehat{Y}\right) \ = \ 0$$

$$\text{(A.6)} \qquad \lambda_s^m \left[\widehat{n} + \mu_s^q \left(\widehat{c_s^q} + \widehat{q}\right) + (1 - \mu_s^q)\, \widehat{c_s^x}\right] + \lambda_s^y \left(\widehat{c_s^y} + \widehat{Y}\right) \ = \ 0$$

Define $|\lambda| = \lambda_s^m \lambda_u^y - \lambda_u^m \lambda_s^y$, which is positive by our factor-intensity assumption. Using (A.1), (A.2), (A.3)and (A.4), eqs. (A.5) and (A.6) imply

$$|\lambda| \frac{\widehat{n} - \widehat{Y}}{\widehat{p}} \ = \ [\lambda_s^m (1 - \mu_s^q)\, \sigma^x \theta_u^x + \lambda_u^m (1 - \mu_u^q)\, \sigma^x \theta_s^x + \lambda_s^m \mu_s^q \sigma^q \theta_u^q + \lambda_u^m \mu_u^q \sigma^q \theta_s^q$$

$$\text{(A.7)} \qquad + \lambda_s^y \sigma^y \theta_u^y + \lambda_u^y \sigma^y \theta_s^y + \left(\frac{L_s^q}{\overline{L}_s} - \frac{L_u^q}{\overline{L}_u}\right) \frac{\theta_s^q - \theta_s^x}{|\theta|} > 0,$$

$$\text{since } \theta_s^q > \theta_s^x, \text{ and } \frac{L_s^q}{\overline{L}_s} > \frac{L_u^q}{\overline{L}_u} \text{ as } \frac{L_s^q}{L_u^q} > \frac{L_s^x}{L_u^x} > \frac{L_s^y}{L_u^y}.$$

The sign of the expression in (A.7) proves that the relative supply curve is upward sloping.

A.2 Trade Pattern

It is straightforward to demonstrate that at the trading equilibirum where FPE and PQE hold, the North is the net exporter of good M.

North's expenditure on good M equals $\gamma \left(w_{sR}\overline{L}_{sN} + w_{uR}\overline{L}_{uN}\right)$. The value of production of good M in the North is equal to $n_N p_R \overline{x}$. Global market clearing of any variety is given by $\overline{x} = (\gamma I_R)/(n_R p_R)$,where $n_R = n_N + n_S$. Hence North is a net exporter of good M if and only if

$$\text{(A.8)} \qquad w_{sR}\overline{L}_{sN} + w_{uR}\overline{L}_{uN} < \frac{n_N}{n_R} \left(w_{sR}\overline{L}_{sR} + w_{uR}\overline{L}_{uR}\right).$$

Here $\overline{L}_{sR} = \overline{L}_{sN} + \overline{L}_{sS}$ and $\overline{L}_{uR} = \overline{L}_{uN} + \overline{L}_{uS}$.

In free trade equilibrium, n_N is solved from the equations, $\overline{c_u} n_N + c_u^y Y_N = \overline{L}_{uN}$ and $\overline{c_s} n_N + c_s^y Y_N = \overline{L}_{sN}$, where $\overline{c}_u$ ($\overline{c}_s$) is the unskilled (skilled) labor used by a firm in sector M and c_u^y (c_s^y) is the unskilled (skilled) labor coefficient in sector Y (see (A.11) and (A.12 later). We have

$$\text{(A.9)} \qquad n_N = \frac{c_u^y \overline{L}_{sN} - c_s^y \overline{L}_{uN}}{\overline{c}_s c_u^y - \overline{c}_u c_s^y}.$$

Given our factor-intensity, the numerator and the denominator are both positive. Likewise, n_R is solved from (A.13) - (A.14) and has the expression:

$$n_R = \frac{c_u^y \overline{L}_{sR} - c_s^y \overline{L}_{uR}}{\overline{c}_s c_u^y - \overline{c}_u c_s^y} \tag{A.10}$$

Substituting (A.9) and (A.10), (A.8) is equivalent to

$$\frac{\omega_R \overline{L}_{sN} + \overline{L}_{uN}}{\omega_R \overline{L}_{sR} + \overline{L}_{uR}} < \frac{\overline{L}_{sN} - k^y \overline{L}_{uN}}{\overline{L}_{sR} - k^y \overline{L}_{uR}} \Leftrightarrow \frac{\overline{L}_{sN}}{\overline{L}_{uN}} - \frac{\overline{L}_{sR}}{\overline{L}_{uR}} > 0, \text{ where } k^y \equiv \frac{c_s^y}{c_u^y}.$$

This is true since the North is relatively more endowed with skilled labor. It then proves that the North is the net exporter of good M.

A.3 Integrated Equilibrium and Pattern of Specialization

In the integrated equilibrium both goods are produced and consumed in the world economy. Define

$$\overline{c}_u(\omega(p_R)) \equiv c_u^q(\omega(p_R))q(\omega(p_R)) + c_u^x(\omega(p_R))f(\overline{x}); \tag{A.11}$$

$$\overline{c}_s(\omega(p_R)) \equiv c_s^q(\omega(p_R))q(\omega(p_R)) + c_s^x(\omega(p_R))f(\overline{x}). \tag{A.12}$$

Then the equations

$$\overline{c}_u(\omega(p_R))n_R + c_u^y(\omega) Y_R = \overline{L}_{uR} \tag{A.13}$$

$$\overline{c}_s(\omega(p_R))n_R + c_s^y(\omega) Y_R = \overline{L}_{sR} \tag{A.14}$$

$$\frac{\gamma\left(w_u \overline{L}_{uR} + w_s \overline{L}_{sR}\right)}{n_R p_R} = x_R, \tag{A.15}$$

determine the world price ratio, p_R, and the world outputs, n_R and Y_R. Letting $\overline{p}_R$, $\overline{n}_R$ and $\overline{Y}_R$ denote the solutions, $\omega(\overline{p}_R) \equiv \overline{\omega}_R$ is the equilibrium relative wage.

Figure 3 around here

In Figure 3, if 0_N is the origin, then 0_S represents the world endowment point. The rays, k_m and k_y, measure the overall skilled to unskilled labor empolyment ratio in sector M and Y respectively.[13] The points B_m and B_y respectively mark the solutions of $\overline{n}_R$ and $\overline{Y}_R$. Alternatively, if 0_S is taken as the origin, 0_N is the world endowment point, and B_m and B_y respectively denote the solutions $\overline{Y}_R$ and $\overline{n}_R$.

Now, if the integrated economy is separated into two countries, North and South, by standard arguments, FPE holds as long as the point representing the endowments of North and South lies within the parallelogram $0_N B_m 0_S B_y$; otherwise, complete specialization occurs in equilibrium in at least one country.

B Oligopoly Model

B.1 Proof of FPE and PQE

The zero-profit condition $c^y(w_s, w_u) = 1$ implicitly gives w_u as a monotonically decreasing function of the relative wage. Substituting this function into $c^x(w_s, w_u)$ and $c^q(w_s, w_u)$ yields c^x and c^q as functions of ω only and as ω increases, c^x and c^q increase, since producing quantity and quality in sector M is relatively more skill intensive than the technology in sector Y. Denote these functions respectively as $\overline{c}^x(\omega)$ and $\overline{c}^q(\omega)$. Define $b(\omega) \equiv \overline{c}^q(\omega)/\overline{c}^x(\omega)$. We have $b'(\omega) > 0$, as quality production is more skill intensive than quantity production.

We now substitute $\overline{c}^x(\omega)$ and $\overline{c}^q(\omega)$ for $c^x(w_{sr}, w_{ur})$ and $c^q(w_{sr}, w_{ur})$ respectively in eqs. (27) - (29).

¿From (27) and (28), eliminate $q_r^\tau p_r^{1-\vartheta}/Z_R$ and obtain

(A.16) $$b(\omega_r)q_r g'(q_r) = \beta x_r, \text{ where } \beta \equiv \tau/(\vartheta - 1).$$

Substitute (A.16) into the zero-profit condition (29) and obtain

(A.17) $$1 + \beta\Psi + \frac{\alpha}{x_r} = \frac{p_r}{\overline{c}^x(\omega_r)}$$

Write (32) as

(A.18) $$q_r^\tau p_r^{-\vartheta} = \frac{x_r}{\upsilon}, \text{ where } \upsilon \equiv \gamma I_R/Z_R.$$

Next substitute (A.17) and (A.18) into (27) and eliminate $q_r^\tau p_r^{-\vartheta}$ and $p_r/\overline{c}^x(\omega_r)$. We obtain

(A.19) $$\frac{\upsilon Z_R}{(\vartheta - 1)\, x_r}\left[\vartheta - \frac{\alpha + (1 + \beta\Psi)\, x_r}{\alpha + \beta\Psi x_r}\right] = p_r.$$

Dividing (A.19) by (A.17) gives

(A.20) $$\overline{c}^x(\omega_r) = \frac{\upsilon Z_R}{(\vartheta - 1)[\alpha + (1 + \beta\Psi)x_r]}\left[\vartheta - \frac{\alpha + (1 + \beta\Psi)x_r}{\alpha + \beta\Psi x_r}\right].$$

Verify that this equation spells a negative relation between ω_r and x_r.

Next, using $g(q) = q^{1+\eta}/(1+\eta)$, we write (A.16) as $q = [\beta x_r/b(\omega_r)]^{1/(1+\eta)}$ and substitute this into the market-clearing condition (A.18). This gives

$$p^\vartheta x^{1-\tau/(1+\eta)} = \upsilon\left[\frac{\beta}{b(\omega_r)}\right]^{\frac{\tau}{1+\eta}}.$$

Now substitute (A.19) into the above and obtain

(A.21) $$\frac{1}{x_r^{\vartheta + \frac{\tau}{1+\eta} - 1}}\left[\vartheta - \frac{\alpha + (1 + \beta\Psi)x_r}{\alpha + \beta\Psi x_r}\right]^\vartheta = \left(\frac{\vartheta - 1}{\upsilon Z_R}\right)^\vartheta \frac{\upsilon\beta^{\frac{\tau}{1+\eta}}}{[b(\omega_r)]^{\frac{\tau}{1+\eta}}}.$$

In this equation, check that ω_r and x_r are positivey related.

We can now prove FPE and PQE. Suppose that in the trading equilibrium $w_N > w_S$. Then (A.20) implies $x_N < x_S$, but (A.21) implies $x_N > x_S$, a contradiction. Similar contradiction arises if $w_N < w_S$. Hence it follows that $w_N = w_S$, implying $x_N = x_S$, $p_N = p_S$ and in particular $q_N = q_S$.

B.2 Slopes of FF_2 and MM_2 Schedules

For notational simplicity, let us drop the 'autarky' subscript a. Totally differentiating the $h(\cdot)$ function in (47),

$$\widehat{n} = \frac{(n-1)\left[n - \tau\Psi\left(n-1\right)\right]\left[\widehat{g} + (1-\mu)\widehat{w}\right]}{n}, \tag{A.22}$$

The FF_2 schedule graphs the full-employment equation (48). Totally differentiating this and utilizing (A.22), we obtain

$$\frac{\tau\Psi\mu\frac{n-1}{n} + \frac{n-1}{n}\left[\tau\Psi + \left(\vartheta - \tau\right)\mu\right]\left[n^2 - \tau\Psi\left(n-1\right)^2\right]}{\left[\tau\Psi + \left(\vartheta - \tau\right)\mu\right]\left(n-1\right) + \mu} \cdot \widehat{g} \tag{A.23}$$
$$= \; -\frac{\left(1-\mu\right)\left[n - \tau\Psi\left(n-1\right)\right]\left\{\left[\vartheta(n-1)^2 - 1\right]\mu + (n-1)^2\tau\Psi(1-\mu)\right\}}{n\{\mu + \left[\tau\Psi + \left(\vartheta - \tau\Psi\right)\mu\right]\left(n-1\right)\}}\widehat{w} + \widehat{\overline{L}}_s$$

The co-efficient of $\widehat{g}$ is positive, while that of $\widehat{w}$ is negative as long as the number of firms, n, is two or higher. This proves that the FF_2 schedule is negatively sloped.

Consider eq. (50), defining the MM_2 schedule. Since $\gamma < 1$, we have

$$\frac{w\overline{L_s} + \overline{L_u}}{w\overline{L_s}} > \frac{\vartheta\left(n-1\right) + 1}{\tau\Psi\left(n-1\right) + \mu\left[\left(\vartheta - \tau\Psi\right)\left(n-1\right) + 1\right]}$$

$$\Leftrightarrow \frac{\overline{L}_u}{w\overline{L}_s + \overline{L}_u} > \frac{\left(1-\mu\right)\left[\left(\vartheta - \tau\Psi\right)\left(n-1\right) + 1\right]}{\vartheta\left(n-1\right) + 1} \tag{A.24}$$

Totally differentiating (50) and using (A.22) once again, we obtain

$$\frac{\tau\Psi\left(1-\mu\right)\left(n-1\right)\left[n - \tau\Psi\left(n-1\right)\right]}{\left[\vartheta\left(n-1\right) + 1\right]\mu + \left(n-1\right)\left[\tau\Psi + \left(\vartheta - \tau\Psi\right)\mu\right]} \cdot \widehat{g}$$
$$= \left\{\frac{\overline{L_u}}{w\overline{L_s} + \overline{L_u}} - \frac{\tau\Psi\left(1-\mu\right)^2\left(n-1\right)\left[n - \tau\Psi\left(n-1\right)\right]}{\left[\vartheta\left(n-1\right) + 1\right]\left\{\mu + \left(n-1\right)\left[\tau\Psi + \left(\vartheta - \tau\Psi\right)\mu\right]\right\}}\right\}\widehat{w} - \frac{\overline{L}_u}{w\overline{L}_s + \overline{L}_u} \cdot \widehat{\frac{\overline{L}_u}{\overline{L}_s}}$$

The coeffient of $\widehat{g}$, and, in view of (A.24), the coefficient of $\widehat{w}$ are both positive, proving that the MM_2 schedule has a positive slope.

NOTES

[1] Copeland and Kotwal (1996) consider a modified Flam-Helpman model, focussing on trade break-downs. Murphy and Shleifer (1997) arrive at similar conclusions (in the context of trade between Eastern and Western Europe), while their model emphasizes demand differences across countries arising out of endowment differences – namely high (low) income countries would tend to produce *and* demand high (low) quality goods.

[2]This means that scale economies are arbitrarily large at a very small level of output, and they are nearly exhausted at an arbitrarily high level of output.

[3]Because $x = \overline{x}$, in view of (7), quality is uniquely related to the relative wage, indicating how factor price equalization implies product quality equalization. But, as we shall see, under oligopoly competition, the long-run output is not invariant, and yet, factor price equalization implies product quality equalization.

[4]The association between trade pattern and quality response is interesting. As trade opens, the North (South), having comparative advantage (disadvantage) in manufacturing, produces more (less) quantity of good M but less (more) quality of the same good!

[5]See, for instance, Das and Donnenfeld (1989) and Zhou et al. (2002).

[6]Since $x'(\omega) > 0$ and $q'(\omega) < 0$, we can define $\Gamma \equiv x(\omega) / ([q(\omega)]^{\tau})$, such that $\Gamma'(\omega) > 0$. The market clearing condition (13) implies that, in the free-trade equilibrium,

$$\frac{\Gamma(\omega_N)}{\Gamma(\omega_S)} = \left(\frac{p_S}{p_N}\right)^{\vartheta}.$$

Suppose $p_N < p_S$.Then the above relation implies $\omega_N > \omega_S$, which is contrary to the Stolper-Samuelson effect implying $\omega_N < \omega_S$. Hence $p_S \not> p_N$. Similarly it can be argued that $p_N \not> p_S$. Thus $p_S = p_N$ and this implies $\omega_N = \omega_S$, $x_N = x_S$ and $q_N = q_S$.

[7]Following the structure of proving the trade pattern in case of monopolistic competition, it is straightforward to prove that in this model too North is the net exporter of good M.

[8]The full-employment of unskilled labor essentially determines the output of the numeraire sector, which is independent of how the other variables in the system are determined.

[9]In the extreme case if sector M uses skilled labor only, the FF_1 curve is vertical, implying $q_{aN} > q_{aS}$ unambiguously.

[10]International trade is also equivalent to an increase in the absolute endowment of unskilled labor in each country. But this is immaterial for changes in the variables we are interested in.

[11]This is different from quality leapfrogging via basic research in discovering new products or catching up in terms of technology.

[12]This paper also cites useful empirical evidences on quality leapfrogging.

[13]Since both quantity and quality productions are more skill-intensive compared to the production of good Y, k_m is steeper than k_a.

REFERENCES

COPELAND, B. and KOTWAL, A. (1996). Product quality and the theory of comparative advantage. *European Economic Review*, **40**, 1745-60.

DAS, S. P. (2003). Trade and relative wages in a global economy. *Review of International Economics*, 11, 397-411.

—— and DONNENFELD, S. (1989). Oligopolistic competition and international trade: quantity versus restrictions. *Journal of International Economics*, 27, 299-318.

DEMING, W. E. (1986). *Out of the Crisis*. MIT Press, Cambridge, MA.

DIXIT, A. and NORMAN, V. (1980). *International Trade*. Cambridge: Cambridge University Press.

FLAM, H. and HELPMAN, E. (1987). Vertical product differentiation and north-south trade. *American Economic Review*, **77**, 810-22.

FALVEY, R. E. (1979). The comparison of trade within import-restricted product categories. *Journal of Political Economy*, **LXXXVII**, 1105-14.

—— and KIERZKOWSKI, H. (1987). Product quality, intra-industry trade and (im)perfect competition. In H. Kierzkowski (eds.), *Protection and Competition in International Trade: Essays in Honour of Max Corden*, Basil Blackwell.

GROSSMAN, G. M. and HELPMAN, E. (1993). *Innovation and Growth in a Global Economy*, The MIT Press.

HERGUERA, H. and LUTZ, S. (1998). Oligopoly and quality leapfrogging. *The World Economy*, 21, 75-94.

KRUGMAN, P. (1979). Increasing returns, monopolistic competition, and International trade. *Journal of International Economics*, **9**, 469-79.

MOTTA, M., THISSE, J.-F. and CABRALES, A. (1997). On the persistence of leadership or leapfrogging in international trade. *International Economic Review*, **38**, 809-24.

MURPHY, K. M. and SHLEIFER, A. (1997). Quality and trade. *Journal of Development Economics*, **53**, 1-15.

RODRIGUEZ, C. A. (1979). The quality of imports and the differential welfare effects of tariffs, quotas, and quality controls as protective devices. *Canadian Journal of Economics*, **12**, 439-49.

SHAKED, A. and SUTTON, J. (1982). Relaxing price competition through product differentiation. *Review of Economic Studies*, **49**, 3-13.

ZHOU, D., SPENCER, B. J. and VERTINSKY, I. (2002). Strategic trade policy with endogenous choice of quality and asymmetric costs. *Journal of International Economics*, **56**, 205-232.