

**Home-biased Asset Holdings  
For Intertemporal Consumption Risk Sharing  
In the presence of Aggregate Uncertainty  
And Home-biased Preferences**

Geun Mee Ahn<sup>\*</sup> · Chung-Han Kim<sup>†</sup>

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**Abstract**

In an open economy model of intertemporal indirect utility maximization with sticky prices and home-biased preferences, we demonstrated that when financial markets are not complete due to the aggregate uncertainty, agents with relative risk aversion greater than unity would have a precautionary motive of asset holdings for their intertemporal consumption risk sharing. Given the stylized facts that consumption pattern of most countries is home-biased and that the fluctuations of exchange rates are far greater than those of goods prices in the short run, they have an incentive to hold assets denominated in each currency proportionally to the expenditure share of goods produced in each corresponding country in their consumption basket for short run consumption stabilization. For the OECD countries of the US, the UK, Japan, Canada, and Germany, we constructed two series of cross-sectional data such as the shares of a foreign asset denominated in each currency in total foreign assets and of imports from each corresponding foreign country in total imports, and examined their correlation. We found a strong and positive correlation between them.

*JEL Classification:* E3, F3, F4, G1

*Keywords:* Aggregate Uncertainty, Home-biased Preferences, Precautionary Motive of Asset Holdings

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<sup>\*</sup> BK21 Research Group, Department of Economics, Korea University, 1 Anam-dong, Sungbuk-gu, Seoul, (136-701), Korea. Phone Number 82-2-3290-2714, Fax Number: 82-2-3290-2716, E-mail: [geunahn@korea.ac.kr](mailto:geunahn@korea.ac.kr), Research funding from BK21 research project is gratefully acknowledged.

<sup>†</sup> Department of International Trade, Kwangwoon University, 447-1 Wolgye-Dong, Nowon-Gu, Seoul, (139-701), Korea. Phone Number 82-2-940-5622, Fax Number: 82-2-942-5329, E-mail: [chk592@daisy.kw.ac.kr](mailto:chk592@daisy.kw.ac.kr),

## I. Introduction

French and Poterba [1991], Tesar and Werner [1995], Lewis [1995, 1999], Karolyi and Stulz [2002], and many others have documented that foreign assets comprise a small share of investors' portfolios across countries.<sup>1</sup> They pointed out that it contradicts the theoretical prediction of finance such that for any given value of standard deviation, an investor would like to choose a portfolio that gives him the highest possible expected rate of return and therefore he always wants a portfolio that lies up along the efficient frontier and the nationality of assets would not matter. For a decade or so via the rapid integration of global financial markets due to the reductions in transaction costs and the degree of capital controls, this stylized pattern has been substantially weakening. It, however, still appears that investors are forgoing the opportunities of pursuing the highest (risk-adjusted) expected rate of return anywhere around the world. This empirical regularity has been dubbed by Obstfeld and Rogoff [2000] as one of the six major puzzles in international macroeconomics.<sup>2</sup>

In finance, asset risk is defined as the variance or the standard deviation of the probability distribution of an asset's return that is assumed to be perfectly known together with the mean return so that portfolio diversification can hedge assets' idiosyncratic risks completely. Aside from this asset risk, initially Knight [1921] and later Samuelson [1969], Merton [1969], and Lucas [1978] introduced the concept of

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<sup>1</sup> French and Poterba [1991] found that U.S. equity traders allocate nearly 94 percent of their funds to domestic securities, even though the U.S. equity market comprises less than 48 percent of the global equity market.

<sup>2</sup> There have been numerous attempts to explain this empirical regularity. One strand of studies has argued that the gains from international diversification are in fact small so that small transaction costs of diversification will lead to heavily concentrated portfolios (Krugman [1981], and Obstfeld and Rogoff [2000]). Others have claimed that the acquisition of information about foreign firms is more costly than for information on home firms (Gehrig [1993], Cooper and Kaplanis [1994], and Brennan and Cao [1997]). Another study points to the hedging demand for assets with stronger negative or smaller positive correlation with domestic state variables such as human capital (Baxter and Jermann [1997], and Bottazzi, Pesenti, and van Wincoop [1996]). Some argue that people simply prefer to deal with familiar situations (Coval and Moskowitz [1999, 2001], and Huberman [2000]).

aggregate uncertainty where the probability distribution of future macroeconomic events in an economy is unknown, and more importantly, where they are impossible to calculate due to the uniqueness or specificity of the situation. Under aggregate uncertainty, unexpected future macroeconomic output and monetary shocks that raise households' future cost of living reduce households' future consumption unexpectedly. When financial markets are incomplete due to aggregate uncertainty,<sup>3</sup> this unexpected change in future consumption is perceived by the households with the degree of relative risk aversion greater than unity in CRRA utility function as un-insurable intertemporal consumption risk. Therefore, to stabilize their future consumption, they would want to hold assets whose returns materialize in the future as 'Precautionary Savings' for future consumption stabilization rather than for (risk adjusted) expected profit maximization, on one hand. On the other hand, they would like to modify their portfolio composition by holding assets whose returns move in the opposite direction of the future economic fluctuation like risk-less assets rather than risky ones whose returns move in the same direction for future consumption stabilization.

When households' preferences or their consumption indices are heterogeneous, another consumption risk, namely, intratemporal consumption risk arises.<sup>4</sup> Unexpected macroeconomic output and monetary shocks alter the cost of living across households, making their consumption level different unexpectedly. Since financial markets are incomplete due to aggregate uncertainty,<sup>5</sup> this unexpected change in future consumption is also perceived by the households with the degree of relative risk

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<sup>3</sup> Financial market incompleteness may be manifested as 'Borrowing Constraints.'

<sup>4</sup> Intratemporal consumption risk is defined as the fluctuations of relative consumption across households while intertemporal consumption risk is as those of intertemporal consumption, for risk-averse households with the degree of relative risk aversion greater than unity.

<sup>5</sup> Due to the uncertainty, the Pareto-efficient Arrow-Debreu state-contingent contracts cannot be arranged in advance at some initial date, the number of risky financial assets is limited to span the space of national output shocks, and the real bonds denominated in consumption are not tradable.

aversion greater than unity in CRRA utility function as un-insurable intratemporal consumption risk. In the long run, households would want to hedge intratemporal consumption risk by holding assets with hedging property of each production risk proportionally to the shares of each goods in their consumption baskets for 'Precautionary Motive.' Even in the short run when the prices of goods are sluggish, unexpected country-specific macroeconomic output and monetary shocks arising in an international economy where consumption indices across countries are heterogeneous alter the exchange rate and hence the cost of living across countries, producing intratemporal consumption risk. Households across countries can hedge this risk by holding assets denominated in each currency in line with the shares of goods produced in each country in their consumption baskets. Further, they have an incentive to hold assets whose returns move in the opposite direction of the future economic condition, if it could be possible to predict it.

In the new open economy macro model of intertemporal indirect utility optimization with aggregate uncertainty, incomplete financial markets, short run price stickiness, monopolistic competition, and home-biased preferences, we demonstrate that risk-averse agents with the degree of relative risk aversion greater than unity have three motives of asset holdings by using a mean-variance approach and solving for the exact solution of the foreign asset share in a closed form. The first is the 'Precautionary Motive' of asset holdings to hedge *unexpected exchange rate uncertainty* in the short run when exchange rates are far more unpredictable than price levels by matching asset shares in different currency denomination with the shares of goods produced in each corresponding countries in the consumption basket. Under the fact that consumption baskets of most countries are home-biased, the compositions of their portfolios would

be also home-biased. The second is the ‘Consumption Risk Hedging Motive’ of asset holdings. When agents with risk aversion greater than unity expect ‘weak currency/a favorable macroeconomic condition’ or ‘strong currency/a depression’ in the future, they would want to hold more of home-currency denominated assets in their portfolio for consumption risk hedging because favorable home output shocks depreciate the exchange rate while adverse ones appreciate it. The third is the ‘Speculative Motive’ to pursue higher (risk adjusted) expected return in terms of the same currency throughout the world, namely, myopic demand for assets.

Our contributions in our paper are as follows: first, we identify three motives of asset holdings numerically by taking a mean-variance approach to indirect utility optimization and computing the exact solution of the share of foreign assets in portfolio in a closed form. We attribute the asset home bias especially to the ‘Precautionary motive’ of risk-averse households with the relative risk aversion greater than unity in CRRA utility for their short run consumption stabilization. Second, to confirm our theoretical prediction on the households’ asset holding pattern, we examine the correlation between the share of each foreign asset in the foreign assets and the share of imports from each corresponding foreign country in the consumption basket in the data of the US, Japan, Canada, Germany and the UK. Our empirical result provides the confirmation that our conjecture is correct. Third, we build a theoretical ground on which an wealth of future researches that can examine how home-biased preferences has an influence on consumption asset pricing and, in turn, the spot exchange rate determination via the international asset market equilibrium.

The organization of our paper is as follows. Literature is reviewed in Section II. Section III derives the optimal foreign asset share of the representative household’s

portfolio by maximizing his indirect lifetime utility in the 2 country open economy macroeconomic model with aforementioned several assumptions. In Section IV, the variance of the spot exchange rate and its covariance with consumption are computed. The empirical test on the correlation between home-biased asset holding pattern and home-biased consumption pattern is performed in Section V. Section VI concludes.

## **II. Literature Review**

In Kouri [1977], an investor's optimal portfolio can be decomposed into a minimum variance portfolio (or equivalently inflation hedge portfolio in Adler and Dumas [1983]) and a speculative portfolio. This decomposition is known as 'two-fund separation' in the theory of finance. The first component is the portfolio of an investor who wants to hold a minimum variance portfolio in terms of real consumption, and the second component is the portfolio of a logarithmic investor who concerns only the means and the variances of the expected returns of assets. Adler and Dumas [1983] build an indirect utility function that depends on both nominal consumption and a random price index (or inflation rate) for the direct utility function that depends on real consumption. They computed the inflation hedge portfolio of the US and France. They showed that an investor's best inflation hedge portfolio (of stocks and bonds denominated in 9 countries' currencies<sup>6</sup>) is almost entirely made up of home currency bank deposits or Treasury Bills. They explained the reason why national investors see home currency deposits or Treasury bills as riskless assets as follows: in the short run, exchange rate or stock price fluctuations are much wider than price level fluctuations and therefore investors with relative risk aversion greater than unity prefer to bear

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<sup>6</sup> These 9 countries are Germany, Belgium, Canada, France, Japan, Netherlands, the United Kingdom, Switzerland, and the United States.

inflation uncertainty rather than to bear exchange rate uncertainty or stock price uncertainty. For the ‘Precautionary Motive’ for unexpected exchange rate uncertainty, this type of agents would like to hold assets denominated in each currency proportionally to the share of goods produced in each country in their consumption basket. Since consumption pattern of most of countries is home biased,<sup>7</sup> their portfolio holding pattern also resembles it. This approach to optimal portfolio choice is called ‘Preferred Local Habitat’ approach.<sup>8</sup>

Samuelson [1969] and Merton [1969, 1971, 1973] have shown that uncertainty implies optimal portfolio strategies for multi-period investors with high risk aversion can be different from those of single-period, or myopic logarithmic investors.<sup>9</sup> Multi-period investors value assets not only for their short-term risk-return characteristics, but also for their ability to hedge consumption against adverse shifts in future investment opportunities. Thus these investors have an extra demand for risky assets that reflects intertemporal consumption hedging. Investors with low relative risk aversion ( $0 < \rho < 1$ ) will choose to consume less now and save more to take advantage of the good investment opportunities where higher yields are available (that is, the substitution effect dominates the income effect). For high risk averters ( $\rho > 1$ ), the reverse is true and the income effect dominates the substitution effect. In the borderline case of Bernoulli logarithmic utility ( $\rho = 1$ ), the income and substitution effect just offset one another. Intertemporal consumption hedging demand for assets is always zero when

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<sup>7</sup> In our paper, we do not try to provide the explanation behind the stylized fact of this ‘Home Bias in Consumption.’

<sup>8</sup> See Solnik [1974], Kouri and de Macedo [1978], Krugman [1981], Dornbusch [1982], Stulz [1983], Adler and Dumas [1983], Branson and Henderson [1985], Dumas [1993] and Uppal [1993] for this approach. Coval and Moskowitz [1999, 2001] observed that the preference for investing close to home also applies to portfolios of domestic stocks, and that the US investment managers exhibit a strong preference for locally headquartered firms, particularly small, highly levered firms that produce non-traded goods for the stable purchasing power of their assets.

<sup>9</sup> For this borderline case of logarithmic utility, see Phelps [1962].

investors have unit coefficients of relative risk aversion ( $\rho = 1$ ). Lucas [1978] introduced consumption CAPM that the risk-averse agents would like to smooth consumption over time by holding more of the assets whose expected returns co-vary with the marginal utility of future consumption and less of the assets whose expected returns have the opposite correlation. This consumption CAPM uses the covariance of the marginal utility of consumption with the asset return to measure the effect of the risk on the returns of assets like the market CAPM uses the covariance of stock returns with the market index return.<sup>10</sup> If the assets whose expected returns are negatively correlated with the marginal utility of future consumption, the asset is not deemed to be as valuable in terms of intertemporal consumption risk sharing. The asset will need a high ex ante return, that is, a low current price in order to entice investors to buy it. If the covariance of an asset's expected return to future consumption is low, however, the asset seems valuable because it delivers a return when the marginal utility of consumption is high so that this asset is demanded more, their price goes up, and their ex ante return gets lower.

### **III. The Optimal Composition of Portfolio**

The composition of portfolio is affected not only by precautionary motive but also by consumption risk hedging and speculative motives as well. To see the factors that affect the portfolio choice, we derive the representative agent's expected indirect utility function that depends on both nominal wealth and a random price index from the direct utility function that depends on consumption. Then, we solve for the optimal

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<sup>10</sup> The static market CAPM states that assets whose returns co-move with the market index return assume higher returns while the assets whose returns' covariance with the market return is negative hold lower returns.



composition by maximizing the representative agent's expected indirect utility function, following Krugman [1981] and Adler and Dumas [1983]. The following is the expected lifetime indirect utility function of the home representative agent derived from the Cobb-Douglas/CRRA utility function.

$$EU_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{W_s^{1-\rho}}{1-\rho} \left[ (P_{H,s})^\gamma (S_t P_{F,s}^*)^{1-\gamma} \right]^{-(1-\rho)} \right\} \quad [1]$$

where  $\beta$  = the rate of time preference

$\rho$  = the degree of relative risk aversion of agents' CRRA utility function

$P \equiv (P_H)^\gamma (S P_F)^{1-\gamma}$  = the aggregate price index

$\gamma$  = the share of home-produced goods in the consumption index

$P_H$  = the price of the home representative good in terms of home currency

$S$  = the spot exchange rate (the price of one-unit of foreign currency in terms of home currency)

$P_F^*$  = the price of the foreign representative good in terms of foreign currency

The representative agent's expected life-time indirect utility is the sum of the present values of future period indirect utilities discounted by the rate of time preference.

Financial markets are assumed to be incomplete, because there are neither Arrow-Debreu type contingent claims, nor real bonds, nor risky assets whose number is enough to span the idiosyncratic output shocks arising in the world economy. Agents can purchase only home and foreign currency denominated nominal bonds that guarantee nominal interest rates already predetermined in the current period in terms of home and foreign currencies. The following is the representative agent's current period budget constraint. His period nominal income is the nominal value of his output plus the gross income from previous investment in home and foreign currency denominated

nominal bonds whose yields are already predetermined in the previous period minus current period investment in home and foreign currency denominated nominal bonds.

$$W_t = P_{H,t-1}(v)Y_{H,t}(v) + (1 + i_{t-1})B_{H,t-1} + (1 + i_{t-1}^*)S_t B_{F,t-1} - B_{H,t} - S_t B_{F,t} \quad [2]$$

where  $P_H(v)Y_H(v)$  = nominal value of each home agent's output

$B_H$  = demand for home bonds by each home agent

$B_F$  = demand for foreign bonds by each home agent

$i$  = nominal return on home bonds

$i^*$  = nominal return on foreign bonds

The representative agent would like to make his life-time consumption and savings decision that maximizes his expected life-time indirect utility subject to his expected life-time budget constraint. The following is the agent's expected life-time utility expressed as a function of real income, which is derived by discounting his nominal income by his cost of living.

$$EU_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \frac{\tilde{W}_s^{1-\rho}}{1-\rho} \right\} \quad [3]$$

where  $\tilde{W}_t = W_t / P_t$  is the level of real income.

When shocks are assumed to occur only in the next period and not to have persistence over time, then, the representative agent's life-time utility optimization problem at period  $t$  can be reduced to maximizing his one-period ahead expected utility subject to his one-period ahead expected budget constraint.

$$EU_t = \frac{E_t(\tilde{W}_{t+1}^{1-\rho})}{1-\rho} \quad [4]$$

We show below that if the exogenous random variables moving the economy have a jointly lognormal distribution, all endogenous variables are lognormal as well. With lognormally distributed variables, the equation [4] has the equivalent representation as follows. Lower cases denote the logarithm of upper cases.

$$EU_t = \exp\left\{(1-\rho)E\tilde{w}_{t+1} + \frac{(1-\rho)^2}{2}\sigma_{\tilde{w}_{t+1}}^2\right\} \quad [5]$$

Taking a logarithm of [5] gives

$$Eu_t = \log EU_t = (1-\rho)\left\{E\tilde{w}_{t+1} + \frac{(1-\rho)}{2}\sigma_{\tilde{w}_{t+1}}^2\right\} \quad [6]$$

From the equation [2], the representative agent's savings are  $[B_{H,t} + S_t B_{F,t}]$ .

Define the expenditure share of foreign currency denominated nominal bonds in the home agent's savings at  $t$  as  $\lambda_t$ .

$$\lambda_t = \frac{B_{F,t}}{B_{H,t} + S_t B_{F,t}} \quad [7]$$

From the equation [2], the  $t+1$  period nominal income can be expressed as follows.

$$E_t W_{t+1} = P_{H,t}(v)E_t Y_{H,t+1}(v) + (1+i_t)B_{H,t} + (1+i_t^*)E_t S_{t+1} B_{F,t} - B_{H,t+1} - E_t S_{t+1} B_{F,t+1} \quad [8]$$

The  $t+1$  period prices of goods are set at  $t$  by each home agent as a monopolistically competitive producer.<sup>11</sup> Assuming that the agent's savings  $[B_{H,t} + S_t B_{F,t}]$  are constant

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<sup>11</sup> For the detailed price determination procedure, see Appendix 2.

at the steady state, the log-linearized  $t + 1$  period expected real income of the agent at  $t$  would be the following.

$$E_t \tilde{w}_{t+1} = [E_t y_{H,t+1}(v) - (1 - \gamma)E_t s_{t+1}] + \{i_t + \lambda_t [i_t^* - i_t] + [\lambda_t - (1 - \gamma)]E_t s_{t+1}\} \quad [9]$$

Taylor-expanding the equation [A.17] in appendix 2 at  $C = C^*$  gives

$$y_{H,t}(v) = p_t + c_t - p_{H,t} \quad [10]$$

where  $p_t = p_{H,t} + (1 - \gamma)s_t + (1 - \gamma)p_{F,t}$

Since  $P_{H,t}$  and  $P_{F,t}$  is determined one-period ahead,  $p_{H,t}$  and  $p_{F,t}$  are zero. Therefore, the equation [10] can be transformed as follows.

$$y_{H,t}(v) = (1 - \gamma)s_t + c_t \quad [11]$$

Substituting the equation [11] into [9] gives the following expected real income change.

$$E_t \tilde{w}_{t+1} = E_t c_{t+1} + \{i_t + \lambda_t [i_t^* - i_t] + [\lambda_t - (1 - \gamma)]E_t s_{t+1}\} \quad [12]$$

From the equation [12], the variance of  $t + 1$  period's real income can be derived as follows.

$$\sigma_{\tilde{w},t+1}^2 = \sigma_{c,t+1}^2 + [\lambda_t - (1 - \gamma)]\sigma_{s,t+1} + [\lambda_t - (1 - \gamma)]^2 \sigma_{s,t+1}^2 \quad [13]$$

Each period, the agent chooses optimal portfolio by mean variance optimization of next period's expected real wealth change. The first order condition for the optimal expenditure share of foreign bonds at  $t$ ,  $\lambda_t$ , is obtained by maximizing the equation [6] given the equations [12] and [13]. The first order condition is as follows.

$$-[i_t - i_t^* - E_t s_{t+1}] = \frac{1}{2}(\rho - 1) \{ \sigma_{cs,t+1} + 2[\lambda_t - (1 - \gamma)] \sigma_{s,t+1}^2 \} \quad [14]$$

The equation [14] tells that if  $\rho = 1$ , there wouldn't exist consumption risk due to the exchange rate fluctuations in the short run so that agents' portfolio decision can be determined solely by the expected profit maximization consideration.

From the first order condition [14], the expenditure share of foreign assets,  $\lambda_t$  in the home agent's portfolio can be solved for as follows.

$$\lambda_t = (1 - \gamma) - \frac{\sigma_{cs,t+1}}{2\sigma_{s,t+1}^2} - \frac{[i_t - i_t^* - E_t s_{t+1}]}{(\rho - 1)\sigma_{s,t+1}^2} \quad [15]$$

The expenditure share of foreign assets,  $\lambda_t$  in the home agent's portfolio is influenced by three motives: precautionary, consumption risk hedging and speculative motives. The first two motives urge investors to hold minimum variance portfolio for consumption risk sharing consideration, whereas the last motive motivates them to follow the highest (risk adjusted) expected return. If  $\rho > 1$ , the home agent wants to stabilize his consumption over time. For the precautionary motive, to share the intratemporal consumption risk due to the short run exchange rate uncertainty, he would like to hold foreign currency denominated assets in his savings proportionally to the share of corresponding foreign country produced goods in his consumption basket. If  $\rho > 1$ , the agent's expectation of either boom/weak currency or recession/strong currency makes his foreign asset holding decrease and his home asset holding increase because home currency denominated assets are more valuable than foreign currency denominated assets. It is because if the elasticity of money demand is less than unity, favorable home productivity shocks reducing home goods' prices depreciate the

exchange rate, reduce the interest rate, and raise consumption while adverse home productivity shocks appreciate the exchange rate, raise the interest rate, and reduce consumption, making home assets perfect consumption risk hedging assets. Even if  $\rho > 1$ , higher relative home interest rate to the foreign rate reduces the foreign asset holdings and increases the home asset holding by the agents for the speculative motive. As  $\rho$  goes down to 1, the speculative motive of the agent overwhelms the other two motives for asset holdings such as the precautionary motive for unexpected consumption risk and the consumption risk hedging motive for the expected consumption risk by the exchange rate fluctuations in the short run.

#### IV. The Variance of the Spot Exchange Rate and its Covariance with Consumption

From Appendix 2, the variance of the spot exchange rate and its covariance with consumption can be solved for as functions of variances of idiosyncratic money and output shocks.

$$\sigma_{s,t+1} = (2\gamma - 1) \left\{ \left( \frac{1}{2\rho} \right) \left( \frac{\varepsilon}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right) \left[ \sigma_{\mu,t+1}^2 + \sigma_{\mu^*,t+1}^2 \right] + \left( \frac{(\varepsilon - 1)}{\varepsilon} \right) \left[ \frac{\gamma}{\rho} \sigma_{\xi,t+1}^2 + \frac{1 - \gamma}{\rho} \sigma_{\xi^*,t+1}^2 \right] \right\} [16]$$

$$\sigma_{s,t+1}^2 = \left( \frac{\varepsilon}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right)^2 \left[ \sigma_{\mu,t+1}^2 + \sigma_{\mu^*,t+1}^2 \right] + \left( \frac{(\varepsilon - 1)(2\gamma - 1)}{\varepsilon} \right)^2 \left[ \sigma_{\xi,t+1}^2 + \sigma_{\xi^*,t+1}^2 \right] [17]$$

From [16], we notice that if agents across countries have identical preferences, idiosyncratic money and output shocks do not have an influence on the covariance between consumption and the spot exchange rate because they do not affect the spot exchange rate. From [17], however, we see that money shocks have an impact on the variance of the exchange rate while output shocks do not.

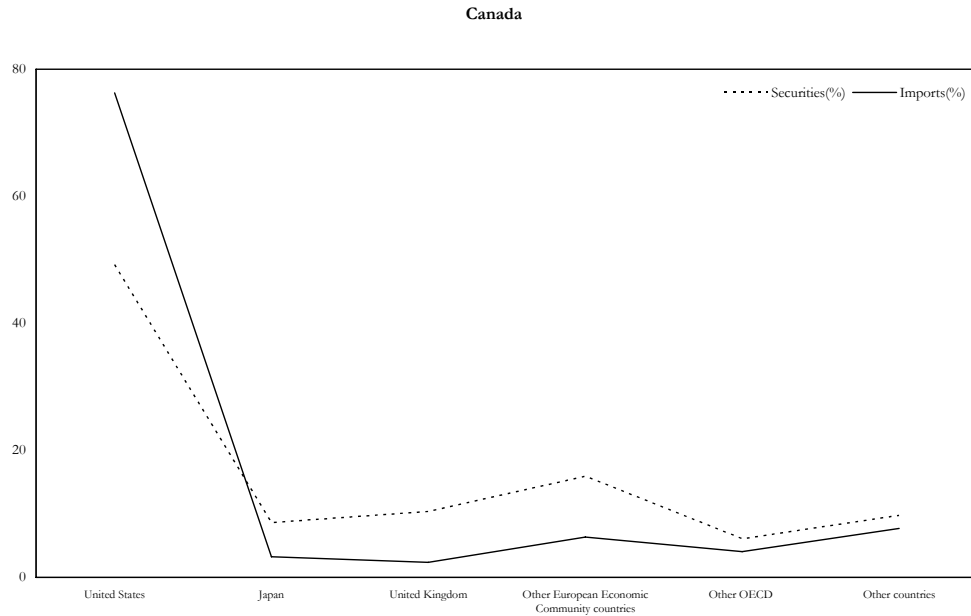
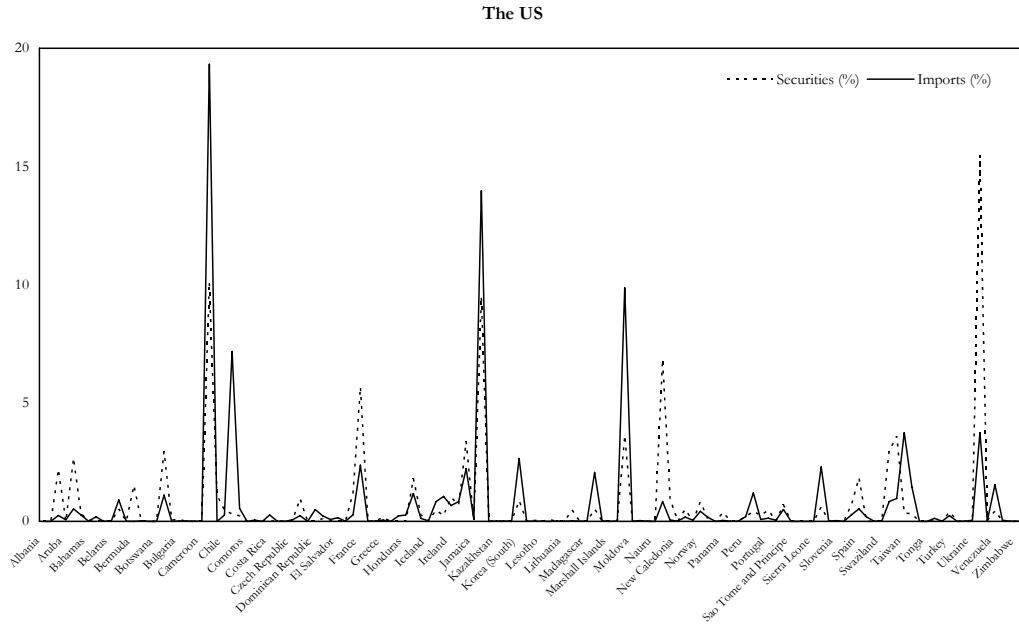
## **V. Data and Estimation**

In this section, we examine the correlation between the portfolio composition and the consumption composition in empirical data to confirm if our theoretical prediction such that the precautionary motive of asset holdings contributes to the home-biased asset holdings given aggregate uncertainty and the home-biased consumption pattern is accurate. For the countries of the US, Japan, Canada, Germany and the UK, two series are constructed: The share of each foreign asset in ‘total foreign assets’ and the share of imports from each corresponding country in ‘total imports.’ They are used instead of the share in total assets and the share in total consumption because the share of total foreign assets in total assets for each country is too small, for example, it is 7% for the US so that foreign asset holding patterns do not stand out when domestic data are included.

In our empirical data, we designate securities that include both bonds and stocks as assets. The US Treasury Department had conducted a comprehensive benchmark survey on “U.S. Holdings of Foreign Long-Term Securities” on December 31, 1997 with the Board of Governors of the Federal Reserve System for more than 130 counterparts. Since the same data for the UK, Japan, Canada and Germany were not available, instead, “changes in foreign portfolio investments [stock data]” from the “International Investment Position” in “National Accounts” gathered by each country according to the IMF’s Special Data Dissemination Standard were used. Canada data on International investment position has only 6 regional asset trading counterparts, while Japan, the UK and Germany have respectively 38, 54 and 32 foreign counterparts for their foreign security investment. Data sources are presented in Appendix 1.

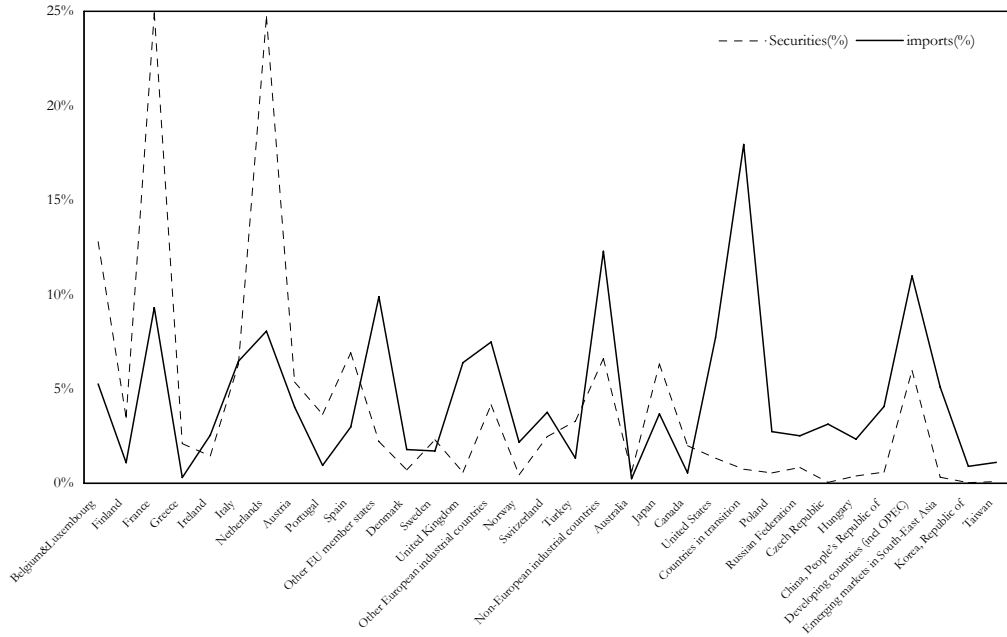
For the US, Japan, Canada, Germany and the UK, two series are plotted together for comparison.

**Appendix 10: Correlation between the Share of Each Foreign Asset in Total Foreign Assets and the Share of Imports from Each Corresponding Foreign Country in Total Imports**

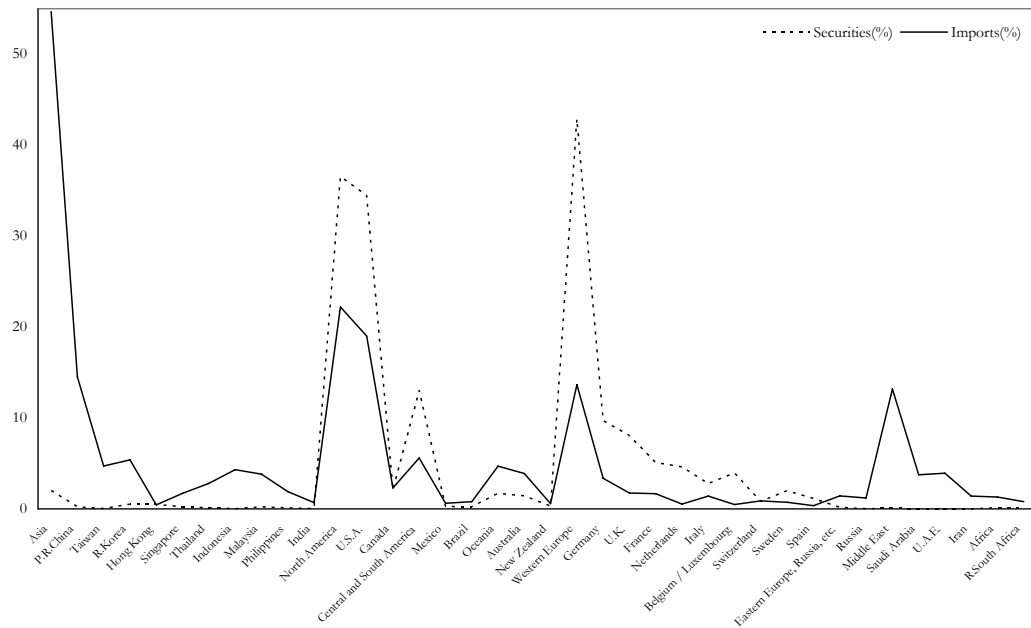


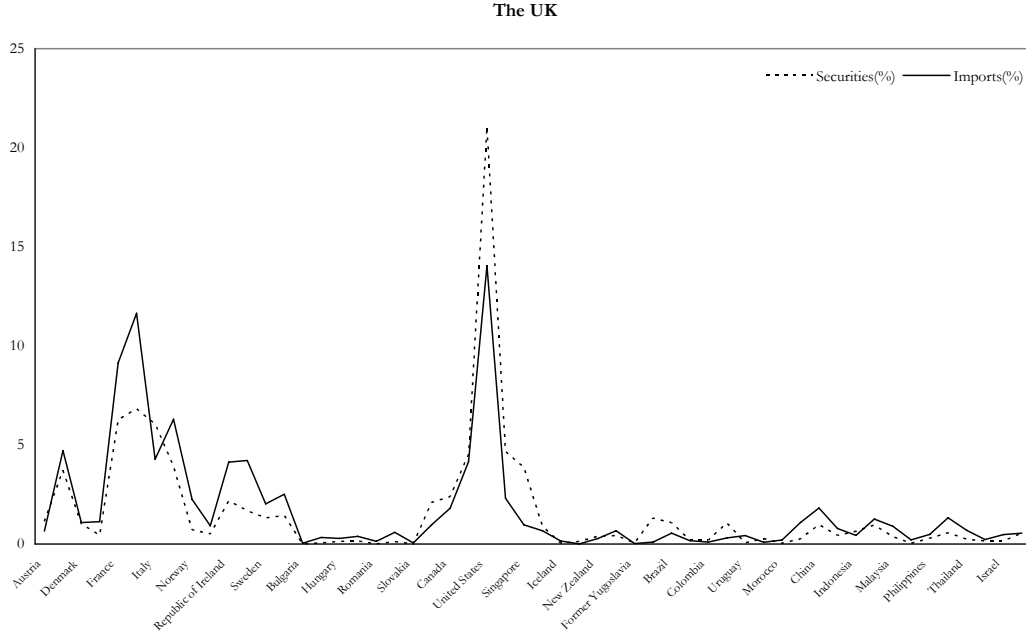


### Germany



### Japan





From the plots, we can notice that two series of the share of each foreign asset in total foreign assets and the share of imports from each corresponding country in total imports seem to be highly correlated.

We compute their correlation coefficients for each country of the US, Canada, Germany, Japan and the UK.

**Table 1: Correlation Coefficients of Foreign Assets and Imports' Shares**

	<i>Correlation Coefficient</i>	<i>Observations</i>
US	0.6632	130
Canada	0.9812	6
Germany	0.3301	33
Japan	0.3675	38
UK	0.8757	54

Canada has the highest correlation between two series, 0.9812, while Germany has the least correlation, 0.3301. For each country, the numbers of observations are not consistent so that it is difficult to compare correlation coefficients directly.

For each country of the US, the UK, Japan, Canada, and Germany, we perform the simple cross-section regression of the share of each foreign asset in total foreign assets on a constant and the share of imports from each corresponding foreign country in total imports. The table below shows the result from the regression.

**Table 2: Regression Result:**

**The Share of Each Foreign Asset in Total Foreign Assets (Dependent Variable)**

Coefficient	The US	Canada	Germany	Japan	The UK
Constant	0.311460** [0.139787]	7.55553** [1.604657]	0.01821 [0.015333]	2.499366 [1.818593]	-0.11172 [0.250634]
Share of Imports	0.568531*** [0.056699]	0.54666** [0.050949]	0.491903* [0.252567]	0.389865** [0.164406]	0.9903*** [0.075708]
R Squared	0.44	0.97	0.11	0.14	0.77
Observations	130	6	33	38	54
Data year	1997	1999	2002	2000	2000

Notes: Standard errors are reported in brackets. \*, \*\*, and \*\*\* indicate statistical significance at the 10, 5 and 1 percent levels respectively.

The dependent variable of the share of each foreign asset holding in total foreign assets appears to be fairly well accounted for by the share of imports from each corresponding foreign country since the coefficients are strictly positive and highly significant for all countries. R squared is distributed between 0.11 [Germany] and 0.97 [Canada].

## VI. Conclusion

In a stochastic dynamic model of intertemporal indirect utility maximization based on New Open Economy Macroeconomic model, theoretically we demonstrated that when financial markets are not complete due to the aggregate uncertainty, agents with relative risk aversion greater than unity would have three motives of asset holdings: precautionary, consumption risk hedging, and speculative motives. Among these motives, the precautionary motive of asset holdings for intertemporal

consumption risk sharing is the most influential. Given the fact that the consumption pattern of most countries is home-biased and the fluctuations of exchange rates are for greater than those of prices in the short run, agents with relative risk aversion greater than unity have an incentive to hold the assets denominated in each currency proportionally to the expenditure share of each good produced in the corresponding countries in their consumption basket for short run consumption stabilization. We also showed that for consumption risk hedging motive, they would have a motivation of holding assets whose returns have a negative correlation with expected future economic conditions for short run consumption stabilization. As the degree of risk aversion converges one, the precautionary and consumption risk hedging motives for consumption risk sharing weaken, while the speculative motive gets stronger. In Section IV, we showed that if households across countries have identical preferences, the variance of the spot exchange rate and its covariance with consumption due to country-specific output shocks would go to zero, which presents another case where the speculative motive of asset holdings prevails.

To check if our theoretical prediction on the precautionary motive of asset holdings is correct in empirical data, first, we constructed two series of the shares of a foreign asset denominated in each currency in total foreign assets and of imports from each corresponding foreign country in total imports for the countries of the US, the UK, Japan, Canada, and Germany. We examined their cross-sectional correlation in three ways: plots, correlation coefficients, and the regression of the foreign asset shares on the constant and the import shares. We found a strong and positive correlation between them. We conclude that under aggregate uncertainty, the precautionary motive of agents with relative risk aversion greater than unity for intertemporal consumption risk

sharing overwhelms other motives of asset holdings, generating the home-biased asset holding pattern given home-biased consumption pattern.

To compare the influences of all three motives of asset holdings on the portfolio compositions, to check how much the consumption risk hedging motive that involves the variance of the spot exchange rate and its covariance with future consumption has an influence, and to see if these influences are persistent over time by controlling the effects of wealth levels, transaction costs, and capital controls from the regression equation, constructing time series data is in the first order. These works are relegated to our future study.

## References

- [1] Adler, M. and Bernard Dumas [1983], "International portfolio Choice and Corporation Finance: A Synthesis," *Journal of Finance*, 38.
- [2] Bottazzi, L., P. Pesenti, and E. van Wincoop [1996], "Wages, Profits and the International Portfolio Puzzle," *European Economic Review*, 40(2).
- [3] Branson, W. H. [1976], "Asset Markets and Relative Prices in Exchange Rate Determination," *IIES Seminar Paper*, No. 66.
- [4] Branson, W. and Henderson [1984], "The specification and influence of asset markets. In *Handbook of International Economics* edited by R. W. Jones and P. B. Kenen, North Holland, Amsterdam.
- [5] Breeden, D. [1979], "An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities," *Journal of Financial Economics*, 7(3).
- [6] Brennan, Michael and Henry Cao [1997], "International Portfolio Investment Flows," *Journal of Finance*, 52.
- [7] Chan, Kalok, Vicentiu Covrig, and Lilian Ng [2005], "What Determines the Domestic Bias and Foreign Bias? Evidence from Mutual Fund Equity Allocations Worldwide," *Journal of Finance*, Vol. LX, no. 3.
- [8] Cole, Harold L., and Maurice Obstfeld [1991], "Commodity trade and international risk sharing," *Journal of International Economics*, 28.

- [9] Cooper, Ian and Evi Kaplanis [1994], "Home Bias in Equity Portfolios, Inflation Hedging, and International Capital Market Equilibrium," *Review of Financial Studies*, vol. 7, no. 1.
- [10] Corsetti, Giancarlo, and Paolo Pesenti [2001], "Welfare and Macroeconomic Inter-dependence," *Quarterly Journal of Economics*, vol. 116, no. 2.
- [11] Coval, Joshua, and Tobias Moskowitz [1999], "Home Bias at Home: Local Equity Preference in Domestic Portfolios," *Journal of Finance*, 54.
- [12] Coval, Joshua and Tobias Moskowitz [2001], "The Geography of Investment: Informed Trading and Asset Prices," *Journal of Political Economy*, 109
- [13] French, Kenneth and James Poterba [1991], "Investor diversification and international equity markets," *American Economic Review*, 81.
- [14] Gehrig, Thomas [1993], "An Information Based Explanation of the Domestic Bias in International Equity Investment," *Scandinavian Journal of Economics*, 21.
- [15] Grossman, Sanford and Robert Shiller [1981] "The Determinants of the Variability of Stock Market Prices," *American Economic Review*, vol. 71, no. 2.
- [16] Hall, Robert E. [1978], "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, vol 86, no 6.
- [17] Helpman, Elhanan and Assaf Razin [1978] *A Theory Of International Trade Under Uncertainty*. New York: Academic Press.
- [18] Huberman, Gur [2000], "Home Bias in Equity Markets: International and Intranational Evidence." In *Intranational Macroeconomics*, eds. Gregory Hess and Eric van Wincoop. Cambridge: Cambridge University Press.
- [19] Karolyi, G. Andrew and Rene M. Stulz [2003], "Are Financial Assets Priced Locally or Globally?" in George Constantinides, Milton Harris and Rene Stulz (eds.), *The Handbook of the Economics of Finance*, New York: North Holland.
- [20] Knight, Frank H. [1921], *Risk, Uncertainty, and Profit*. Boston: Houghton Mifflin.
- [21] Kouri, Pentti [1976], "The Exchange Rate and the Balance of Payments in the Short Run and in the Long Run: A Monetary Approach," *Scandinavian Journal of Economics*, vol. 78, no. 2.
- [22] Krugman, Paul [1981], "Consumption Preferences, Asset Demands, and Distribution Effects in International Financial Markets," *NBER Working Paper*, No. 0651.

- [23] Lewis, Karen [1995], "Puzzles in International Financial Markets," *Handbook of international economics*, Volume 3.
- [24] Lewis, Karen [1999], "Trying to explain home bias in equities and consumption," *Journal of Economic Literature*, 37.
- [25] Lewis, Karen [2000], "Why Do Stocks and Consumption Imply Such Different Gains from International Risk Sharing?" *Journal of International Economics*, 52 (1).
- [26] Lucas, Robert [1978], "Asset Prices in an Exchange Economy," *Econometrica*, Vol. 46, No. 6.
- [27] Mankiw, Gregory and Stephen Zeldes [1991], "The Consumption of Stockholders and Nonstockholders," *Journal of Financial Economics*, vol. 29, no. 1.
- [28] Merton, Robert C. [1969], "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, Vol. 51, No. 3.
- [29] Merton, Robert C. [1971], "Optimum Consumption and Portfolio Rules in A continuous-Time Model," *Journal of Economic Theory*, 3.
- [30] Merton, Robert C. [1973], "An Intertemporal Capital Asset Pricing Model," *Econometrica* 41(5).
- [31] Obstfeld, Maurice [1986], "Capital Mobility in the World Economy: Theory and Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 24.
- [32] Obstfeld, Maurice [1995], "International Capital Mobility in the 1990s," *Understanding interdependence: The macroeconomics of the open economy*.
- [33] Obstfeld, Maurice and Kenneth Rogoff [1998], "Risk and Exchange Rates," *NBER Working paper*, 6694.
- [34] Obstfeld, Maurice, and Kenneth Rogoff. [2000], "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?" in *NBER Macroeconomics Annual*.
- [35] Phelps, Edmond [1962], "The Accumulation of Risky Capital: A Sequential Utility Analysis," *Econometrica*, 30.
- [36] Pesenti, P., and E. van Wincoop [2002], "Can Nontradables Generate Substantial Home Bias?" *Journal of Money, Credit, and Banking*, 34.
- [37] Rubinstein, Mark [1976], "An Aggregation Theorem for Securities Markets," *Journal of Financial Economics*, vol. 1, no. 3.

- [38] Samuelson, Paul [1969], “Lifetime Portfolio Selection By Dynamic Stochastic Programming,” *The Review of Economics and Statistics*, Vol. 51, No. 3.
- [39] Solnik, B. [1974], “Why Not Diversify Internationally Rather than Domestically?” *Financial Analyst Journal*, 30
- [40] Stulz, Rene [1983], “The Demand for Foreign Bonds,” *Journal of International Economics*, 15
- [41] Tesar, Linda and Ingrid Werner [1995] “Home bias and high turnover,” *Journal of International Money and Finance*, 14.
- [42] Uppal, Raman [1993], “A General Equilibrium Model of International Portfolio Choice,” *Journal of Finance*, 48.

### Appendix 1: Data Sources

	<b>Securities</b>	<b>Imports</b>
<b>The US [1997]</b>	Report on U.S. Holdings of Foreign Long-Term Securities as of December 31, 1997 published by Department of the Treasury	NIPA data at Bureau of Economic Analysis
<b>The UK [2000]</b>	Consolidated external claims and unused commitments of UK-owned banks and their branches and subsidiaries worldwide	UK Balance of Payment [Pink book] for 2000
<b>Japan [2000]</b>	Regional Direct Investment Position and Regional Portfolio Investment Position [2000] released by Bank of Japan	Table 12-2: Value of Japan Imports by principal country of origin [1975-2000] in Japan Statistical Yearbook
<b>Canada [1999]</b>	Canada's international investment position published by statistics Canada in 1999	Statistics Canada, CANSIM II, tables 228-0001, 228-0002 and 228-0003, 1999
<b>Germany [2002]</b>	Balance of payments statistics published by Deutsche Bundesbank	Balance of payments statistics published by Deutsche Bundesbank

### Appendix 2: The Basic Model

#### Preferences

In the world economy, there are two countries of the same economic size, Home and Foreign. In Home and Foreign, there are continuums of identical households,



$0 \leq \nu \leq 1$  and  $1 \leq \nu \leq 2$  respectively, each of who specializes in a single differentiated product indexed by  $\nu$ . The representative household  $\nu$  in Home is assumed to maximize his lifetime utility given by

$$U_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ \frac{C_s^{1-\rho}}{1-\rho} + \frac{\chi}{1-\varepsilon} \left( \frac{M_s}{P_s} \right)^{1-\varepsilon} - \eta_s Y_s(\nu) \right] \right\}, 0 \leq \beta \leq 1, 0 \leq \rho < \infty, 0 < \varepsilon < \infty \text{ [A.1]}$$

where  $Y(\nu)$  is the amount of the representative product  $\nu$  produced by the representative household  $\nu$ .  $\beta$  denotes the time discount rate, and  $\rho$  is the degree of relative risk aversion of CRRA utility function.  $C$  is the index of per capita consumption. Real money holding  $M/P$  provides a liquidity service via the reduction of transaction costs of goods and assets. The inverse of the elasticity of money demand with respect to consumption is  $\varepsilon$ , and  $\chi$  is some constant. Technology shows constant returns to scale so that  $Y(\nu) = L(\nu)$ , where  $L(\nu)$  denotes the amount of labor supplied by the representative household,  $\nu$ .  $\eta$  is an expected adverse output shock arising in the home country that adversely affects home households' utility.

Households' preferences across countries are identically asymmetric since the weights on domestically produced goods and imports,  $\gamma$  and  $1-\gamma$ , are the same. The indexes of per capita consumption of home and foreign countries are the following.

$$C \equiv \frac{C_H^\gamma C_F^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}; C^* \equiv \frac{(C_H^*)^{1-\gamma} (C_F^*)^\gamma}{\gamma^{1-\gamma} (1-\gamma)^\gamma}, 0 \leq \gamma \leq 1 \quad \text{[A.2]}$$

where  $C_H$  and  $C_F$  are respectively the representative home household's consumption of home and foreign produced goods, and  $C_H^*$ , and  $C_F^*$  are the representative foreign

household's consumption of home and foreign produced goods respectively.

The sub-indexes of per capita consumption of home and foreign goods in home and foreign countries are respectively,

$$C_H = \left[ \int_0^1 C_H(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} ; C_F = \left[ \int_1^2 C_F(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} \quad [\text{A.3}]$$

$$C_H^* = \left[ \int_0^1 C_H^*(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} ; C_F^* = \left[ \int_1^2 C_F^*(v)^{\frac{\theta-1}{\theta}} dv \right]^{\frac{\theta}{\theta-1}} \quad [\text{A.4}]$$

where  $C_H(v)$  and  $C_F(v)$  are respectively the representative home household's consumption of home and foreign produced goods, and  $C_H^*(v)$ , and  $C_F^*(v)$  are the representative foreign household's consumption of home and foreign produced goods respectively. The elasticity of substitution between goods produced within the same country is  $\theta$  that is assumed to be greater than 1, while the elasticity of substitution between goods produced in Home and Foreign,  $\sigma$  is assumed to be 1.

### **Cost of Living of the Representative Households in Home and Foreign**

The consumption-based price indexes of home and foreign countries are as follows.

$$P \equiv (P_H)^\gamma (P_F)^{1-\gamma} ; P^* \equiv (P_H^*)^{1-\gamma} (P_F^*)^\gamma \quad [\text{A.5}]$$

where  $P_H$  and  $P_F$  are home country's price indexes for the goods produced in home and foreign countries, and  $P_H^*$  and  $P_F^*$  are foreign country's price indexes for the goods produced in home and foreign countries.

The sub-price indexes for home and foreign goods are respectively,

$$P_H = \left[ \int_0^1 P_H(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}}; P_F = \left[ \int_1^2 P_F(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad [\text{A.6}]$$

$$P_H^* = \left[ \int_0^1 P_H^*(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}}; P_F^* = \left[ \int_1^2 P_F^*(v)^{1-\theta} dv \right]^{\frac{1}{1-\theta}} \quad [\text{A.7}]$$

where  $P_H(v)$  and  $P_F(v)$  are the prices of the representative goods produced in home and foreign countries in the home country, while  $P_H^*(v)$  and  $P_F^*(v)$  are the prices of the representative goods produced in home and foreign countries in the foreign country, respectively. The law of one price is assumed to hold for each individual good so that  $P(v) = SP^*(v)$ ,  $\forall v \in [0,2]$ , where  $S$  is the spot exchange rate of home currency to foreign currency. For the sub-price indexes such as  $P_H$ , and  $P_F$ , consumption-based purchasing power parity holds so that  $P_H = SP_H^*$ , and  $P_F = SP_F^*$ . Because home and foreign households do not have an identical preference on home and foreign-produced goods, consumption-based purchasing parity for overall consumer price indexes,  $P \neq SP^*$ , does not hold.

### Goods Market Equilibrium

Under sub-demand functions [A.3] and [A.4], optimal intratemporal consumption choices for each differentiated goods are as follows.

$$C_H(v) = \left[ \frac{P_H(v)}{P_H} \right]^{-\theta} C_H; C_F(v) = \left[ \frac{P_F(v)}{P_F} \right]^{-\theta} C_F \quad [\text{A.8}]$$

$$C_H^*(v) = \left[ \frac{P_H^*(v)}{P_H^*} \right]^{-\theta} C_H^*; C_F^*(v) = \left[ \frac{P_F^*(v)}{P_F^*} \right]^{-\theta} C_F^* \quad [\text{A.9}]$$

where  $C_H(v)$  and  $C_F(v)$  are the demand for the representative home and foreign goods

of the home representative household, while  $C_H^*(v)$  and  $C_F^*(v)$  are the demand for the representative home and foreign goods of the foreign representative household.

The Cobb-Douglas overall consumption indexes imply that the demands for home and foreign goods,  $C_H$ ,  $C_F$ ,  $C_H^*$ , and  $C_F^*$  are given by

$$C_H = \gamma \left( \frac{P_H}{P} \right)^{-1} C; C_F = (1 - \gamma) \left( \frac{P_F}{P} \right)^{-1} C \quad [\text{A.10}]$$

$$C_H^* = (1 - \gamma) \left( \frac{P_H^*}{P^*} \right)^{-1} C^*; C_F^* = \gamma \left( \frac{P_F^*}{P^*} \right)^{-1} C^* \quad [\text{A.11}]$$

Combining [A.8] and [A.10], and [A.9] and [A.11] respectively gives

$$C_H(v) = \gamma \left[ \frac{P_H(v)}{P_H} \right]^{-\theta} \left( \frac{P_H}{P} \right)^{-1} C; C_F(v) = (1 - \gamma) \left[ \frac{P_F(v)}{P_F} \right]^{-\theta} \left( \frac{P_F}{P} \right)^{-1} C \quad [\text{A.12}]$$

$$C_H^*(v) = (1 - \gamma) \left[ \frac{P_H^*(v)}{P_H^*} \right]^{-\theta} \left( \frac{P_H^*}{P^*} \right)^{-1} C^*; C_F^*(v) = \gamma \left[ \frac{P_F^*(v)}{P_F^*} \right]^{-\theta} \left( \frac{P_F^*}{P^*} \right)^{-1} C^* \quad [\text{A.13}]$$

The world consumption for each individual good produced in home and foreign countries is defined as follows.

$$C_H^W(v) = C_H(v) + C_H^*(v); C_F^W(v) = C_F(v) + C_F^*(v) \quad [\text{A.14}]$$

where  $C_H^W(v)$ , and  $C_F^W(v)$  represent total world consumption for each individual good produced in Home and Foreign countries respectively. Plugging [A.12] and [A.13] into [A.14] gives

$$C_H^W(v) = \gamma \left[ \frac{P_H(v)}{P_H} \right]^{-\theta} \left( \frac{P_H}{P} \right)^{-1} C + (1 - \gamma) \left[ \frac{P_H^*(v)}{P_H^*} \right]^{-\theta} \left( \frac{P_H^*}{P^*} \right)^{-1} C^* \quad [\text{A.15}]$$

$$C_F^W(v) = (1 - \gamma) \left[ \frac{P_F(v)}{P_F} \right]^{-\theta} \left( \frac{P_F}{P} \right)^{-1} C + \gamma \left[ \frac{P_F^*(v)}{P_F^*} \right]^{-\theta} \left( \frac{P_F^*}{P^*} \right)^{-1} C^* \quad [\text{A.16}]$$

The goods market for each individual good produced in home and foreign countries clears when the demand equals the supply. Taking into account of the population of two countries and evaluating it at the symmetric equilibrium, where,  $P_H(v) = P_H$ , and  $P_F(v) = P_F$ , we obtain the world market clearing condition for each individual good produced in home and foreign countries as follows.

$$C_H^W(v) = \left\{ \gamma \left( \frac{P_H}{P} \right)^{-1} C + (1 - \gamma) \left( \frac{P_H^*}{P^*} \right)^{-1} C^* \right\} = Y(v) \quad [\text{A.17}]$$

$$C_F^W(v) = \left\{ (1 - \gamma) \left( \frac{P_F}{P} \right)^{-1} C + \gamma \left( \frac{P_F^*}{P^*} \right)^{-1} C^* \right\} = Y^*(v) \quad [\text{A.18}]$$

### The Budget Constraint

Given intra-temporal consumption choices, the budget constraint of the representative household in the home country is as follows.

$$P_t C_t + M_t + Q_t B_{H,t} + S_t Q_t^* B_{F,t} = M_{t-1} + P_{H,t}(v) Y_t(v) + B_{H,t-1} + S_t B_{F,t-1} + T_t \quad [\text{A.19}]$$

where  $Q_t$  and  $Q_t^*$  are the prices of home and foreign currency denominated bonds.  $T_t$  is the monetary transfer from the government to each citizen. Only domestic currency is assumed to be held by the household in each country.

The government budget constraint is given as follows. The change in the money supply by the government is transferred directly to the each household. There are no government expenditures over time.

$$M_t = M_{t-1} + T_t \quad [\text{A.20}]$$

### First Order Conditions for the Representative Households in Home and Foreign

First order conditions for the representative home household are as follows.

$$Q_t = \beta \frac{E_t(P_{t+1}^{-1} C_{t+1}^{-\rho})}{P_t^{-1} C_t^{-\rho}} \quad [\text{A.21}]$$

$$Q_t^* = \beta \frac{E_t(S_{t+1} P_{t+1}^{-1} C_{t+1}^{-\rho})}{S_t P_t^{-1} C_t^{-\rho}} \quad [\text{A.22}]$$

$$\left( \frac{M_t}{P_t} \right)^\varepsilon = \frac{\chi C_t^\rho}{1 - Q_t} \quad [\text{A.23}]$$

First order conditions for the representative foreign household are as follows.

$$Q_t = \beta \frac{E_t(S_{t+1}^{-1} P_{t+1}^{*-1} C_{t+1}^{*-\rho})}{S_t^{-1} P_t^{*-1} C_t^{*-\rho}} \quad [\text{A.24}]$$

$$Q_t^* = \beta \frac{E_t(P_{t+1}^{*-1} C_{t+1}^{*-\rho})}{P_t^{*-1} C_t^{*-\rho}} \quad [\text{A.25}]$$

$$\left( \frac{M_t^*}{P_t^*} \right)^\varepsilon = \frac{\chi C_t^{*\rho}}{1 - Q_t^*} \quad [\text{A.26}]$$

### Consumption Risk Sharing Condition

From the first order conditions for bond holdings, [A.21], [A.22], [A.24], and [A.25], equating two equations [A.21] and [A.24], and [A.23] and [A.25] respectively since the price of one-period nominal bonds denominated in each currency is the same across countries, gives the following two equations.

$$Q_t = \beta \frac{E_t(P_{t+1}^{-1} C_{t+1}^{-\rho})}{P_t^{-1} C_t^{-\rho}} = \beta \frac{E_t(S_{t+1}^{-1} P_{t+1}^{*-1} C_{t+1}^{*-\rho})}{S_t^{-1} P_t^{*-1} C_t^{*-\rho}}; \quad [\text{A.27}]$$

$$Q_t^* = \beta \frac{E_t(S_{t+1} P_{t+1}^{-1} C_{t+1}^{-\rho})}{S_t P_t^{-1} C_t^{-\rho}} = \beta \frac{E_t(P_{t+1}^{*-1} C_{t+1}^{*-\rho})}{P_t^{*-1} C_t^{*-\rho}} \quad [\text{A.28}]$$

Assuming that both countries are initially in the symmetric situation where  $P_t^{-1}C_t^{-\rho} = (P_t^*)^{-\rho}(C_t^*)^{-\rho}$  gives the following expression.

$$\Rightarrow \frac{C_t^{-\rho}}{P_t} = \frac{(C_t^*)^{-\rho}}{S_t P_t^*} \quad \forall t \quad [\text{A.29}]$$

Rearranging the equation [A.29] gives the following ‘consumption risk sharing condition.’

$$\Rightarrow \left( \frac{C_t}{C_t^*} \right)^\rho = \frac{S_t P_t^*}{P_t} \quad \forall t \quad [\text{A.30}]$$

### Short Run Inflexible Prices in Goods Markets

The monopoly prices of the representative consumer-producers in home and foreign countries,  $P_{H,t-1}(v)$  and  $P_{F,t-1}^*(v)$  are determined by maximizing their lifetime expected utility, [A.1], given the information at time  $t-1$  and their life-time budget constraint at the symmetric equilibrium where  $P_{H,t-1}(v) = P_{H,t-1}$  and  $P_{F,t-1}(v) = P_{F,t-1}$ .

$$P_{H,t-1}(v) = P_{H,t-1} = \left( \frac{\theta}{\theta-1} \right) \frac{E\{\eta Y_t(v)\}}{E\left\{ \frac{Y_t(v)}{P_t C_t^\rho} \right\}} \quad [\text{A.31}]$$

$$P_{F,t-1}^*(v) = P_{F,t-1}^* = \left( \frac{\theta}{\theta-1} \right) \frac{E\{\eta^* Y_t^*(v)\}}{E\left\{ \frac{Y_t^*(v)}{P_t^* C_t^{*\rho}} \right\}} \quad [\text{A.32}]$$

### Permanent Consumption and Country-specific Output and Monetary Shocks

Suppose that the growth of output and money supply in both home and foreign countries follows a random walk.

$$\log \eta_t = \log \eta_{t-1} + \xi_t; \log \eta_t^* = \log \eta_{t-1}^* + \xi_t^* \quad [\text{A.33}]$$

$$m_t = m_{t-1} + \mu_t; m_t^* = m_{t-1}^* + \mu_t^* \quad [\text{A.34}]$$

where  $m_t = \log M_t$ ,  $m_t^* = \log M_t^*$ , and  $\xi_t, \xi_t^* \sim N(0, \sigma_\xi^2)$ , and  $\mu_t, \mu_t^* \sim N(0, \sigma_\mu^2)$  for every date  $t$ .

Log-linearizing money demand equations for the home and foreign countries, [A.23] and [A.26] at a non-stochastic steady state where  $i = i^* = \bar{i}$ <sup>12</sup> gives the following expressions.

$$\mathcal{E}\{m_t - p_t\} = \log \chi - \log(1 - Q_t) + \rho c_t \quad [\text{A.35}]$$

$$\mathcal{E}\{m_t^* - p_t^*\} = \log \chi^* - \log(1 - Q_t^*) + \rho c_t^* \quad [\text{A.36}]$$

Adding two equations [A.35] and [A.36] under the assumption that at the initial equilibrium, two countries have the same money supply and consumption price index gives the following expression.

$$\rho\{c_t + c_t^*\} = \mathcal{E}\{\mu_t + \mu_t^*\} - \mathcal{E}\{p_t + p_t^*\} \quad [\text{A.37}]$$

$$\text{where } \rho(c_t - c_t^*) = (2\gamma - 1)(s_t + p_{F,t-1}^* - p_{H,t-1}) \quad [\text{A.38}]$$

$$p_t = \mathcal{P}_{H,t-1} + (1 - \gamma)s_t + (1 - \gamma)p_{F,t-1}^* \quad [\text{A.39}]$$

$$p_t^* = (1 - \gamma)p_{H,t-1} - (1 - \gamma)s_t + \mathcal{P}_{F,t-1}^* \quad [\text{A.40}]$$

Combining equations [A.37], [A.38], [A.39], and [A.40] gives

$$c_t = \frac{(2\gamma - 1)}{2\rho} \{s_t - (p_{H,t-1} - p_{F,t-1}^*)\} + \left(\frac{\mathcal{E}}{2\rho}\right) \{\mu_t + \mu_t^*\} - \left(\frac{\mathcal{E}}{2\rho}\right) \{p_{H,t-1} + p_{F,t-1}^*\} \quad [\text{A.41}]$$

$$c_t^* = -\frac{(2\gamma - 1)}{2\rho} \{s_t - (p_{H,t-1} - p_{F,t-1}^*)\} + \left(\frac{\mathcal{E}}{2\rho}\right) \{\mu_t + \mu_t^*\} - \left(\frac{\mathcal{E}}{2\rho}\right) \{p_{H,t-1} + p_{F,t-1}^*\} \quad [\text{A.42}]$$

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<sup>12</sup> Or equivalently,  $Q = Q^* = \bar{Q}$ .



Subtracting the equation [A.41] from [A.42] under the assumption that at the initial equilibrium, two countries have the same money supply and consumption price index gives the following expression.

$$\rho\{c_t - c_t^*\} = \varepsilon\{\mu_s - \mu_s^*\} - \varepsilon\{p_t - p_t^*\} \quad [\text{A.43}]$$

Combining equations [A.43], [A.38], [A.39] and [A.40] gives the following.

$$s_t = \left( \frac{\varepsilon}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right) \{\mu_t - \mu_t^*\} - \left( \frac{(\varepsilon - 1)(2\gamma - 1)}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right) (p_{H,t-1} - p_{F,t-1}^*) \quad [\text{A.44}]$$

Taking a log of [A.31] and [A.32] in the current period gives

$$p_{H,t-1} = \log \eta_t + p_t + \rho c_t \quad [\text{A.45}]$$

$$p_{F,t-1}^* = \log \eta_t^* + p_t^* + \rho c_t^* \quad [\text{A.46}]$$

Subtracting [A.46] from [A.45] gives the ex ante terms of trade as follows.

$$p_{H,t-1} - p_{F,t-1}^* - E s_t = -(\xi_t - \xi_t^*) \quad [\text{A.47}]$$

$$\text{where } E s_t = - \left( \frac{(\varepsilon - 1)(2\gamma - 1)}{(2\gamma - 1) + 2\varepsilon(1 - \gamma)} \right) (p_{H,t-1} - p_{F,t-1}^*)^{13} \quad [\text{A.48}]$$

Combining [A.47] and [A.48] gives

$$p_{H,t-1} - p_{F,t-1}^* = - \left( \frac{(2\gamma - 1) + 2\varepsilon(1 - \gamma)}{\varepsilon} \right) (\xi_t - \xi_t^*) \quad [\text{A.49}]$$

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<sup>13</sup> This expected exchange rate is derived by taking the expectation of the equation [A.40] under the assumption that future period monetary surprises are not expected by private agents.

Adding [A.45] and [A.46], and using the equation [A.37], price indexes and the consumption risk-sharing condition [A.38] at the initial symmetric equilibrium give

$$p_{H,t-1} + p_{F,t-1}^* = (\mu_t + \mu_t^*) - \frac{1}{\varepsilon}(\xi_t + \xi_t^*) \quad [\text{A.50}]$$

Plugging [A.49] and [A.50] into [A.41] and [A.42] respectively give permanent consumption for home and foreign agents as functions of country specific output money and output shocks.

$$c_t = \frac{(2\gamma-1)}{2\rho} \left( \frac{\varepsilon}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) \{ \mu_t - \mu_t^* \} + \frac{\gamma}{\rho} \{ \xi_t \} + \frac{(1-\gamma)}{\rho} \{ \xi_t^* \} \quad [\text{A.51}]$$

$$c_t^* = -\frac{(2\gamma-1)}{2\rho} \left( \frac{\varepsilon}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) \{ \mu_t - \mu_t^* \} + \frac{(1-\gamma)}{\rho} \{ \xi_t \} + \frac{\gamma}{\rho} \{ \xi_t^* \} \quad [\text{A.52}]$$

### The Spot Exchange Rate in the Foreign Exchange Market

Combining [A.44] with [A.49] gives the nominal exchange rate as a function of relative money supply and relative expected productivity shocks.

$$s_t = \left( \frac{\varepsilon}{(2\gamma-1) + 2\varepsilon(1-\gamma)} \right) (\mu_t - \mu_t^*) - \left( \frac{(\varepsilon-1)(2\gamma-1)}{\varepsilon} \right) (\xi_t - \xi_t^*) \quad [\text{A.53}]$$